Changing a half-baked 3d navigational game

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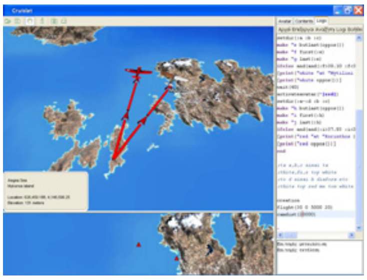
Abstract

A number of research studies focus on the ways digital games can provide a rich context for students to experiment, explore and engage in meaningful formalism. These researches suggest that games can help students develop valuable skills in the transitional stage between intuitions (informal) and formal mathematics. This can be done either by playing, designing games, or reflecting on game's rules (Kafai, 1996). In this paper we endeavour to combine these powerful approaches on learning with games, using half-baked games, an idea based on that of half-baked microworlds (Kynigos, 2001). These are games that incorporate an interesting game idea, but they are incomplete *by design.* Students while playing the game reflect on rules and change them in order to create a new game of their own choice. Thus they engage both in the process of instrumentation while using the game and in the process of instrumentalisation while changing the game resulting in a variety of artefacts (Guin and Trouche, 1999).

In present research study we used a half-baked game built on the Cruislet environment, a 3d digital medium that incorporates both GIS and Logo programming. It is designed for mathematically driven navigations in virtual 3d geographical spaces and thereby enables the user to explore navigation in 3d geographical space, spatial visualization and mathematical concepts embedded in a realistic context. Navigation with Cruislet is feasible by defining objects' (airplanes) displacements, either by using an iconic interface or using Logo. The Logo editor provides opportunities to run and edit programs and thus create multiple behaviours of objects, or game rules.

In this small scale research, two 17th year old students worked collaboratively and engaged in processes of playing the game, reflecting on its rules, de-coding the rule and intervening into the Logo code in order to change the game and create something of their own choice. Findings indicate that the half-baked game and the representations embedded in the Cruislet environment provided a meaningful context in which students engage in processes of mathematisation of geographical space.



Keywords

3d environment, half-baked games, navigation in geographical space, Logo

Introduction

Lately there is a growing interest about the ways in which game–based learning environments facilitate new ways of learning (Gee, 2003). The key feature of this approach is that playing games can provide a context for the development of valuable skills (Kirriemuir and McFarlane et al., 2004) in the transitional stage between intuitions (informal) and formal mathematics. Using games with an appropriate set of tasks and pedagogy, students can be engaged in exploration, problem solving, rule-based thinking and other forms of mathematical thinking (Goldstein et al, 2001). Although game-play could provide a rich context for students to experiment, explore and engage in mathematical expression, usually students adopt the role of passive consumer of the game, in contrast to the active role of learning when students construct their knowledge. Through the process of designing games and reflecting on game rules, students adopt an active role and engage in meaningful expressions (Kafai, 1996).

In order to combine these two approaches, we use the idea of half – baked games, an idea taken from microworld design (Kynigos, 2001). These are games that incorporate an interesting game idea, but they are incomplete *by design* in order to encourage students to change their rules. Students play *and* change them and thus adopt both roles of player and designer of the game (Kafai, 2006). In order to address our design rationale and our study of half-baked game use by students, we found it useful to employ an idea coming from cogntitive ergonomy which distinguishes the object game from the mental construct created by each student through the process of using it. In articulating this idea, Rabardel (1995) uses the terms 'artefact' to denote the tangible game and 'instrument' to denote this construct which he called 'schemes of use of the artefact'. Later, Guin and Trouche (1999) went further in using the term 'instrumentalisation' to denote the process by which students. transform the artefact for specific uses. From this point of view, the process of designing game change and play activities can be seen as the design of the terrain within which the instrumentation/ instrumentalisation processes may take place by students' interactions with the game and domain specific rules and concepts embedded in it. To support the construction of such games, we use an environment where rules are built within Logo procedures and thereby students are able to intervene into the structure of games' rules and change them. Thus, Logo provides opportunities to express ideas in meaningful ways and in this way it can be seen as a medium in the transitional stage between intuitions and meaningful formalism (Dubinsky, 2000).

In this particular study, children's engagement with a number of tasks focuses on the exploration of the concept of navigation in 3d geographical space, a concept which we consider as complicated; as it is strongly interrelated with several mathematical concepts and it is also depended on the situations that the concept appears (Vergnaud, 1991). Espousing Vergnaud's view that a mathematical concept is not isolated from other concepts, we consider navigation as a *conceptual field* strongly related to concepts such as geographical coordinates, spatial visualization, covariation and the interplay between them. Using this approach to navigation, we endeavour to study the mathematical concepts that are integrated with geo-spatial representations and information, providing opportunities for processes of mathematisation of geographical space. The mathematical concepts involved in the particular tasks are those underlying the use of issues regarding 3d space, including linear functions, as the displacement of objects (in this case, airplanes) could be conceived as the co-variation or 'rate of change' of geographical coordinates. As Tall (1996) suggests enactive sensations of moving objects may give a sense that "continuous" change implies the existence of a "rate of change".

The Cruislet environment

The 'Cruislet' environment is a digital medium based on GIS (Geographic Information Systems) technology that incorporates a Logo programming language. It is designed[1] for mathematically driven navigations in virtual 3d geographical spaces and is comprised of two interdependent representational systems for defining a displacement in 3d space, a polar coordinate and a geographical coordinate system. In the present study we'll focus on the geographical coordinate system and the ways this is used to navigate objects (airplanes) in 3d space.

The environment enables the user to explore spatial visualization and mathematical concepts by controlling and measuring the behaviours of objects. The objects are airplanes and their displacement is represented by a vector. The user is able to navigate airplanes either by using the Avatar tab or through the use of a Logo editor Tab (see Figure 1). In using the Avatar Tab, the user can create airplanes and define their position in either of the following ways:, a) by determining the latitude, the longitude and the height in which the airplane will be placed or b) by determining the vector of displacement, i.e. the angles r and f of two vertical planes and the length r of the displacement, i.e. the length of the vector. When a parameter from one of the ways of displacement is changed the parameters in the other are dynamically changed as well.. In the Logo Tab the user can actually edit and run Logo programs and thus create multiple or relative displacement rules in 3d space. The Logo programmability is considered necessary as it provides users with the option to actually anticipate the result of their action and engage in expression of mathematical ideas through meaningful formalism by means of programming. In this sense, Cruislet can be conceived as a constructionist medium (Kafai & Resnick, 1996) in that the user can construct flights and build dependency between flights.

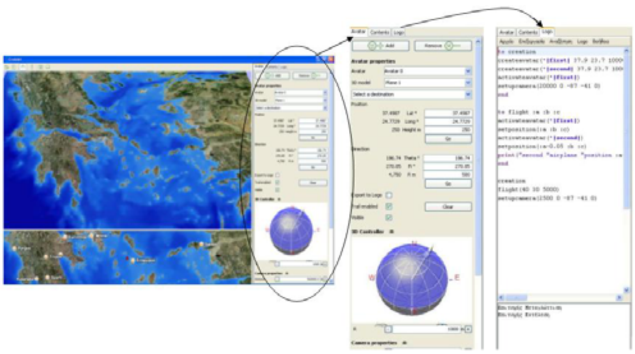


Figure 1. Cruislet environment – Avatar Tab – Logo Tab

Our initial aim when designing Cruislet was to create a realistic context such as a 3d geographical map where geo-coded data will be integrated with mathematic representations, in which students would be able to explore, experiment and construct virtual reality microworlds. Students' constructions in these microworlds are airplanes' trips as well as rules of airplanes' displacements that are defined in Logo procedures. In that sense, navigations in virtual 3d geographical spaces within Cruislet could be conceived as game play simulations, as users manipulate objects in a 3d game-like environment according to rules built on this which are defined by the designer of each microworld.

Task design

Students carried out three tasks, each one of which aimed to capture different aspects of interaction with the environment. We use the term interaction to define the ways students utilize

[1] 'Remath' – Representing Mathematics with Digital Media FP6, IST-4, STREP 026751 (2005 – 2008), http://remath.cti.gr

the environment either by using the representations provided, playing the game built on Cruislet or changing the game. In each of these tasks, the artefact becomes an instrument (Verillon and Rabardel, 1995) through the process of instrumentation (Guin &Trouche, 1999) as students work and play with Cruislet. According to these, in the first task which was considered introductory to the environment, we asked students to create an airplane and make a flight to Athens or any other city of their choice, by defining the geographical coordinates of each city. The task had two phases, where students had to do this using the avatar tab or edit Logo commands to carry out the same flight. This activity aimed atintroducing students to Cruislet and the provided representations as well as editing basic Logo commands

Our approach in designing the second and third tasks promoted investigation through the design of activities that offer a research framework to investigate purposeful ways that allow children to appreciate the utility of mathematical ideas (Ainley et al., 2006). In the second task we created game rules built in Logo procedures, in order to engage students with Logo programming and simultaneously to embed mathematical concepts in game rules. The task was based on the idea of the "Guess my function" game, in order to provoke children to discuss, compare and experiment with dependence relations such as linear functions. In particular, students were asked to study the existence of a rate of change between the displacements of twoairplanes which are defined in the geographical coordinate system.

Thesecond airplane's position depends in a certain way on the first airplane's position, according to the 'distance rule', as the second airplane (spy or escort) follows the first one. Students must find the dependence relation by moving the first airplane and seeing what happens to the other one in order to decode the rule. The object of the game is to place the first airplane in a position where the spy will be placed at Thessaloniki (city on the north of Greece). A Logo procedure (named flight) determines the function, which is hidden to students and they have to guess it based on repeated displacements of the first airplane and observations of the relative positions and displacements of the first and the second airplane (spy). Thus defining the first's airplane's position using this procedure, the environment gives them feedback about second airplane's position. The procedure that defines the rule is:

```
to flight :a :b :c
activateavatar("|first|)
setposition(:a :b :c) → define first plane's position (Lat Long Height)
activateavatar("|second|)
setposition(:a-0.05 :b+0.05 :c-1000)
print("second "airplane :a-0.05 :b+0.05 :c-1000) → print second plane's position
end
```

For example, if students write:
flight(37 28 10000) → define first plane's position (Lat Long Height) and the displacement of the first airplane is executed as well as the displacement of the second airplane and
they will get the answer:
Second airplane (36.95   28.95   9000)

With this task, students get involved with the concept of function as a dynamic process of covariation using the geographical coordinates as a system of reference. In particular, students are actually asked to study the existence of a rate of change of relative displacements of points on the space as moving the first airplane between successive positions has as a result the displacement of the second plane between corresponding successive values (Confrey and Smith, 1995).

In third task we wanted to provoke students' interest, in a way that this could give them a motive to engage in the task. Based on Ainley, Pratt and Hansen work (2006, p. 35) we took into account three critical aspects involved in the tasks, in order to stress purpose and utility: i) we wanted an explicit end product created by students, ii) we told students to create something for another audience and iii) we created tasks in order to give opportunities to students to make meaningful decisions regarding mathematical concepts involved in the tasks. Thus, in the third task students have to formulate another rule and change the functional relations of plane coordinates by intervening in the Logo code and by creating through this process another game to challenge another team. The process of finding another rule, alternate the game and challenge others could engage students with mathematical and geographical concepts as well as with Logo while experimenting with the environment. While students reflect on rules, engage in the process of instrumentalisation (Guin &Trouche, 1999), since displacement rules can be questioned and re-defined by them resulting in a variety of artefacts, which are actually the different games they create. We should mention here, that although the game is relying on mathematics and is very different to digital games students are used to play with, in this small scale study we endeavoured to create a rule within the activity, that could be considered by students as a game. Additionally we wanted to engage them in the activity of creating other rules and through this procedure, create a game.

Research setting and methodology

In our research we used a design-based research method (Cobb et al., 2003) which entailed the ‘engineering’ of tools and task, as well as the systematic study of the forms of learning that took place within the specific context as defined by the means of supporting it. Our approach is also based in iterative design (diSessa, 1989) and initially our aim was to carry out a small scale study and observe students using and modifying the environment as this would give as feedback into the next iteration of Cruislet development and task design.

The research reported herein took place in the computer lab of a public vocational school in Athens. Two 17th year old students, used the Cruislet environment and worked collaboratively in a set of activities which lasted 6 hours in total over 2 days. Two researchers participated in the experiment, espousing the role of naïve and participant observer allowing for the data to structure the results and to pilot their analysis. For data collection we used a camera and a tape-recorder in order to capture students’ dialogues and their interaction with the environment. Background data were collected, such as students’ notes on papers and one researcher was collecting observational notes as they occurred. All audio-recordings were analyzed verbatim. In analyzing the data we focused on two different processes in relation to students’ actions, through which students interacted with game’s rules: a) the process of instrumentation, and b) the process of instrumentalisation of the artefact. In the first case we looked for instances where meanings related to navigation and geospatial representations were expressed by students while playing the game. In the second case we tried to identify critical episodes during their experimentation, where students by reflecting on the rules constructed meanings of the relative displacements of the airplanes and actually changed the game constructing their own rules.

Playing the game

While students were playing the game several meanings emerged from their interaction with Cruislet environment regarding the inter-dependent relationship between geo-spatial representations and mathematics. Here, we focus on meanings where mathematical issues are integrated with geo-spatial representations and information, providing opportunities to students for processes of mathematisation of geographical space.

The students participated in our experiment were using their intuitions to anticipate the relation between airplanes and thus to find the linear function. In order to decode the rule, they experimented by giving various values to coordinates (Lat, Long, Height) that define the position of the first plane, they communicated their observations about the position of the second plane and they formed conjectures about the relationship between planes’ positions. Initially they have started thinking of this relationship, visualizing the result of airplane’s position (figure 2).

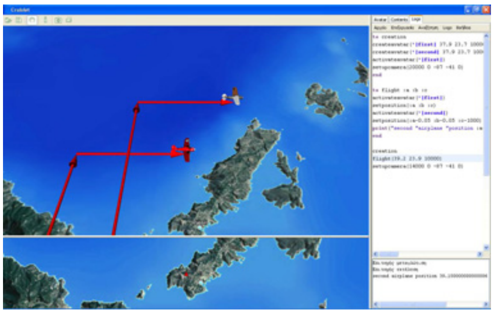


Figure 2. Airplane's displacement in Cruislet environment.

Although the feedback from the environment was mathematically represented, that is the 3 coordinates of each airplane's position, they preferred to think of it as a relationship between two objects in 3d space rather than a function between coordinates. Thus, when they were experimenting giving several coordinates in the first airplane and observing the second airplane's position, they translated addition and subtraction in functions as front, behind, left and right depending on airplanes' positions. Using direction to specify airplanes' relationship was easier for them, rather than using mathematical symbols, like plus or minus. In the following instances of an episode, the researcher asked them to explain how they correlate direction with airplanes position and coordinates.

R1 Can we understand if it's back, front, right or left, looking at these? (refers to airplanes' positions)

S1 If someone knows where the coordinates are, that is how these are defined from our point of view. How these are defined? I mean if it goes right with plus or minus or if it goes left with plus or minus.

R1 So, how do you know if it's right or left? How do you think plus and minus are related to front or back, left or right?

S1 Plus, if we see lat (refers to latitude), minus is behind and plus is in front.

R1 Yes…

S1 According to our position. In long (refers to longitude), plus is right and minus is left.

…

S1 It is 5 meters in the right and behind us. It's 0.05 in latitude, 0.05 in longitude when we use integers. And it's 1000 feet underneath us.

The interesting point here is that students preferred to use an experiential egocentric system of reference in order to find the relationship rather than using mathematical formalism, although they were accustomed in using mathematical symbols and working with functions. The way they specified a position in 3d space, that is their system of reference, is related to the existence of a stationary point (first airplane) and a sense of direction (right, left, etc). However, because they were flying towards north of Greece, "behind" for them meant minus in the function, although this isn't the case. We think that this indicates the strong sense of the ego-centric system of reference that students have. The episode also indicates that students have realized that the system of geographical coordinates is an absolute fixed one, as they say that in order to correlate direction with signs "someone has to know where the coordinates are", implying that there is a convention in the geographical system of reference.

The Cruislet environment provided opportunities to students to associate issues regarding both navigation in geographical space and mathematical concepts underlying 3d space. Within this scope they explored concepts regarding 3d space and especially the third dimension, height. In the following episode students are trying to fly upon Greece and land in a city, but there are constraints that don't allow the flight. Students managed to overcome the problem that occurred, following their intuitions and using the representational issues (the 3d map of Greece) provided by the environment.

S2 The information we gave, with these coordinates, the airplane can't go there. Do we have to give…

R Is there another reason that the airplane can't fly?

S1 No, that's the reason why. Because we had it at low height, we had it at low meters. The airplane was at very low height.

R And?

S2 It couldn't. The program warned us that there are mountains in front of us, so the airplane couldn't fly. It has to do with the airplane's course. This is nice, ha? In case you are a pilot.

R And what did you do?

S1 We just raised meters. We had it in 1000 meters and we raised it at 5000 meters, which was the initial height.

Despite the fact that the environment didn't give a hint about the problem occurred, students found at once that the constraints had to do with height. We think that this is important as geographical information provide a context to explore representational issues regarding 3d space. Although in this case the third coordinate was manipulated effectively, students many times forgot to edit height or they thought of it as unnecessary when they were using the Logo editor Tab and had to specify a specific place in space. The following episode indicates that students got confused with the third coordinate.

S1 And the other one is 22.9367. Close the bracket. (after editing lat and long)

S2 What about height?

S1 Height is fixed at 12000. Change it?

S2 I don't know. No. Let it. Or we should edit height? (he is asking the researcher)

R Do you want to change height?

S2 No. Do we have to edit it or it remains fixed?

Although students knew that in order to specify a position in space they had to edit three coordinates, in this case they didn't want to change height, as they considered it as unnecessary. We could say that may be this is because they were not familiar with the environment and thought that the environment 'remembers' previous positions or coordinates. But this is not the case as such confusion occurred only with height and not with other coordinates, even if one of them remained stable. A possible interpretation about this confusion is that students are accustomed to 2d representations where they manipulate only two magnitudes and this is the reason why they usually preferred to fly at a fixed height. On the other hand if we accept the view of Dalgarno et al. (2002) that we understand 3D models through multiple 2D representations, maybe students had focused subconsciously on a simplified 2D way of visualising the displacements of the airplanes. We consider our findings compatible with another research conducted by Kynigos and Latsi (2005) in relation to 3D representation of vectors that have also shown that children have focused in a simplified 2D representation.

Changing the game

Our initial aim was to ask students to reflect on the game and change it if there was something they didn't like, in order to create another game for the next couple of students. During the experiment we experienced a pleasant surprise when students suggested that they would like to change something on the game and see what happens. With this pretext we encouraged them to reflect on the rules and thus make their own game and challenge other students. In this paper we focus on students' modifications of the game either at the imaginative level where students reflect on rules and use their imagination to create new ones, or at the creative level, where students edit their rules using Logo and through this process create a new game. The two levels of game design that we categorized our findings are similar to Goldstein et al (2001) distinction between 'Game Inside' and 'Game Formal' respectively, as in the first case we refer to students' ideas while in the second to rules that students actually created. These two games differ as 'the expression and application of rules are mediated by the tools available' (Hoyles, et al, 2001) and as students were not familiar with Logo, the implementation of their ideas was limited.

Reflecting on rules

We refer to students' reflection on the rules as it is considered indicative of their way of thinking. Here we report students' ideas while they were thinking of alternative rules in order to change the game.

- Easy function

While playing the game students considered it as an easy one and the rules built upon airplanes' displacements as easy to decode, so they reflected on the rules in order to make a more realistic and difficult game for others to play. The first thing that considered as really easy was function's numbers.

S2 We would use more difficult numbers for the other to find. Instead of 5 we would use a decimal number. That is 56, the difference not to be easy.

R1 You mean the variation to be a difficult number?

S2 Yes.

This excerpt indicates that students were thinking that the function was easy because it involved easy numbers. This is interesting as they didn't thought for another non-linear function, but a linear function with complex numbers.

- Easy rule

Before playing the game students found that the way the second airplane followed the first one was not realistic as the second one should try to hide from the first, in order to avoid get caught by the first airplane. Thus, they thought that it would be realistic if the second one was moving not according to a specific plan or he was moving according to a complicated rule.

S1 If the spy (means: the second airplane) changed his place, it would be difficult to find the rule.

R1 I don't understand what you 're saying. Do you want to explain it?

S1 He (the spy) was in a position wherever we went; he was in a fixed position behind or in front of us, wherever we went. If his position was moving it wouldn't be easy for someone to find.

R1 How the position would be moving? Do you want to say to me?

S1 For example, if it goes right, left, or near us or far away form us, according to the place we go.

- Insert boundaries

Students wanted to limit the way the first airplane was flying. Specifically they wanted the first airplane to fly only above Greece and limit its possible displacements. With this rule students were thinking about how they could limit the coordinates that would be permissible and by this they were referring to the domain of numbers that the function could get.

Create the rules

After reflecting on rules, students focused on a specific rule and tried to edit the rule and extend the predefined Logo procedure. Their idea was to divide Greece into two segments where while the first airplane would fly upon one, the spy would follow him according to a function, and while the first would fly upon the other segment, the spy would follow him according to another function. We think that students chose to implement only this rule because they were not experienced with Logo and this constrained the implementation of their ideas and the 'translation' of them in Logo commands.

During the process of making the rule, students were involved with the process of mathematisation of the geographical space as they wanted to divide geographical coordinates in two segments. Thus they searched for a city in the middle and cities in the north and south end of Greece and found their coordinates. In the following experts students are trying to explain the rule.

S2 I'm saying that from Heraklio (south) to Athens (middle)…

R Yes, nice.

S2 The second airplane has a distance 'a' from us.

R Ok.

S2 And from Athens to Drama (north), it has a different distance from the previous one.

…

S1 We have divided Greece in two segments and we have determined the boundaries. With some coordinates. That there…that in these coordinates are our boundaries. Up to there, the second one can be that close and in the rest of Greece the second will be in a longer distance.

R In the rest of Greece. You're saying that you have divided into two segments.

S1 In two segments.

S2 In two segments. Namely, what he looks (meaning the second airplane) is the length, its a (meaning the latitude).

The episode is interesting as it reflects students' thoughts about the rule involving both geographical concepts such as finding cities that are in the north, middle and south of Greece and mathematical concepts such as coordinates and functions. Here, geographical coordinates of Greece, could be seen as the domain of the function, where the function changes according to the domain it is applied and additionally it is limited by specific boundaries.

In order to specify and implement the rule, students had to extend the initial Logo code that was a black box to them when they were playing the game. It was really surprising to us that although students were not experienced with Logo programming, they successfully managed to decode the Logo code and reflect upon its structure. A possible reason for this is that Logo commands are based upon everyday language, resulting in an easy way of exploring the operation of each of them as well as of learning to use them.

The rule that students selected to implement was comprised of two conditions and the corresponding actions. Initially they were confused about the way they should express the rule in Logo as they didn't know how to edit Logo commands. Researchers told them to express the rule in a way independently of Logo and that they would help them with Logo. Surprisingly students chose the 'if' command, as they knew it from Excel and edited the rule, thus it was easy to experiment and finally edit the Logo code. The following procedure in Logo is the final product that students created. (Their contribution in the initial Logo procedure we gave them is marked in bold.)

to flight :a :b :c
activateavatar("|first|)
setposition(:a :b :c)
if :a<38.0551
[activateavatar("|second|)
setposition**(:a-0.05 :b+0.05 :c-1000)**
print("second "airplane "position **:a-0.05 :b+0.05 :c-1000)]**
if :a>38.0551

```
[activateavatar("|second|)
setposition(:a-0.07 :b-0.07 :c-1500)
print("second "airplane "position :a-0.07 :b-0.07 :c-1500)]
end
```

Another interesting idea that students had, was to divide Greece in 'four quadrants' as they said, so the second airplane would have to check for the longitude of the first airplane as well. When the researchers told them to write this idea in their Logo procedure, they were puzzled as they didn't know if they had to use more 'if' in the procedure or not. Finally they agreed that they had to use 2 additional 'if' in their procedure, as the segments they wanted to divide Greece were four. A possible explanation about students' difficulties to extend the rule could be their inexperience in Logo that they had to use in order to express the conditions and actions of the rule. Nevertheless, they seemed to have developed their own rule regarding the relative displacements of the two airplanes as co-variation within the frame of a function. Moreover, they tried to make more complicated the rate of change and the domain of the function, probably aiming to construct a more difficult rule, less recognisable by their classmates.

Conclusions

Students' engagement with the game built on Cruislet environment seemed to promote their thinking about a variety of concepts related to the concept of navigation, such as geographical coordinates and mathematical concepts regarding 3d space and linear functions. However, our aim was not to study a specific mathematical or geographical concept and student's way of thinking about this. We rather tried to focus on processes where students engage in meaningful game-play that provides opportunities for meaning making.

Initially, student's tried to find out and understand the rule of relative displacements of the two airplanes. Cruislet environment provided opportunities to students to associate issues regarding both navigation in geographical space and mathematical concepts. They experimented by altering the geographical coordinates of the first airplane aiming to correlate the displacements of the two airplanes. They also explored the functionalities of the microworld and identified the rule of the game by relating it with the rate of change of the geographical coordinates of the two airplanes.

Having explored and internalized the functionalities of the microworld and consequently the rule of the game students started to reflect on the microworld and actually building their own games within the mathematical context of geographical coordinates, 3d space and function. Through the utilization of the simulation, information handling, geographical systems and Logo programming students gradually developed their own game play environment. They built functional relationships between the relative displacements of the two airplanes initially by varying the rate of change and later by altering the domain of the function.

It seems that the half-baked game built on Cruislet environment provided a meaningful context in which students explored several meanings regarding both mathematical and geospatial concepts embedded in the rules of the game and in the representation provided by the environment. The key feature of the game was that it combined an interesting game idea with a mathematical and geographical domain and inviting at the same time changes to both. These changes where feasible using Logo, which is considered as an essential feature of the environment as it allows users to intervene into the structure of the game, find and decode its rules and change them.

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