

Procedural building-up of geometrical object in concepts and strategies of primary school pupils

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Abstract

The article concerns the research of the geometry perception teaching of primary school pupils by means of computer application involving turtle graphics. Authors deal with problems and the research of LOGO environment usage for creation of ideas of dividing and assembling the complicated geometrical figures at the primary school pupils. The main goal was to encrypt which strategies do children use by discovering which part of given shape is capable to draw by applying an algorithm built from commands of several given types.

The application called Obkreslovačka (Encircler) has been created in the Imagine Logo for testing in schools. Turtle painting commands have been limited only to STEP, LEFT and RIGHT which only allow encircling the rectangular shapes. Some operating commands for the cycle have been added. Writing the turtle program by the commands typing has been replaced by the arrangement of command buttons due to the age of tested pupils. They had to pass several tasks to fill a given figure. They could create and use the turtle program to encircle the closed rectangular shape and to pave the rest of the figure by triangle-shaped tile objects. They were forbidden from filling the whole figure only by paving without using the turtle program.

The course of research tries to catch in what manner and what strategy a pupil divides the figure into parts created by procedural way and how to discover the symmetry of the shape parts and its reflection in the written program, what help gives the elementary geometrical shapes from which the figure is possible to compose. Based on these tests results a new set of tests has been prepared for the next testing in some South Bohemian schools, in the classes from three to five grades. The results of the main tests will be presented at the conference.

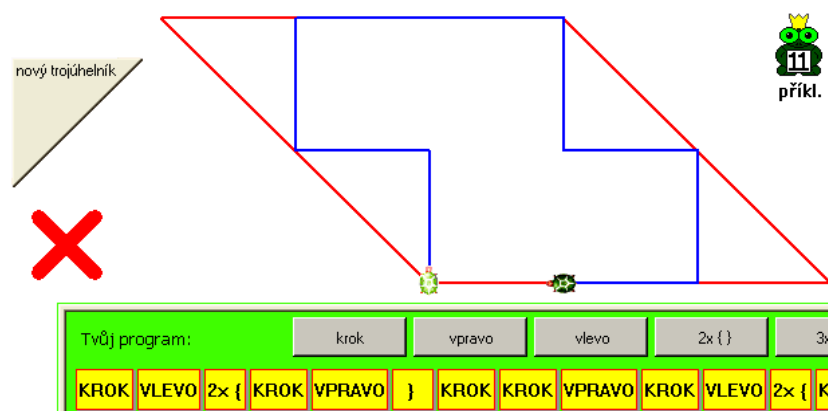


Figure 1. The environment of the testing application of Encircler

Keywords

geometry, primary school, building-up of geometrical object

Introduction

The contribution of micro-world Logo for geometrical imagination development and perception of space and shapes (turtle coordinates, creation of geometrical shape by commands for movement of an object defining the shape) is really well known. The modeling and drawing in the environment of turtle graphics is contributory to the teaching geometry based on the activity of a pupil and also to the expansion of the tasks repertory being available for the teacher. This leads to the more precise pupil conceptions creation.

Our concern is to find out how primary pupils are able to perceive geometrical figure as divided into two parts of various nature: a) part created by the procedural process (in terms of turtle graphics commands) and b) part paved with given basic tile objects. That was the question how pupils will discover which part of a figure can be drawn procedure-aided and in which method pupils discover this shape in each specific task and what strategies to solve such tasks will be created by them. Another question was how pupils form the appropriate procedure to create the shape.

We have started from the requirements of Czech and Slovak experts who declare the geometry teaching in primary school is based on the pupils experience obtained during solving various problems. For example František Kuřina says: "Geometry in primary school has not even a preparatory deductive character but it is markedly operational." (Hejný and Kuřina, 2001) Hejný argues similarly: "Experimentation is a basic and irreplaceable way to obtain geometrical experience. Pupils in primary schools have a natural ambition to investigate world by their own activity" (Hejný et al., 2004). Kuřina worked out a theoretic analysis of primary pupil geometrical experience and expressed four didactical principles that should form didactical structure for geometry teaching in primary schools (Kuřina, 2001). The presence of these principles is commented in the described investigative project.

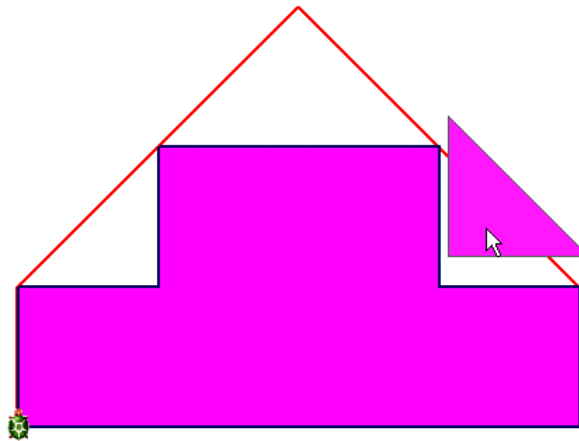


Figure 2. Procedural part of the figure and the rest paved by triangle tile object

Space Partition

It is applied when introducing the new notions into geometry (line divides a plane into two parts). Pupils meet the space partition principle from the babyhood. The space partition principle is included in that part of the project when a given figure is divided into the parts according to the way of creation, i.e. into the part that can be modeled by the use of the set of turtle graphics commands, and into the part that can be paved by predetermined basic tile objects.

Space Filling

The fact that the figure besides its sides also contains other points of the space is emphasized by the space filling principle. The areas of geometrical figures can be measured by filling the space. Calculations form traditionally a part of geometry teaching but it is also important to teach the pupils to observe and to guess. Although the assignment of testing tasks is to encircle a given figure the pupil fills the shape with objects created by procedure or paves the tile objects.

Construction

Constructions mean not only solving the geometrical tasks by means of drawing but also various ways of geometrical figures representation (for example construct a building from two, three, four bricks of the children building set). Many questions concerning the ordinary life also have a constructional character (determination of the journey from the place A to the place B). Modeling the figure by use of writing the program brings the possibility of a deeper understanding geometrical figure attributes. Above all the use of cycle leads to the idea of symmetry and its discovering both in shapes of parts of objects and in shapes of their perimeter lines.

Movement in the space

The idea of the direct movement and experience with it is good developed at a normal child aged of six years in our conditions. It can be checked on the tasks in which we search the traces on the plan. The movement in the space can be interpreted as the imagination of turtle movement when encircling the geometrical figure. Here the procedural construction of figure (or procedural shape) means the shape created by commanding, by means of which a given figure will be encircled (figure 2).

Main research objectives

Aspects of the process of polygon conception understanding

- How affects the computer and turtle graphics use a deeper comprehension of the polygon conception
- Understanding of the polygon as a part of the plane (it is not an outline demarcating it, but it is also created by the points from inside)
- propaedeutics of the symmetry conception as one of the polygon attributes

Geometrical transformation from a dynamical point of view

- paving of triangle tiles, propaedeutics of the geometrical transformation conception (congruency, translation, rotation, axial reflection)
- how will a procedural part of a given figure be found (i.e. the ability to find a figure of a specific shape in the given figure, particularly the shape of rectangle, which they are able to model by means of given set of motion commands) in a given polygon
- which shapes seem to be simple and which difficult and why to children
- which strategies to discover a procedural created shape will be chosen

Testing environment

An application called Obkreslovačka (Encircler) to test pupils in schools has been created in the Imagine Logo environment (figure 3). We have chosen this environment thanks to its three advantages: Imagine Logo naturally involves turtle graphics, enables to access object-oriented (Blaho and Kalaš, 2001) and is accessible to Czech schools.

The commands of turtle painting have been limited only to KROK (STEP, go ahead a constant distance), VLEVO (LEFT, turn left 90 degrees) and VPRAVO (RIGHT, turn right 90 degrees) because primary school pupils are not familiarized with an angle conception and miss the required skills to work with angle size and the research has been intended for schools without any prior experience with Logo. So the limited commands of turtle graphics only allowed encircling the rectangular shapes put together from vertices of a base grid of a graphic screen.

Some steering commands for cycle REPEAT 2x, REPEAT 3x, REPEAT 4x and brackets for defining lists of repeated commands were added to the commands controlling the turtle movement. Of course, to nest cycle commands was possible.

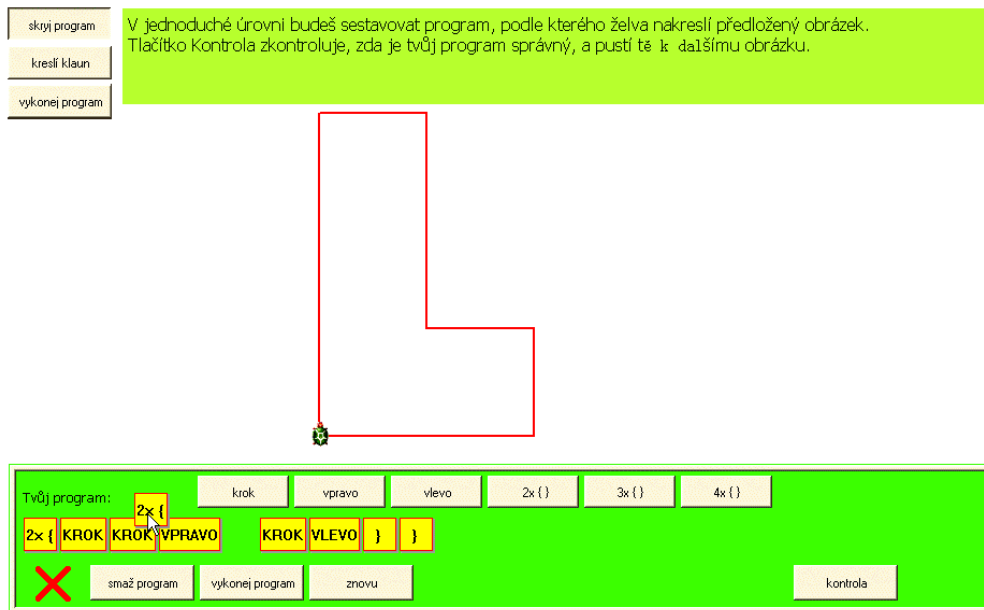


Figure 3 – The environment of the testing application, the level No. 1

To encircle the given geometrical shape in the Encircler environment was the typical task of testing. That meant to fill the given figure or its part with a rectangular polygon created according to the “turtle program” made by the tested pupil. In case of a turtle return to a start-up position after executing the program, an inside of the delineated polygon was coloured and could fill the whole object or its part.

Some given figures have not only the rectangular shape, but there are some skew line segments, so it is impossible to fill the whole figure only by using the turtle program. In this case, the user was allowed to add some other tile objects to the figure to fill gaps in the given polygon. These tile objects had exclusively the shape of right-angled and isosceles triangle with legs of the length of STEP, so that the two triangles could fill a basic square of the grid together.

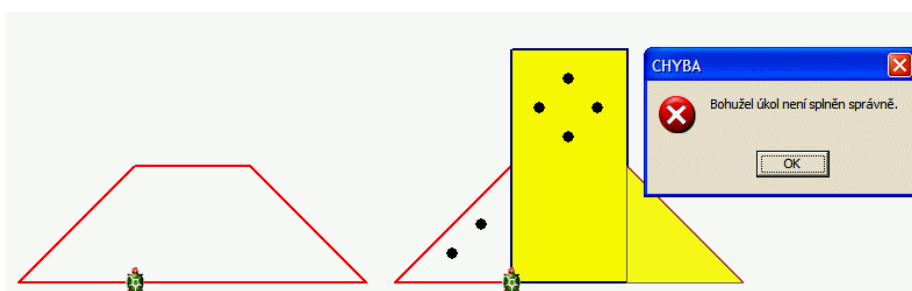


Figure 4. A settings of the task and a result of children solving check – there is one tile object missing and the turtle program is wrong.

The user could move such objects in this manner: to drag and drop through the grid, to turn right 90 degrees by a right-click, to remove it. It was not allowed to place two triangles in such position to fill a basic grid square. Thereby we prevent the pupil from paving the whole shape of given polygon by these tiles without creating a program.

Due to the age of tested pupils, writing of the turtle program by typing the commands was replaced by arrangement of command buttons. The user could modify his program by dragging buttons and could try and check it visually at his discretion; he could send his work to be checked only if he was sure that the whole figure was filled. After the request to check, computer checked whether the pupil filled a given figure correctly and any triangle-shaped tile object does not overlap the polygon made by the turtle; eventually it marked some mistakes and returned the task to be corrected by the user (figure 4). The tested person had the unlimited number of attempts to correct his solving, but he was allowed not to finish the task after the first unsuccessful check and start another task.

Testing tasks levels

The testing tasks have been divided into three groups varying by a difficulty of the given objects:

- The lowest level included only figures of simple rectangular shape made of base grid squares, e. g. rectangles of different shapes, shapes of L and U letters and bigger squares with startup positions on the vertices or the centres of the sides (the startup position is a place where a turtle is at start of executing the pupil's program). This level served mainly as a mean to familiarize with the application environment and the type of testing tasks. We used the same shape (e.g. square) several times with other starting point, which produced a couple of new tasks. We did not interest in the correct writing of the program encircling the object because our main point of interest was how children could divide given shape into the parts described above. In other words, we interested how children could find rectangular shape in the given object which was able to be drawn by the given turtle commands. So we could prepare more different situation looking like similar only by moving starting point of encircling on the same object.

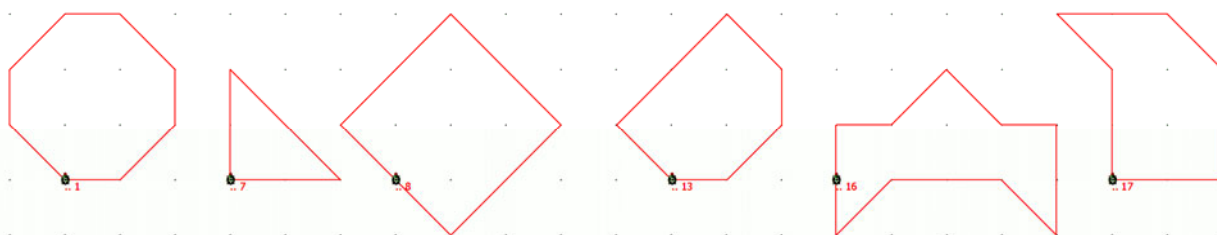


Figure 5. The second level shapes – in these shapes children had to discover a hidden part to encircle it by the turtle program.

- The shape of the setting figures used in the second level involved horizontal, vertical and transversal sides with a slant angle of 45 degrees, e. g. rhombs and rhomboids, big right triangles, octagonal shape of the traffic sign Stop, square staying on its top, a pinwheel etc. In this level, the first time pupils used tile objects to fill the figure. Each task had only one solution of a question - which part of a given figure is procedural (i. e. which part is creatable by a turtle program) - though algorithms and pupils programs leading to this solution were different (and were not evaluated). This level was pivotal and the most important for the research because it was the first time when pupils decided which part of the shape was procedural and which could be made by paving of triangle tile objects.
- The third level of the game was the hardest one. The tasks involved figures and environment of the same type as the second one, but the shapes of the figures were more difficult to make a turtle program for description of their procedural part. Only simple widening command buttons to a program row would lead to too long programs with a risk of overlapping a given length of the row. The pupil had to use more complicated structures, i. e. nesting cycles, or non-typical traces of depicting the turtle polygon, which had not to be always convex or coherent.

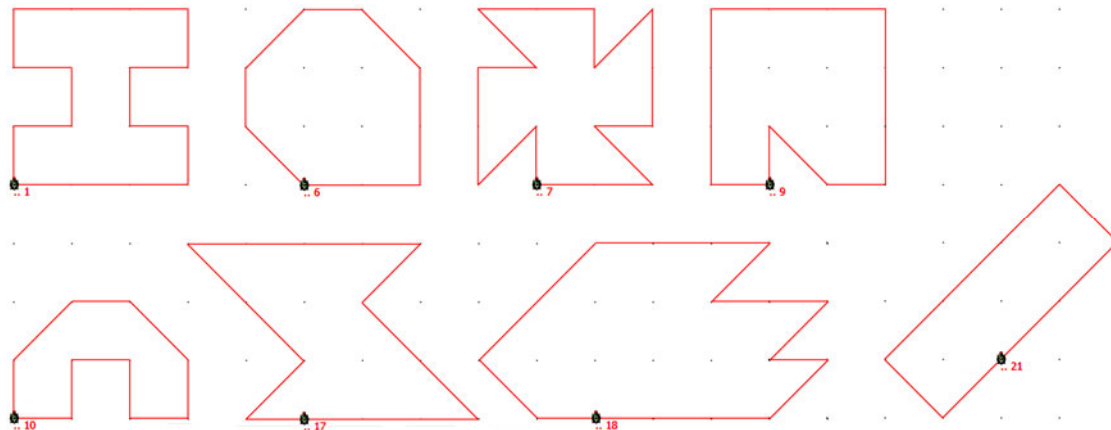


Figure 6. Shapes used in third level of the test.

Outlets and their analysis

After finishing each task level, following data used to the latest analysis of each tested pupil work have been saved:

- the given figure of task settings, the turtle program, the polygon painted by the turtle and position of each used triangle tile object have been saved after each request to execute the turtle program
- the same data have been saved after each request to check the task solution
- pupil answer to the question about the area of the given object has been saved when transitioning to a new task

The testing application enabled to show the same situation on the screen which was visible in the time of any request of the pupil to execute or check what is useful for the latest analysis of children process of the task solution creation. An examiner could see the same situation as the pupil whenever the pupil asked turtle to depict the program (e. g. when he was not sure or before checking his solution) and could check both the program and the shape of the polygon painted by the turtle. It enabled to find typical mistakes made by children and their strategies when encircling the given figure.

Course the test

The test consisted of an introductory training and three levels of the “game” in the Encircler environment. At each level, the tested pupil had to pass five tasks to fill a randomly chosen figure from a set of shapes for this level. Each task was finished with one question about the area of the given figure couched in squares of base grid or in the amount of triangle tile objects. The reason for including this question in the test was both checking the estimation in case of manipulated objects and a small relaxation before the next task and the extension of concentration time of the tested person.

Prior to the first test level pupils worked at the level of “training”, at which they could create simple turtle programs without any specific figure to encircle. The main purpose of the training was acquainting pupils with the environment of the Encircler application and introducing them to the turtle graphics and geometry by a simple way of using experiments and manipulations. This tutorial took about 20 minutes and the tutor had to lead and motivate pupils with the instructive tasks (to paint a square, to try to use the cycle buttons, to create stairs, to return the turtle to the start-up position) and help them individually.

After introducing to the environment, pupils have already worked without any help at the other levels of the “test game”. The tutor only checked the situation and tried to catch any information which the testing application could not have recorded. Tutors notes might be very useful for the later completion of the application. No limit has been determined for the time of the test.

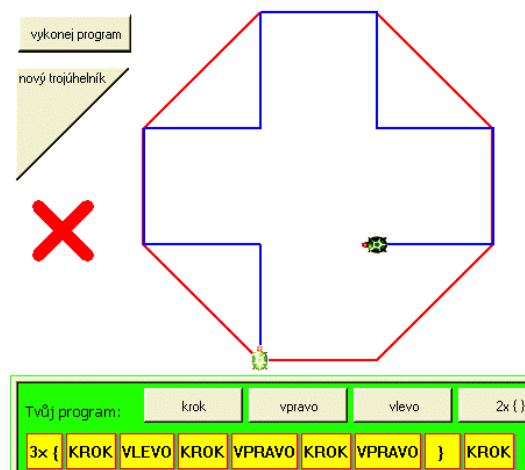


Figure 7. The second testing level of the test. The turtle creates the procedural part of the figure according to the program below

The objective of the first level (encircling the figures only by the turtle program use) was to introduce tested persons to the course of the test, type of testing tasks and to improve their skills to create the turtle program. Only the second level was pivotal from the research point of view. Pupils were examined how they divide a given shape to the part creatable by turtle painting and to additional parts created by paving the tile objects. The third level observed whether the pupils were able to divide more difficult figures and how they created programs for such a difficult procedural shape of the polygon.

The pretest – debug environment checking

The first objective of the pretest was to confirm the testing application in school conditions and the organization of the test. The second objective was to find out if qualitative data obtained within the testing are sufficient for the analysis of the way of pupil thinking and the observing his/her strategy.

The pretest took place in the primary school ZŠ Mladé, the suburban school in České Budějovice with pupils from fourth grade. These pupils have never worked in the environment of Logo before. 14 pupils were present within the pretest. They were divided into seven groups which appeared to be suitable especially in the first part of test – during the training and in the first level of test. The time period of the test was less than two school hours, i.e. approximately 80 minutes. The test was interrupted by the break with regard to the low age of pupils.

It also proved that random choosing of shapes from the set of figures creates (even with a big care of choosing) the non-comparable conditions for various pupils in particular at the first task. The result was the necessity to create a strict sequence of testing tasks even though it will be with regard to a various speed of pupils at the expense of the objectivity, because pupils of this age tend to advise in the event that they see the same task which they have already solved at their neighbor, even though they are not asked for it.

Observing the pupils in the second level of the pretest resulted in the conclusion that pupils approximately after two experiments moved to the strategy first to pave tile objects to a figure (some pupils immediately from the first task, one group at the last one). In this way they used the computer for drawing a rectangle as a rest of figure after paving tile object and for the ensued rest they wrote a program which was drawn by the turtle (figure 9). This meant for the following elaboration of the application the necessity to extend saved data concerning information, if at the beginning of each task a pupil first created a program or paved tile objects.

The first testing results show us the main goals for further improvement and arrangements of the testing application Encircler:

- it is necessary to add another testing level in which pupils are first allowed to pave tile objects to the figure only after finalizing the turtle program without any possibility to change it later
- to reduce the excessive check of a created program we ponder to add any form of counting the requests of the pupils (we also hope that pupils will be hereby motivated to make less attempts without thinking during creation of the program)
- to add information on the overlapping the maximal allowed length of the turtle program to the test record, which can be a trigger to shorting process of the programs by using cycles
- to add information to the test record whether the pupil started a new task solving by writing the program or paving a tile objects to the figure to get the required rectangle for turtle painting).

The test

Based on the pretests results a new set of tests has been prepared for the next testing in the classes from three to five grades. As the strategy “first to complete paving tile objects and then to write a program” showed to be a winner among the tested pupils, we suppose in the newest version of the Encircle a creation of another extra level of test in which the pupil will have to create first a program and then he/she will be allowed to pave tile objects. We suppose the pupils will be forced to think more abstract. This level was placed between second and third level.

The main tests took place in sixteen South Bohemian primary schools during two month, May and June 2007. Two hundred pupils were present within the test, 113 girls and 87 boys. Tests were realized by couples of teacher students who had been prepared at the faculty for leading initial training and for special technical and pedagogical situations.

The test proved that pupils are able to follow students’ instructions and manage to command the application without any marked difficulties that could misrepresent the outcomes of test by the fact that disability to command the application would have been the reason for failure during the testing.

Outcomes of the test

Our testers referred that children which all hadn’t programmed before were more kind and skilful than were expected. After preliminary evaluation of the test records and tester’s messages next findings can be framed:

Strategy of polygon creation

Some pupils verified their program too often, especially in the initial level and especially girls. After each change they verified the program that way that they let turtle draw an outline. Some of them evoked the feeling that they used a method try-error: they chose randomly any new command and added it at the end of the program. They verified their program immediately and then erased last button or left it. A typical problematic example was to turn the turtle in a concave angel of rectangle (figure 8). While observing them we have found that a part of pupils who seldom used verifying their program by the drawing the outline helped itself with the illustration of supposed turtle motion by a finger on the monitor.

The fact that girls were much more cautious is probably conformable with other researches, from which follows, that girls have not so direct relationship to the computer. There is a question, if it causes in virtue of the medium of computer or to the environment of the application and the character of the exercises.

It appeared that pupils didn’t understand their programs well because they corrected longer parts more often than it was essentially necessary. Another well used strategy was to delete the program and to begin creating it again but for the inverse turtle way through a figure. It seemed

like children supposed that opposite direction allowed turtle to reach the opposite side of the figure more easily. Children may have understood the tasks as not to encircle but to do through.

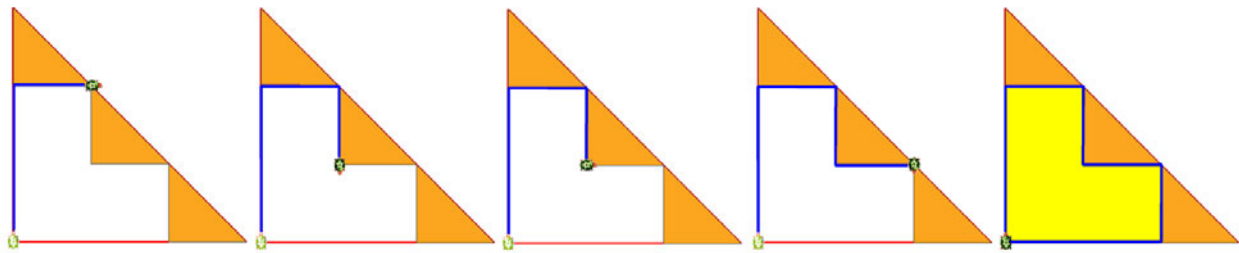


Figure 8. A recorded process of a turtle polygon creation. Maruška was a little cautious and checked it after any changes of the program.

The total feeling of the main test was that pupils have bigger difficulties to write a program than to discover division of a figure into a procedural and a tile object part. We could see many times that tile objects were dislocated correctly but the pupils were not able to write a program that fills the remaining rectangle shape. It appears that a longer time period within which the pupils will handle the turtle and write a program will be required before allowing the pupils to solve tasks of the main test. This can be supported with a longer and goal-directed training and a longer first level of testing.

The extra level of the test in which pupils have first to create a program and then to add pave tile objects caused big problems. The most of the children did not divide given shapes correctly even if they had a feedback. There was visible that even after several attempts drawn shapes did not get closer to the correct shape.

According to the achievement, we can divide pupils into three frictional groups. Beside expected groups of very successful and unsuccessful children one group more appeared. These children first work very slowly, with mistakes and a lot of checking attempts. About in the middle of the second level of the test they probably got familiarize with the environment because they start to work more quickly and most of the tasks were solved at once and without mistakes.

Cycle and symmetry

The analysis of the most difficult level of the test resulted in the finding that pupils begin to use a cycle for notation of program only when they are forced to shorten their program. Although the advantages of such notation were demonstrated to them in the introduction of work with Encircle the pupils give commands in a sequential consecutive manner. The restriction of the program length caused difficulties only in the most complicated level where the shapes, which were impossible to encircle by mere consecutive giving the commands, were included. Therefore the impulse for the structure program use was not the effort to simplify the program or to make it understandable but the effort to shorten it.

The pupils got themselves into the above mentioned situation unexpectedly even in more simple level when they needlessly drag out a program by composing the nonfunctional sequences of a program (for example LEFT RIGHT LEFT and etc.). Then because of the abbreviation they were adding a cycle only to the last part of a program instead of the whole program control.

Some pupils used the cycle only for repeating one command (REPEAT 3x [STEP]). Other pupils that inserted more commands into a cycle (for example ... REPEAT 2x [STEP LEFT STEP RIGHT] ...) could find a repeating part of a figure or the repeating sequences of program. The observation resulted in the finding that the approach described as the second one was prevailing.

Some pupils discovered the connection between the central symmetry and the program which was all written as a cycle (for example REPEAT 2x [STEP STEP LEFT STEP LEFT]). These

pupils tried to discover symmetry in further pictures and to use a cycle as a general approach to write a program for such kind of picture. One group discovered axial reflection at one picture but left the task without any trying to find another solution because it was not able to write a program as cycle. It looks like that they misinterpret this attribute also for axial reflection.

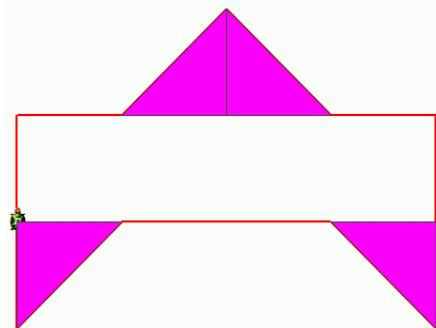


Figure 10. A strategy of paving tile objects first to get the shape of turtle painting.

Conclusion

The testing results show us the main goals for further improvement:

- Most of tested pupils have a problem with finding procedural part of the given shape. One of the future aims could be to discover which types of geometrical shapes are easier (and good for initial training of dynamical geometrical imagination) and which of them are more difficult.
- There is an interesting point, how children get the same object with different starting point of the drawing. We haven't got any records where pupils automatically use prepared program used in the task before in the case of two tasks with the same object and different starting point. But the research was not targeted to this question. Therefore it could be interesting to allow children to set the initial direction and place of the turtle.
- It could be useful to arrange used figures of the first and second testing level not only firmly according to the difficulty, but to the rate of repeating of the outline fragments and to the symmetry of the figure.
- Some other direction of the research could go to applying of other grid than rectangular. How difficult could be shapes drawn into a triangular grid for the pupils?
- It is necessary to train tutors for testing so that they are able to serve the application and computer in a technical manner and to lead the initial training and introduce children to the environment of the testing application. They need to be more supplied with the schemes for the turtle painting (i.e. to place turtle at the specific position, to encircle symmetrical figures, to create non-coloured figure though the turtle finished its program at the start-up position) and a methodology how to explain the technology and advantage of using cycles in a program to pupils. They need to repeat the commands and explanations more often.

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