A provenly correct translation of *Fickle* into Java^{*}

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Abstract. We present a translation from *Fickle*, a small object-oriented language allowing objects to change their class at run-time, into Java. The translation is provenly correct, in the sense that we have shown it to preserve the static and dynamic semantics. Moreover, it is compatible with separate compilation, since the translation of a *Fickle* class does not depend on the implementation of used classes. Based on the formal system, we have developed an implementation.

The translation turned out to be a more subtle problem than we expected, and the proofs helped us uncover several subtle errors. In this paper, we discuss four different possible approaches we considered for the design of the translation and justify our choice, we present formally the translation and the proof of preservation of the static and dynamic semantics, and we discuss the prototype implementation.

The language *Fickle* has undergone, and is still undergoing several phases of development. In this paper we are discussing the translation of *Fickle*_{II}.

1 Introduction

Dynamic object re-classification is a programming language feature which allows an object to change its class membership at run-time while retaining its identity. Thus, one can express fundamental change of an object's behavior (*e.g.*, non-empty lists becoming empty, iconified windows getting expanded, *etc.*) through *re-classification*, rather than through replacing objects of the old class by objects of the new class. Lack of re-classification primitives has long been recognized as a practical limitation of object-oriented programming. *Fickle* [10–12, 7–9] is a small Java-like language which supports dynamic object re-classification, aiming to demonstrate how object re-classification could extend an imperative, typed, class-based, object-oriented language.

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Several other approaches to the expression of fundamental change of behaviour have been suggested, see [6, 13, 17]. However, we claim that *Fickle* is more in line with the mainstream object oriented approach, whereby an object's behaviour is determined by its class membership. Moreover, *Fickle* is type-safe, *i.e.*, any type correct program (in terms of the *Fickle* type system) is guaranteed never to attempt to access non-existing fields or methods.

Indeed, *Fickle* stands for a whole family of languages, which differ in the extent of the use of re-classifyable types, in the treatment of variables, in the support for class loading and the presence of threads. All *Fickle* languages support object re-classification, and are expression-oriented¹.

Fickle, see [10, 11] is a Java-like language with a *re-classification* operation that changes the class membership of an object while preserving its identity. The basic problem in the design of the language is to have a sound type system, that is, a type system that, even in the presence of object re-classification insures that no attempt is made at accessing non existing members. This is obtained by tracking the type of the variables possibly denoting a re-classified objects. An object *o* of class *c* could also be denoted by a field *f* of type *d* (of some other object *o'*), where *d* is a superclass of *c*. Therefore, to achieve type soundness, we must forbid re-classification from (a subclass of) *d* to a class which is not a subclass of *d* (otherwise subsequent accesses to members of *o'.f* existing in class *d* could fail). Classes for which this does not happen are said to be “respected” by re-classification and can be safely used as types for fields. In *Fickle* re-classifiable classes are explicitly marked as *state classes*. State classes are the possible sources and targets of re-classifications, and *root* classes are the superclasses of state classes and declare all the members common to them. Only classes that are not subclasses of root classes can be used as types for fields (this classes are always respected by re-classification). Effects are sets of root classes, $\{c_1, \dots, c_n\}$, meaning that there could be a re-classification between two subclasses of a class in the set. Re-classification is traced by *effects*, and methods are annotated with the effects that may be caused by the execution of their body. In *Fickle* the mutation operation is only possible on the metavariable **this**, and in typing an expression we keep track of the current class of the metavariable **this** and the effects that are caused by the execution of the expression.

In *Fickle_{II}* [12] re-classification is possible on all the variables (not only on **this** as before). So in the type system we have to keep track of the current class of all the variables in the environment.

In *Fickle_{III}* [7] we refined the effect system with effects that are sets of pairs. The meaning of $\{(c_1, c'_1), \dots, (c_n, c'_n)\}$ is that there could be a re-classification from c_i to c'_i . *Fickle*-style effect annotations can be coded into *Fickle_{III}*-style effect annotations, but not vice versa, and *Fickle_{III}* allows to correctly type meaningful programs which *Fickle* rejects.

The *Fickle_{III}*-style of effects is used in *Fickle₃* [8], to eliminate the explicit marking of classes for which re-classification is allowed (state and root), since this

¹ We chose to express our ideas in an expression-oriented setting, as this greatly facilitated the formal treatment.

information can be inferred from the effects. In this way we obtain a language that is syntactically simpler, and allows *unanticipated re-classification*. However, this has a drawback: when linking a class c with an existing set of classes we have to check on one side that the effects of the methods of c respect the classes used as types of fields of the preexisting classes, and on the other that the effects of the methods of the preexisting classes respect the classes used as types of fields of c .

Finally in $\mathcal{Fickle}_{MT}$ [9] we consider the issue of dealing with re-classification in the presence of multi-threading. To this aim we define a multi-threaded extension of the language $\mathcal{Fickle}$ (in which re-classification is only allowed on **this**), that we call $\mathcal{Fickle}_{MT}$. In $\mathcal{Fickle}_{MT}$ the expression **spawn**(e) starts the evaluation of the expression e in a new thread while the current thread continues by evaluating the expression following **spawn**(e). The new and the current thread work on a common heap containing the set of defined objects, and, through aliasing, re-classification may change the class membership of objects across threads. To achieve soundness we prevent executions in which a thread re-classifies an object while another is executing a method on the object. This is done by combining the static type and effect system (defined for $\mathcal{Fickle}$) with a *synchronization mechanism* based on *effect information* that delays the execution of possibly dangerous threads. We can prove that, on one side no execution of a well-typed expression can cause the access to non-existing members of objects, and on the other the delays introduced do not cause deadlocks.

In this paper we investigate the practicality of the ideas around $\mathcal{Fickle}$, through the discussion of an implementation, which maps $\mathcal{Fickle}$ onto Java. From the languages in the $\mathcal{Fickle}$ family, we chose $\mathcal{Fickle}_{II}$, which in some sense is the “essence” of re-classification, as it allows parameters and return types of re-classifyable type, but does not support class loading or threads. We chose this version of $\mathcal{Fickle}$, because it is the most expressive language from the $\mathcal{Fickle}$ family which is solely concerned with re-classification for object-oriented languages,² and also because we have published a complete semantics and proofs of soundness for $\mathcal{Fickle}_{II}$ [12].

We wanted to study the problem of simulating re-classification from its theoretical and its practical aspect. Therefore, we proceeded by developing in parallel both a formalization of the translation, and a prototype implementation. The translation turned out to be a more complex task than we had originally anticipated, and several subtle issues had to be considered. Thus, the formalization served to elucidate the design, and the properties of the formalization helped us in identifying and addressing the aforementioned subtle issues. The formalization maps $\mathcal{Fickle}_{II}$ into $\mathcal{Fickle}_{II}^-$, where $\mathcal{Fickle}_{II}^-$ is a fragment of $\mathcal{Fickle}_{II}$ without re-classification – we chose $\mathcal{Fickle}_{II}^-$ because this allowed us to use the same type system and operational semantics as for $\mathcal{Fickle}_{II}$. Our prototype implementation [5] maps $\mathcal{Fickle}^{st}$ onto Java. $\mathcal{Fickle}^{st}$ is a statement oriented version of $\mathcal{Fickle}_{II}$.

² $\mathcal{Fickle}_{II}$ is most expressive with regards to re-classification, but does not have the extra features supporting unanticipated software evolution, nor has it got the multithreading support as in $\mathcal{Fickle}_{III}$ and $\mathcal{Fickle}^m$.

The translation is provenly correct: We present here the proofs that it preserves the static and dynamic semantics – *i.e.*, well-formed $\mathcal{Fickle}_{\text{II}}$ programs are translated into well-formed $\mathcal{Fickle}_{\text{II}}^-$ programs which behave “in the same way”.

Moreover, the translation is compatible with separate compilation, in the sense that the translation of a $\mathcal{Fickle}_{\text{II}}$ class does not depend on the implementation of any classes *it* uses. Therefore, our translation could form the basis for an extension of a Java compiler; namely any type information needed by the translation can be retrieved from type information stored in binary files, as is done, indeed, by Java compilers for ordinary compilation.

For the design of the translation we had to consider the following issues:

1. an appropriate encoding for re-classifiable objects,
2. the relation between the types of a $\mathcal{Fickle}_{\text{II}}$ expression and the corresponding translated Java expression,
3. ensuring that a translated Java expression of class type will always denote the object in its most current state,
4. the fact that a standard Java class c can be extended by a re-classifiable class, possibly after c has been translated (*i.e.*, compiled),
5. making the translation compatible with separate compilation.

Concerning point 1), the basic idea is to represent each re-classifiable $\mathcal{Fickle}_{\text{II}}$ object o through a pair $\langle id, imp \rangle$ of Java objects. Roughly speaking, id provides the (immutable) *identity* of o , whereas imp is the *implementor object* of the id object, and provides its (mutable) *behavior*. A re-classification of o changes imp but not id , and method invocations are resolved by imp .

Concerning point 2), our initial idea was that there should be an isomorphism between the types of $\mathcal{Fickle}_{\text{II}}$ expressions and the types of the translated Java expressions. However, as we shall argue in section 7, this lead to a complex system, and made the issues around point 3) more difficult. In our current solution all $\mathcal{Fickle}_{\text{II}}$ expressions of class type are translated to Java expressions of the same type, namely **Identity**.

Concerning point 3), originally we were maintaining a chain of objects where each was delegating to the next, more recent implementation object. Thus, every field access or method call needed to follow the chain in order to find the most recent implementation object. However, this solution grew rather complex, and we abandoned it for the pair $\langle id, imp \rangle$ described above.

Concerning points 4), and 5), we decided to represent *all* objects, even the non-re-classifiable ones, through such pairs $\langle id, imp \rangle$. Thus, all Fickle classes, even the ones that describe objects which may not be re-classified, are translated in a uniform way. Also, field accesses or method calls need to find the implementation object, and are therefore translated in a uniform way, independently of whether the receiver belongs to a re-classifiable class, or not. Thus, a class may be translated without internal knowledge of the classes it is using.

Lastly, we also had to reconcile the requirements for the production of efficient Java code, the simplicity of the translation, and simplicity of the proofs. We

decided to make the translation as simple as possible, neglecting efficiency in favor of uniformity (*i.e.*, fewer different cases) and simplicity.

The work presented in this paper comes out of the collaboration of three different research groups and is based on their previous experience in the design and implementation of Java extensions [3, 12]. All sites have been involved in the design of the translation and its properties.

The formal work has been taking place in parallel with the development of prototype implementations, and there was considerable cross-fertilization. The first implementation, *Carmela* [4], together with an extensive test-suite, was developed largely before the formalization, but as it grew rather complex we introduced the concepts of the formalization to reason about it. We then uncovered some problems, and modified the approach [1, 2]. The current paper is the culmination of our experience with various approaches, and presents the simplest design. The approach has been implemented in *Isabella* [5]. We have developed full proofs of correctness.

The paper is organized as follows: In Section 2 we introduce *Fickle_{II}* informally in terms of an example. In Section 3 we give a brief formal description of *Fickle_{II}* (syntax, operational semantics, and typing) and state the soundness of the type system. In Section 4 we give an informal overview of the translation, while in Section 5 we give the formal description. In Section 6 we state the properties of the translation (preservation of static and dynamic semantics) and illustrate the compatibility of the translation with Java separate compilation. In Section 7 we discuss the various design alternatives considered for the translation. In Section 8 we describe our current implementation of the translation [5]. We conclude by summarizing the relevance of this work and discussing further research directions. Some technical definitions are illustrated in Appendix A. Proofs of the main results are given in Appendices B and C.

2 *Fickle_{II}*, an informal overview

In this section we introduce *Fickle_{II}* informally using an example.

The *Fickle_{II}* example in Fig. 1 describes accounts which belong to people, and which may be daily accounts, or savings accounts. It consists of a class **Person**, and a class **Account**, with subclasses **SavingsAccount** and **DailyAccount**. An **Account** belongs to a person (field **Person owner**), and holds some money (field **int amount**). **Accounts** implement the method **transact(int x)** which increments or decrements the amount. If the amount exceeds the threshold of 1000, then the account turns into a **SavingsAccount**. Similarly, if the amount falls below the threshold then the account turns into a **DailyAccount**. Savings accounts pay interest.³

Class definitions may be preceded by the keyword **state** or **root** with the following meaning:

³ The example would have been expressed more naturally if we could use abstract classes, and exceptions, but these are not part of *Fickle_{II}*.

```

class Person {
    int age;
    String name;
}

root class Account {
    int amount;
    Person owner;
    void transact(int x)    {Account} {}
    int interest() {} { 0 }
}

state class DailyAccount extends Account {
    int overdraft;
    void transact(int x) {Account} {
        amount +=x;
        if (amount > 1000) {
            this!!SavingsAccount; this.interestRate = 10;
        } else {
            if (amount < overdraft ) {
                ...
            }
        }
    }
}

state class SavingsAccount extends Account {
    real interestRate;
    void transact(int x) { Account } {
        amount +=x;
        if (amount < 200) this!!DailyAccount; this.overdraft = -400 ; }
    int interest() { } { interestRate*amount }
}

```

Fig. 1: Program Account - accounts with re-classifications

- *state classes* describe the properties of an object while it satisfies some conditions; when it does not satisfy these conditions any longer, it can be explicitly re-classified to another state class. For example, `DailyAccount` describes daily accounts; when these hold more than 1000 drachmas, then they are re-classified to `SavingsAccount`.

We require state classes to extend either root classes or state classes.

- *root classes* abstract over state classes. Objects of a state class `C1` may be re-classified to a class `C2` only if `C2` is a subclass of the uniquely defined root superclass of `C1`. For example, `Account` abstracts over `DailyAccount` and `SavingsAccount`; objects of class `DailyAccount` may be re-classified to `SavingsAccount`, and vice versa.

We require root classes to extend only non-root and non-state classes.

Objects of a non-state, non-root class `C` behave like regular Java objects, that is, are never re-classified. However, since state classes can be subclasses of non-state, non-root classes, an object bound to a variable `x` of type `C` *may* be re-classified. Namely, if `C` had two state subclasses `C1` and `C2` and `x` referred to an object `o` of class `C1`, then `o` may be re-classified to `C2`.

Objects are created in the usual way, by the expression `new C()`, regardless of whether `C` is a state, or root, or non-state and non-root, class.

A *re-classification statement* has the form `id!!C`, and sets the class of `id` to `C`, where `C` must be a state class with the same root class of the static type of `id`. The re-classification operation preserves the types and the values of the fields defined in the root class, removes the other fields, and adds the fields of `C` that are not defined in the root class, initializing them in the usual way. Re-classifications may be caused by re-classification statements, *e.g.*, `this!!SavingsAccount` in method `transact` in class `DailyAccount`, or, indirectly, by method calls, like `a.transact(...)`.⁴ At the start of method `transact` of class `DailyAccount` the receiver is an object of (a subclass of) `DailyAccount`, therefore it has the fields `amount`, `owner` and `overdraft`, while it does not have the field `interestRate`. After execution of `this!!SavingsAccount` the receiver is of class `SavingsAccount`, the fields `amount` and `owner` retain their values, the field `overdraft` disappears, and the field `interestRate` becomes available.

Fields must have declared types which are not state classes; we call these types *non-state types*. Thus, fields may denote objects which do change class, but these changes do *not* affect their type. Instead, the type of `this`, a parameter or a local variable *may be a state class* and *may change*.

Annotations like `{}` and `{Account}` before the method bodies⁵ are called *effects*. Similarly to exceptions in `throws` clauses, effects list the root classes of all objects that may be re-classified by execution of that method. Methods annotated with the empty effect `{}`, like `interest`, do not cause any re-

⁴ In the example, only `this` is re-classified; note that *Fickle_{II}* also allows re-classification of parameters and local variables.

⁵ In a fully developed language these would appear before or immediately after the `throws` clauses.

classification. Methods annotated with non-empty effects, *e.g.*, **transact** annotated with $\{\text{Account}\}$, may re-classify objects of (a subclass of) a class in their effect – in the example, of **Account**.

A method annotated with effects can be overridden only by methods annotated with the same or fewer effects⁶. By relying on effects annotations, the type and effect system of $\mathcal{Fickle}_{\text{II}}$ ensures that re-classifications will not cause accesses to fields or methods that are not defined for the object.

3 The language $\mathcal{Fickle}_{\text{II}}$, a formal description

3.1 Syntax

The syntax of $\mathcal{Fickle}_{\text{II}}$ is specified in Fig. 2. We use standard extended BNF, where a $[-]$ pair means optional, A^* means zero or more repetitions of A , and A^+ means one or more repetitions of A . We follow the convention that non terminals appear as *nonTerm* and terminals appear as **term**. We omit separators like “;” or “,” where they are obvious (note that “;” in $\mathcal{Fickle}_{\text{II}}$ is a separator rather than a terminator as in Java).

```

p      ::= class*
class  ::= [root | state] class c extends c' {field* meth*}
field  ::= t f
meth   ::= t m(par*)  $\phi$  block
t      ::= bool | int | c
par    ::= t x
 $\phi$     ::= {c*}
block  ::= {var* e+}
var    ::= t x
e      ::= id | sval | isnull(e) | e.f | (c)e | new c |
           id!!c | x = e | e.f = e1 | e.m(e*) |
           if e then e1 else e2 | block
id     ::= x | this
sval   ::= true | false | null | n

```

Fig. 2: $\mathcal{Fickle}_{\text{II}}$ syntax

⁶ This means that adding a new effect in a method of a class c does not require any change to the subclasses of c , but may require some changes to its superclasses, and the classes using them. Note also that effects are explicitly declared by the programmer rather than inferred by the compiler. Even though effects inference could be implemented in practice, more flexibility in method overriding can be achieved by requiring the programmer to annotate methods with more effects than those that would be inferred (similarly to what happens with **throws** clauses for exceptions).

Metavariables c , f , m and x range over sets of *class names*, *field names*, *method names* and *variables*, respectively. We assume a distinguished class name **Object** which cannot be used as name of a declared class.

A program is a sequence of class definitions. A class definition may be preceded by the keyword **root** or **state**. As already explained, state classes describe the properties of an object while it satisfies some conditions, whereas root classes abstract over state classes.⁷ Any subclass of a state or a root class must be a state class. Objects of a state class c may be re-classified to class c' , where c' must be a subclass of the uniquely defined root superclass of c .

A class specifies its superclass and declares a sequence of fields and methods. The type of fields may be either a primitive type or a non-state class; we call such types *non-state types*. Thus, fields may point to objects which *change class*, but these changes do *not affect* their type. In contrast, the type of identifiers (**this**, parameters and local variables) may be a state or root class.

Method declarations have the shape:

$$t \ m \ (t_1 \ x_1, \dots, t_q \ x_q) \ \{c_1, \dots, c_n\} \ block$$

where t is the result type, m the name, $t_1, \dots, t_q$ are the types of the formal parameters $x_1, \dots, x_q$, and *block* is the body. The effect consists of root classes $c_1, \dots, c_n$, with $n \geq 0$.

A block consists of a possibly empty sequence of local variable declarations and a sequence of expressions. Expressions include identifiers (that is, **this**, parameters and local variables), source language values (that is, constants of primitive types and **null**), field selection, casting, object creation, re-classification, assignment to a variable, assignment to a field, method call, conditional and block.

In object creation **new** c , c may be *any* class, including a state class. Re-classification expressions, $id !! c$, set the class of id to c – c must be a state or a root class.

We require the inheritance hierarchy to be a tree, root classes to extend only non-root and non-state classes, and state classes to extend either root classes or state classes.

3.2 Operational semantics

We give a structural operational semantics that rewrites pairs of expressions and stores into pairs of values, exceptions, or errors, and stores - in the context of a given program p . The signature of the rewriting relation $\rightsquigarrow$ is:

⁷ Notice that our proposal is orthogonal to the “abstract superclass rule” discussed in [15]. In fact, root classes are not necessarily abstract classes, and state classes may be superclasses only of other state classes.

$\frac{}{p, \gamma \vdash \mathbf{true} : \mathbf{bool} \parallel \gamma \parallel \{ \}}$ $\frac{}{p, \gamma \vdash \mathbf{false} : \mathbf{bool} \parallel \gamma \parallel \{ \}}$ $\frac{}{p, \gamma \vdash n : \mathbf{int} \parallel \gamma \parallel \{ \}}$ $\frac{}{p, \gamma \vdash id : \gamma(id) \parallel \gamma \parallel \{ \}}$	$\frac{p \vdash c \Diamond_{ct}}{p, \gamma \vdash \mathbf{null} : c \parallel \gamma \parallel \{ \}}$ $\frac{}{p, \gamma \vdash \mathbf{new} \ c : c \parallel \gamma \parallel \{ \}}$
$\frac{p, \gamma \vdash e : c' \parallel \gamma' \parallel \phi}{(p \vdash c' \leq c \text{ or } p \vdash c \leq c')} \quad \frac{}{p, \gamma \vdash (c)e : c \parallel \gamma' \parallel \phi}$	$\frac{p, \gamma \vdash e : c \parallel \gamma' \parallel \phi}{\mathcal{F}(p, c, f) = t} \quad \frac{}{p, \gamma \vdash e.f : t \parallel \gamma' \parallel \phi}$
$\frac{p, \gamma \vdash e : c \parallel \gamma_1 \parallel \phi_1 \quad p, \gamma_1 \vdash e_1 : t \parallel \gamma_2 \parallel \phi_2 \quad \mathcal{F}(p, \phi_2 @_p c, f) = t' \quad p \vdash t \leq t'}{p, \gamma \vdash e.f = e_1 : t \parallel \gamma_2 \parallel \phi_1 \cup \phi_2}$	$\frac{p, \gamma \vdash e : t \parallel \gamma' \parallel \phi \quad \gamma'(x) = t' \quad p \vdash t \leq t'}{p, \gamma \vdash x = e : t \parallel \gamma' \parallel \phi}$
$\frac{p, \gamma \vdash e : c \parallel \gamma' \parallel \phi}{p, \gamma \vdash \mathbf{isnull}(e) : \mathbf{bool} \parallel \gamma' \parallel \phi}$	
$\frac{p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0 \quad p, \gamma_{i-1} \vdash e_i : t_i \parallel \gamma_i \parallel \phi_i \ (i \in \{1, \dots, n\}) \quad \mathcal{M}(p, (\phi_1 \cup \dots \cup \phi_n) @_p c, m) = t \ m(t'_1 x_1, \dots, t'_n x_n) \ \phi \{ \dots \} \quad p \vdash (\phi_{i+1} \cup \dots \cup \phi_n) @_p t_i \leq t'_i \ (i \in \{1, \dots, n\})}{p, \gamma \vdash e.m(e_1, \dots, e_n) : t \parallel \phi @_p \gamma_n \parallel \phi \cup \phi_0 \cup \dots \cup \phi_n}$	
$\frac{p, \gamma \vdash e : \mathbf{bool} \parallel \gamma_0 \parallel \phi_0 \quad p, \gamma_0 \vdash e_1 : t_1 \parallel \gamma_1 \parallel \phi_1 \quad p, \gamma_0 \vdash e_2 : t_2 \parallel \gamma_2 \parallel \phi_2}{p, \gamma \vdash \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2 : t_1 \sqcup_p t_2 \parallel \gamma_1 \sqcup_p \gamma_2 \parallel \phi_0 \cup \phi_1 \cup \phi_2}$	
$\frac{\gamma_0 = \gamma[x_1 \mapsto t_1, \dots, x_s \mapsto t_s] \quad p, \gamma_{i-1} \vdash e_i : t'_i \parallel \gamma_i \parallel \phi_i \ (i \in \{1, \dots, n\})}{p, \gamma \vdash \{t_1 x_1; \dots t_s x_s; e_1; \dots e_n\} : t'_n \parallel \gamma_n[x_1 \mapsto \gamma(x_1), \dots, x_s \mapsto \gamma(x_s)] \parallel \phi_1 \cup \dots \cup \phi_n}$	
$\frac{p \vdash c \Diamond_{rt} \quad \mathcal{R}(p, c) = \mathcal{R}(p, \gamma(id))}{p, \gamma \vdash id !! c : c \parallel (\{ \mathcal{R}(p, c) \} @_p \gamma)[id \mapsto c] \parallel \{ \mathcal{R}(p, c) \}}$	

Fig. 3: *Fickle*_{II} – typing rules for expressions

$$\leadsto \quad : \quad p \longrightarrow e \times \text{store} \longrightarrow (\text{val} \cup \text{dev}) \times \text{store}$$

$$\begin{aligned} \text{store} &= (\{\text{this}\} \longrightarrow \text{addr}) \cup (x \longrightarrow \text{val}) \cup (\text{addr} \longrightarrow \text{object}) \\ \text{val} &= \text{sval} \cup \text{addr} \\ \text{dev} &= \{\text{nullPtrExc}, \text{castExc}, \text{stuckErr}\} \\ \text{object} &= \{ \llbracket f_1 : v_1, \dots, f_r : v_r \rrbracket^c \mid f_1, \dots, f_r \text{ are field identifiers,} \\ &\quad v_1, \dots, v_r \in \text{val}, \text{ and } c \text{ is a class name} \} \end{aligned}$$

The store is a finite partial function with finite domain, which maps **this** to an address, variables to values, and addresses to objects. Values are the source language values in Section 3.1, or addresses. Addresses may point to objects, but *not* to other addresses, primitive values, or **null**. Thus in *Fickle_{II}*, as in Java, pointers are implicit, and there are no pointers to pointers. As we will show, the execution of a well-typed expression does never produce a stuck error, although it may throw a null pointer exception or a cast exception. We denote stores with σ , and addresses with ι .

We need some operations on objects and stores.

For object $o = \llbracket f_1 : v_1, \dots, f_l : v_l, \dots, f_r : v_r \rrbracket^c$, store σ , value v , address ι , identifier or address z , field identifier f , value or object w , we define:

$$\begin{aligned} - \text{field access} \quad o(f) &= \begin{cases} v_l & \text{if } f = f_l \text{ for some } l \in 1, \dots, r, \\ \text{Udf} & \text{otherwise} \end{cases} \\ - \text{object update} \quad o[f \mapsto v] &= \begin{cases} \llbracket f_1 : v_1, \dots, f_l : v, \dots, f_r : v_r \rrbracket^c & \text{if } f = f_l \text{ for some } l \in 1, \dots, r, \\ \text{Udf} & \text{otherwise} \end{cases} \\ - \text{store update} \quad \sigma[z \mapsto w](z) &= w, \quad \sigma[z \mapsto w](z') = \sigma(z') \text{ if } z' \neq z. \end{aligned}$$

Also, we follow the convention that $\sigma(\iota)(f) = \text{Udf}$ whenever $\sigma(\iota) = \text{Udf}$.

Figures 14, 15, 16, and 17 from Appendix A list all the rewrite rules of *Fickle_{II}*. We discuss the two most significant rewrite rules of *Fickle_{II}*: method call and re-classification.

For method calls, $e.m(e_1, \dots, e_n)$, we evaluate the receiver e , obtaining an address, say ι . We then evaluate the arguments, $e_1, \dots, e_n$. We find the appropriate body by looking up m in the class of the object at address ι – we use the term $\mathcal{M}(p, c, m)$ that returns the definition of method m in class c going through the class hierarchy in p , if needed (see Appendix A). We execute the body after substituting **this** with the current object, and assigning to the formal parameters the values of the actual parameters. After the call, we restore the receiver and parameters to the values they had immediately before execution of the body.⁸

For re-classification expressions, $id !! d$, we find the address of id , which must point to an object, say of class c . We replace the original object by a new object of

⁸ We restore the references, but not the contents: thus, after a method call the side effects caused by execution of the method body survive. Note also that if one of the method parameters was undefined before the call, then it will be undefined after the call as well.

class d . We preserve the fields belonging to the root superclass of c and initialize the other fields of d according to their types. The term $\mathcal{R}(p, t)$, defined by

$$\mathcal{R}(p, t) = \begin{cases} c & \text{if } t \text{ is a state class and } c \text{ is the root superclass of } t \\ t & \text{otherwise,} \end{cases}$$

denotes the least superclass of t which is not a state class, if t is a class, and denotes t itself if t is not a class or a non-state class. Moreover, $\mathcal{F}s(p, c)$ denotes the set of all fields (either directly defined or inherited) of class c , and $\mathcal{F}(p, c, f)$ the type of field f in class c (see Appendix A). Note that we do not allow hiding of fields in $\mathcal{Fickle}_{\text{II}}$ ⁹

$$\begin{array}{l}
\mathcal{C}(p, c) = [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \dots \} \\
\forall f : \mathcal{FD}(p, c, f) = t_f \implies p \vdash t_f \diamond_{f_t} \quad \text{and} \quad \mathcal{F}(p, c', f) = \text{Udf} \\
\forall m : \mathcal{MD}(p, c, m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ \text{block} \implies \\
\quad p \vdash \phi \diamond \\
\quad p, t_1 \ x_1, \dots, t_n \ x_n, \ c \ \text{this} \vdash \text{block} : t' \parallel \gamma' \parallel \phi' \\
\quad p \vdash t' \leq t \\
\quad \phi' \subseteq \phi \\
\quad \mathcal{M}(p, c', m) = \text{Udf} \text{ or } (\mathcal{M}(p, c', m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi'' \{ \dots \} \text{ and } \phi \subseteq \phi'') \\
\hline
p \vdash c \diamond
\end{array}$$

$$\begin{array}{l}
\vdash p \diamond_h \\
\forall c : \mathcal{C}(p, c) \neq \text{Udf} \implies p \vdash c \diamond \\
\hline
\vdash p \diamond
\end{array}$$

Fig. 4: $\mathcal{Fickle}_{\text{II}}$ – rules for well-formed classes and programs

3.3 Typing

Widening, environments, effects The following assertions, defined in Fig. 19 of Appendix A, describe kinds of classes, and the widening relationship between types:

– $p \vdash c \diamond_{ct}$ means that c is any class,

⁹ In well-typed programs, $\mathcal{R}(p, c) = \mathcal{R}(p, d)$ always holds, and c and d must be state or root classes. This implies that re-classification depends only on the target class d , not on the class c of the receiver. Therefore, a compiler could fold the type information into the code, by generating specific re-classification code for each state class. The rule for re-classification uses the types of the fields to initialize the fields, similarly to the rule for object creation.

- $p \vdash c \diamond_{rt}$ means that c is a re-classifiable type, *i.e.*, either a root or a state class,
- $p \vdash t \diamond_{ft}$ means that t is a non-state type, *i.e.*, `bool`, or `int`, or a non-state class, and
- $p \vdash t \leq t'$ means that type t' widens type t , *i.e.*, t is a subclass of, or identical to, t' .

Environments, γ , map parameter names and local variables to types, and the receiver `this` to a class. They have the form $x_1 : t_1, \dots, x_n : t_n, \text{this} : c$. Lookup, $\gamma(id)$, and update, $\gamma[id \mapsto t]$, have the usual meaning, and are defined in Fig. 20 of Appendix A.

An effect, ϕ , is a set $\{c_1, \dots, c_n\}$ of root classes; it means that any object of a subclass of c_i (including c_i itself) may be re-classified. The empty effect, $\{\}$, guarantees that no object is re-classified. Effects are well-formed, *i.e.*, $p \vdash \{c_1, \dots, c_n\} \diamond$, iff $c_1, \dots, c_n$ are distinct root classes. Thus, $p \vdash \{c_1, \dots, c_n\} \diamond$ implies that c_i are not subclasses of each other.

$$\begin{array}{c}
\frac{v = \text{true or } v = \text{false}}{p, \sigma \vdash v \prec \text{bool}} \quad (\text{bool } \prec) \qquad \frac{v = n}{p, \sigma \vdash v \prec \text{int}} \quad (\text{int } \prec) \\
\\
\frac{p \vdash c \diamond_{ct}}{p, \sigma \vdash \text{null} \prec c} \quad (\text{null } \prec) \qquad \frac{\sigma(\iota) = [[\dots]]^c \quad p \vdash c \leq c'}{p, \sigma \vdash \iota \prec c'} \quad (\iota \prec) \\
\\
\frac{p, \sigma \vdash v \prec t \quad v \in \text{sval}}{p, \sigma \vdash v \triangleleft t} \quad (\text{sval } \triangleleft) \qquad \frac{\sigma(\iota) = [[\dots]]^c \quad p, \sigma \vdash \iota \prec c' \quad \forall f \in \mathcal{Fs}(p, c) : p, \sigma \vdash \sigma(\iota)(f) \prec \mathcal{F}(p, c, f)}{p, \sigma \vdash \iota \triangleleft c'} \quad (\iota \triangleleft) \\
\\
\frac{\sigma(\text{this}) \neq \text{null and } \sigma(\iota) = [[\dots]]^c \implies p, \sigma \vdash \iota \triangleleft c \quad (\text{for all addresses } \iota) \quad \gamma(id) \neq \text{Udf} \implies p, \sigma \vdash \sigma(id) \triangleleft \gamma(id) \quad (\text{for all identifiers } id)}{p, \gamma \vdash \sigma \diamond} \quad (\diamond) \\
\\
\frac{\sigma(\text{this}) = \sigma'(\text{this}) \quad \sigma(\iota) = [[\dots]]^c \implies \sigma'(\iota) = [[\dots]]^{c'}, \quad \phi @_p c = \phi @_p c'}{p, \phi \vdash \sigma \triangleleft \sigma'} \quad (\sigma \triangleleft)
\end{array}$$

Fig. 5: Agreement between programs, stores, and values

Typing rules The typing rules are given in Fig. 3. We use the look-up functions $\mathcal{F}$ and $\mathcal{M}$, which search for fields and methods through the class hierarchy (see Appendix A). In the rules, $t \sqcup_p t'$ is the least upper bound of t and t' w.r.t. the

$\leq$ relation between types in p , and $\gamma \sqcup_p \gamma'$ associates with an identifier the least upper bound in p of its types in γ and γ' . (The formal definitions can be found in Fig. 20 of Appendix A.) Moreover, we define the application of effects to types:

$$\{c_1, \dots, c_n\} @_p t = \begin{cases} c_i & \text{if } \mathcal{R}(p, t) = c_i \text{ for some } i \in 1, \dots, n \\ t & \text{otherwise} \end{cases}$$

Note that, if

$$p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$$

is a derivable judgment, then the environments γ , and γ' are defined for the same set of identifiers, and any differences in the types associated with an identifier in γ and γ' are due to the effect ϕ . So, if e is a $\mathcal{Fickle}_{\text{II}}^-$ expression, then $\gamma = \gamma'$ and $\phi = \{\}$.

Well-formed Programs A program is well formed (written $\vdash p \diamond$) if the inheritance hierarchy is well-formed ($\vdash p \diamond_h$) and all its classes are well-formed ($p \vdash c \diamond$): Methods may override superclass methods only if they have the same name, argument, and result type, and their effect is a subset of that of the overridden method. Method bodies must be well formed, return a value appropriate for the method signature, and their effect must be a subset of that in the signature. See Fig. 4, where $\mathcal{C}(p, c)$ returns the definition of class c in program p , and the look-up functions $\mathcal{FD}(p, c, f)$, $\mathcal{MD}(p, c, m)$ search for fields and methods only in class c (see Appendix A).

Soundness The main property of the type system is type soundness. This result will be needed in the proof of correctness of the translation.

In Fig. 5 we introduce the agreement relations between programs stores and values.

- $p, \sigma \vdash v < t$ means that value v has type t in p, σ . When the value is an address ι , then t must be a class type c , and the store σ must map ι to an object of class c whose fields have the right type. The definition of $p, \sigma \vdash v' < t'$ is needed to break the circularity of the definition, since the value of one of the fields of the object could be ι .
- $p, \gamma \vdash \sigma \diamond$ means that: for all addresses ι , if $\sigma(\iota) = [[\dots]]^c$ (c is the class of the object to which ι is mapped in σ , then ι has type c in p, σ (see the previous definition). Moreover, the value of the identifiers and of **this** in the store σ agree with their type in the environment γ .
- $p, \phi \vdash \sigma < \sigma'$ means that store σ' is the store that may be obtained from σ after the evaluation of an expression whose effects are ϕ . So the address bound to **this** is not changed, and only objects that may have being re-classified are the objects that have class which is a subclass of one of the classes in ϕ .

The following theorem asserts that the evaluation of a well-typed expression in a store that agrees with the typing environment, results either in an exception, or in a value that is of the right type. Moreover, the store resulting from the evaluation agrees with the environment resulting from the typing judgment. That is the evaluation of a well-typed expression cannot produce `stuckErr`. The proof of the theorem can be found in [12].

Theorem 1. *Let $p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$. If $p, \gamma \vdash \sigma \diamond$ and $e, \sigma \xrightarrow{p} w, \sigma'$, then*

- $w = v$, $p, \gamma' \vdash \sigma' \diamond$, $p, \sigma' \vdash v \triangleleft t$, and $p, \phi \vdash \sigma \triangleleft \sigma'$, or
- $w \in \{\text{castExc}, \text{nullPtrExc}\}$.

4 Translating $\mathcal{Fickle}_{\text{II}}$ into $\mathcal{Fickle}_{\text{II}}^-$: an informal overview

In this section we give an informal overview of the translation; we outline the encoding of objects (Section 4.1), and then discuss an example (Section 4.2).

4.1 Encoding of objects

The translation is based on the idea that each object o of a state class c can be encoded in $\mathcal{Fickle}_{\text{II}}^-$ by a pair $\langle id, imp \rangle$ of objects; we call id the *identity object of imp* and imp the *implementor object of id* . Roughly speaking, id provides the identity of o , and imp the behavior of o , so that any re-classification of o changes imp but not id and method invocations are resolved by imp . Hence, two implementors paired with the same identity represent the same object at different execution stages.

An object o which is not an instance of a state class does not need to be encoded in principle; however, for uniformity, the same kind of encoding described above is adopted also in this case, so that during the execution of a translated program there will be *exactly an identity object for any $\mathcal{Fickle}_{\text{II}}$ object*. Note that, while there could be more than one implementor encoding a $\mathcal{Fickle}_{\text{II}}$ object, say $\langle id, imp \rangle$ and $\langle id, imp' \rangle$, the converse cannot be true: if $\langle id, imp \rangle$ and $\langle id', imp \rangle$ are pairs encoding $\mathcal{Fickle}_{\text{II}}$ objects, then $id = id'$.

Re-classification of objects can be exemplified by the diagram in Fig. 6. As shown there, the identity object is the same during the lifetime of a $\mathcal{Fickle}_{\text{II}}$ object, whereas at different times the implementor object can change; the right implementor can always be recovered by the `imp` field of the identity object (dotted lines in the figure show obsolete values of this field).

Classes are translated according to the following two rules:

- each $\mathcal{Fickle}_{\text{II}}$ class (including `Object`) is translated into exactly one $\mathcal{Fickle}_{\text{II}}^-$ class (whose instances are implementors);
- the translation preserves the inheritance hierarchy.

We illustrate the above in terms of the classes in Example 1.

Example 1. The following $\mathcal{Fickle}_{\text{II}}$ program defines the classes `P`, `R`, `S1`, and `S2`.

Fig. 6: Re-classification of objects

```

class P extends Object
{ int f1;
  R m1(){R s; s = new S1;}
  int m2(S1 x){R}{x!!S2; 1}
  int m3 (S1 x){R} { x.m(this.m2(x));}
  int m4 (S1 x){R} { x.f1 = this.m2(x);}
  int m(int x){}{this.f1 = x; }
}

root class R extends P {
}

state class S1 extends R {
  int m4(S2 x){R}{this!!S2; this.f2 = x.f2;}
}

state class S2 extends R{
  int f2;
  int m(int x){}{this.f2 = x; }
}

```

After the instruction

```
s=new S1;
```

in the body of `m1`, the *Fickle_{II}* object referred by `s` is encoded in the translation, as sketched in Fig. 7, by the two *Fickle_{II}* objects `o` and `o1` in which the field `imp` of `o` points to `o1` and the field `id` of `o1` points to `o`.

The object `o1` has two fields: `id` of type *Identity* (inherited from class *FickleObject*, see Section 5.2), and `f1` (inherited from `P`). The fields `id` and `imp` are used to recover the identity and the implementor of an object, respectively. In this case the field `id` points to the object `o` of class *Identity* referred by `s`.

Fig. 7: Encoding of the *Fickle_{II}* object referred by *s*

Assume now that the object referred by *s* is re-classified to *S2*, *e.g.* through *s!!S2*. Then, a new object, *o2*, of class *S2* is created, and the field *imp* of the identity *o* points to the new object *o2*.

4.2 An example of translation

In this section we illustrate the translation of the classes in Example 1. Below we give the result of the translation of class *P*, together with the definitions¹⁰ of classes *Identity* and *FickleObject* which are added in the translation of any program.

```
class Identity extends Object{FickleObject imp;}
class FickleObject extends Object{Identity id;}
class P extends FickleObject{
  int f1;
  Identity m1(){}{
    Identity s;
    s = {
      Identity theId; S1 theImp; theId=new Identity; theImp=new S1;
      theImp.id=theId; theId.imp=theImp;
      theId
    }
  }
  int m2(Identity x){}
  {
    Identity theId; S2 theImp; R theLastImp;
```

¹⁰ the declarations are just the signatures in some cases

```

        if (isNull(theId=x)) then null
        else {
            theImp=new S2; theLastImp=(R)theId.imp; theImp.id=theId;
            theId.imp=theImp; theImp.f1=theLastImp.f1
        }
        theId
    }
    1
}
int m3(Identity x){} {
    {
        Identity theId; int arg;
        if (isNull(theId=x) then null.m(arg)
        else {
            arg={
                Identity theId1; Identity arg1;
                if (isNull(theId1= this.id)) then null.m2(arg1)
                else {arg1=x; ((P)(theId1.imp)).m2(arg1)}
            }
            ((R) (theId.imp)).m(arg);
        }
    }
}
int m4(Identity x){}{
    int rightval;
    rightval = {((P)(this.id.imp)).m2(x)}
    ((R) (x.imp)).f1 = rightval
}
int m(int x){this.id.imp.f1=x}
}

```

The translation maps the *Fickle*_{II} class P into the Java class P, which extends *FickleObject*, and hence inherits field *Identity id*.

We now consider the translation of method m1. First of all, note that type R in the result type of the method and in the local variable declaration is translated to *Identity*. Indeed, a property of the translation is that expressions of a class type are translated to expressions of type *Identity* (see Theorem 2).

The translation of the method body also demonstrates the encoding of the creation of a new object of class S1: As explained above, two new objects (the identity and the implementor) are created, which point to each other through their fields *imp* and *id*, respectively. Then, the identity object is returned.

The translation of method m2 demonstrates the encoding of the re-classification of the parameter *x* to S2. First of all, it is necessary to check whether *x* is *null*, since in this case the re-classification will have no effect. Otherwise, a new implementor object of class S2 is created and this implementor object and the identity object are made to point to each other. Moreover, all fields common to the new and old implementor object¹¹, are copied from the old (*theLastImp*) to the new (*theImp*) implementor object.

The translation of method m3 demonstrates the encoding of method calls. Consider the external method call *x.m(...)*. First, it is necessary to check that the receiver *x* is

¹¹ Common fields are those inherited from their most specific common superclass.

not null. Then, the argument (which is in turn a method call `this.m2(x)`) is evaluated. Finally, the current implementor, `theId.imp`, of the receiver is selected and the method `m` is invoked on it. Note, that the implementor can be correctly selected only after the evaluation of the argument, because this evaluation could re-classify the receiver object. This is exactly what happens in this case: namely, the receiver is reclassified from `S1` to `S2`, and thus the method `m` from class `S2` has to be executed; if we selected the implementor earlier, then the method `m` from class `P` would be executed instead, which contravenes the $\mathcal{Fickle}_{II}$ semantics. This is the reason for the introduction of the auxiliary variables `theId` and `arg`. For the same reason, there is a cast to `R` (indeed, since evaluation of the argument could re-classify the receiver, we can only assume that the implementor has type `R`).

The internal method call is translated in the same way; note that in this case the `isNull` test is useless, since the receiver is `this`, hence could be removed in an optimized translation. Note that the cast to `P` is necessary because the field `imp` has type `FickleObject`.

The translation of method `m4` demonstrates the encoding of field assignment. The schema is analogous to that for method calls, except that here we have optimized the translation omitting unnecessary blocks, local variables and `isNull` tests. However, as in method `m3`, it is necessary to select the implementor only after evaluation of the right hand side of the assignment, because this evaluation could also re-classify the object containing the field; otherwise, `1` would be assigned to the field of the old implementor.

The translation of the method `m` has been optimized too, as well as the translation of the classes `R`, `S1` and `S2` below:

```
class R extends P {
}

class S1 extends R {
  int m4(Identity x){}{
    {
      Identity theId; S2 theImp; R theLastThis; theId=this.id; theImp=new S2;
      theLastThis=(R)(theId.imp); theImp.id=theId;
      theId.imp=theImp;theImp.f1=theLastThis.f1;
      theId;
    }
    ((S2)this.id.imp).f2= ((S2)x.imp).f2;
  }
}

class S2 extends R{
  int f2;
  int m(int x){}{((S2)this.id.imp).f2 = x}
}
```

5 Translation of $\mathcal{Fickle}_{II}$ into $\mathcal{Fickle}_{II}^-$: a formal definition

In this section we give the formal definition of the translation.

5.1 Translation of programs

The translation of a $\mathcal{Fickle}_{II}$ program p consists of the declaration of the two special classes **FickleObject** and **Identity**, together with the translation of all classes declared in p . Since the translation of expressions depends on their types, the program p is passed as parameter to the translation function for classes.

$$\begin{aligned} \llbracket p \rrbracket_{prog} \triangleq & \text{class Identity extends Object}\{\text{FickleObject imp};\} \\ & \text{class FickleObject extends Object}\{\text{Identity id};\} \\ & \llbracket class_1 \rrbracket_{class}(p) \dots \llbracket class_n \rrbracket_{class}(p), \quad \text{where } p = class_1 \dots class_n \end{aligned}$$

For simplicity, here we are implicitly assuming no name conflicts between the classes and fields declared in p and the names **FickleObject**, **Identity** and **id**¹²; however, such conflicts could be always avoided by a slightly more complex translation where class names and fields are suitably renamed.

5.2 Translation of classes

As previously said, each translated class extends class **FickleObject**. An object o which needs to be re-classified to a state class c (recall that in the translation every class except for **Identity** is subclass of **FickleObject**), and which is encoded by the pair $\langle id, imp \rangle$, is transformed into $\langle id, imp' \rangle$, where imp' denotes the new implementor of class c (provided by a proper constructor of c ; see definition below). Fields are initialized so that the identity and the new implementor point to each other. We introduce the translation of types.

Definition 1. *Given a type t and a class c define:*

- *theType(t) = Identity if t is a class, and theType(t) = t otherwise, and*
- *theName(c) = FickleObject if $c = \text{Object}$, and theName(c) = c otherwise. I replaced c' by c*

Each $\mathcal{Fickle}_{II}$ class c is translated into a single $\mathcal{Fickle}_{II}^-$ class containing the translation of all field and method declarations of c .

The translation of fields and methods is the same for any kind of class. Since the translation of expressions depends on their types, the program p and the class c defining the type of **this** is passed as parameter to the translation function for methods.

$$\begin{aligned} \llbracket [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \text{field}_1 \dots \text{field}_m \text{ meth}_1 \dots \text{meth}_n \} \rrbracket_{class}(p) \triangleq \\ \text{class } c \text{ extends theName}(c') \{ \llbracket \text{field}_1 \rrbracket_{field} \dots \llbracket \text{field}_m \rrbracket_{field} \\ \llbracket \text{meth}_1 \rrbracket_{meth}(p, c) \dots \llbracket \text{meth}_n \rrbracket_{meth}(p, c) \\ \} \end{aligned}$$

5.3 Translation of field and variable (i.e. parameter or local variable) declarations

$$\begin{aligned} \llbracket t \rrbracket_{field} & \triangleq \text{theType}(t) \text{ } f \\ \llbracket t \rrbracket_{var} & \triangleq \text{theType}(t) \text{ } x \end{aligned}$$

¹² Field **imp** of class **Identity** does not conflict since no translated class extends **Identity**.

5.4 Translation of method declarations

Translating methods consists of translating their bodies. Effects are omitted, whereas the signatures remain the same. Since the translation of expressions depends on their types, the program p and the environment γ defining the type of the parameters and of **this** must be passed as argument to the corresponding translation functions.

$$\begin{aligned} \llbracket t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ block \rrbracket_{meth}(p, c) &\triangleq \\ &\text{theType}(t) \ m(\llbracket t_1 \ x_1 \rrbracket_{var}, \dots, \llbracket t_n \ x_n \rrbracket_{var}) \{ \} \llbracket block \rrbracket_{expr}(p, \gamma) \\ \text{where } \gamma &= t_1 \ x_1, \dots, t_n \ x_n, \ c \ \text{this} \end{aligned}$$

5.5 Translation of expressions

Values, variables, this, null test, field selection, and cast In our encoding, in order to access the current implementor of an object we have to select the implementor currently pointed to by the identity of the object.

$$\begin{aligned} \llbracket sval \rrbracket_{expr}(p, \gamma) &\triangleq sval \\ \llbracket x \rrbracket_{expr}(p, \gamma) &\triangleq x \\ \llbracket \text{this} \rrbracket_{expr}(p, \gamma) &\triangleq \text{this.id} \\ \llbracket \text{isnull}(e) \rrbracket_{expr}(p, \gamma) &\triangleq \text{isnull}(\llbracket e \rrbracket_{expr}(p, \gamma)) \\ \llbracket e.f \rrbracket_{expr}(p, \gamma) &\triangleq ((\text{theName}(c)) (\llbracket e \rrbracket_{expr}(p, \gamma).\text{imp})).f \\ \text{where } p, \gamma \vdash e : c \parallel \gamma' \parallel \phi. \end{aligned}$$

Downcasting to c is needed because field **imp** has type `FickleObject`.

$$\begin{aligned} \llbracket (c)e \rrbracket_{expr}(p, \gamma) &\triangleq \{ \text{Identity } x; \\ &\quad \text{if } (\text{isnull}(x = \llbracket e \rrbracket_{expr}(p, \gamma))) \\ &\quad \quad \text{then null} \\ &\quad \quad \text{else } ((\text{theName}(c))(x.\text{imp})).\text{id} \\ &\quad \} \\ \text{where } \gamma(x) &= \mathcal{U}df \end{aligned}$$

Variable assignment, field assignment, and method call Field f of the object denoted by the translation of e is accessed through the implementor of its identity. However, e_1 could re-classify the object, therefore the selection of field **imp** is correct only after evaluating the translation of e_1 . This is achieved by introducing auxiliary local variables. The same idea is applied to the translation of method call.

$$\begin{aligned} \llbracket x = e \rrbracket_{expr}(p, \gamma) &\triangleq x = \llbracket e \rrbracket_{expr}(p, \gamma) \\ \llbracket e.f = e_1 \rrbracket_{expr}(p, \gamma) &\triangleq \{ \text{Identity } x; \\ &\quad \text{theType}(t) \ x_1; \\ &\quad \text{if } (\text{isnull}(x = \llbracket e \rrbracket_{expr}(p, \gamma))) \\ &\quad \quad \text{then null.f} = x_1 \\ &\quad \quad \text{else } \{ x_1 = \llbracket e_1 \rrbracket_{expr}(p, \gamma_1); \\ &\quad \quad \quad ((\phi_2 @_p \text{theName}(c))(x.\text{imp})).f = x_1 \\ &\quad \quad \} \\ &\quad \} \end{aligned}$$

where $p, \gamma \vdash e : c \parallel \gamma_1 \parallel \phi_1$ and $p, \gamma_1 \vdash e_1 : t \parallel \gamma_2 \parallel \phi_2$, and $\gamma(x) = \gamma(x_1) = \mathcal{U}df$.

$$\begin{aligned}
\llbracket e.m(e_1, \dots, e_n) \rrbracket_{expr}(p, \gamma) \triangleq & \{ \text{Identity } x; \\
& \text{theType}(t_1) \ x_1; \\
& \dots \\
& \text{theType}(t_n) \ x_n; \\
& \text{if } (\text{isnull}(x = \llbracket e \rrbracket_{expr}(p, \gamma))) \\
& \quad \text{then null.m}(x_1, \dots, x_n) \\
& \quad \text{else } \{ x_1 = \llbracket e_1 \rrbracket_{expr}(p, \gamma_0); \\
& \quad \quad \dots \\
& \quad \quad x_n = \llbracket e_n \rrbracket_{expr}(p, \gamma_{n-1}); \\
& \quad \quad (((\phi_1 \cup \dots \cup \phi_n) @_p \text{theName}(c))(x.\text{imp})).m(x_1, \dots, x_n) \\
& \quad \} \\
& \}
\end{aligned}$$

where $p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0$, for all $i \in \{1, \dots, n\}$ $p, \gamma_{i-1} \vdash e_i : t_i \parallel \gamma_i \parallel \phi_i$,
and $\gamma(x) = \gamma(x_1) = \dots = \gamma(x_n) = \text{Udf}$.

Object creation and re-classification According to the *Fickle_{II}* semantics, only the fields of the root superclass are preserved by re-classification.

$$\begin{aligned}
\llbracket \text{new } c \rrbracket_{expr}(p, \gamma) \triangleq & \{ \text{theName}(c) \ \text{theImp}; \\
& \text{Identity } \text{theId}; \\
& \text{theId} = \text{new Identity}; \\
& \text{theImp} = \text{new theName}(c); \\
& \text{theImp.id} = \text{theId}; \\
& \text{theId.imp} = \text{theImp}; \\
& \text{theId} \\
& \} \\
\text{where } \gamma(\text{theImp}) = \text{Udf}, \gamma(\text{theId}) = \text{Udf}
\end{aligned}$$

$$\begin{aligned}
\llbracket \text{this}!!c; \rrbracket_{expr}(p, \gamma) \triangleq & \{ \text{Identity } \text{theId}; \\
& \text{theName}(c) \ \text{theImp}; \\
& \mathcal{R}(p, c) \ \text{theLastThis}; \\
& \text{theId} = \text{this.id}; \\
& \text{theImp} = \text{new theName}(c); \\
& \text{theLastThis} = (\mathcal{R}(p, c))(\text{theId.imp}); \\
& \text{theId.imp} = \text{theImp}; \\
& \text{theImp.id} = \text{theId}; \\
& \text{theImp.f}_1 = \text{theLastThis.f}_1; \\
& \dots \\
& \text{theImp.f}_r = \text{theLastThis.f}_r; \\
& \text{theId} \\
& \} \\
\text{where } \{f_1, \dots, f_r\} = \mathcal{Fs}(p, \mathcal{R}(p, c)) \text{ and } \gamma(\text{theImp}) = \gamma(\text{theId}) = \gamma(\text{theLastThis}) = \text{Udf}
\end{aligned}$$

$$\llbracket x!!c; \rrbracket_{expr}(p, \gamma) \triangleq \{ \text{Identity } theId; \\ theName(c) \text{ theImp}; \\ \mathcal{R}(p, c) \text{ theLastImp}; \\ \text{if } (isnull(theId = x)) \\ \text{then null} \\ \text{else } \{ \text{theImp} = \text{new } theName(c); \\ theLastImp = (\mathcal{R}(p, c))(theId.\text{imp}); \\ theId.\text{imp} = theImp; \\ theImp.\text{id} = theId; \\ theImp.f_1 = theLastImp.f_1; \\ \dots \\ theImp.f_r = theLastImp.f_r \\ \}; \\ theId \}$$

where $\{f_1, \dots, f_r\} = \mathcal{Fs}(p, \mathcal{R}(p, c))$ and $\gamma(theImp) = \gamma(theId) = \gamma(theLastImp) = \mathcal{Udf}$

Conditionals and blocks

$$\llbracket \text{if } e \text{ then } e_1 \text{ else } e_2 \rrbracket_{expr}(p, \gamma) \triangleq \\ \text{if } \llbracket e \rrbracket_{expr}(p, \gamma) \text{ then } \llbracket e_1 \rrbracket_{expr}(p, \gamma_0) \text{ else } \llbracket e_2 \rrbracket_{expr}(p, \gamma_0) \\ \text{where } p, \gamma \vdash e : \text{bool} \parallel \gamma_0 \parallel \phi_0$$

$$\llbracket \{t_1 \ x_1; \dots; t_s \ x_s; \ e_1; \dots; e_n\} \rrbracket_{expr}(p, \gamma) \triangleq \{ \llbracket t_1 \ x_1 \rrbracket_{var}; \\ \dots \\ \llbracket t_s \ x_s \rrbracket_{var}; \\ \llbracket e_1 \rrbracket_{expr}(p, \gamma_0); \\ \dots \\ \llbracket e_n \rrbracket_{expr}(p, \gamma_{n-1}) \}$$

where, $\gamma_0 = \gamma[x_1 \mapsto t_1, \dots, x_s \mapsto t_s]$ and for all $i \in \{1, \dots, n\}$ $p, \gamma_{i-1} \vdash e_i : t'_i \parallel \gamma_i \parallel \phi_i$

6 Properties of the translation

In this section we formalize and prove the good properties of the translation previously mentioned.

6.1 Preservation of static semantics

For any environment γ , its translation $\llbracket \gamma \rrbracket$ is defined by

$$\llbracket \gamma \rrbracket = \{ theType(t) \ x \mid \gamma(x) = t \} \cup \{ theName(\gamma(\text{this})) \ \text{this} \}.$$

Theorem 2. *Let p be a program s.t. $\vdash p \Diamond, \gamma$, and γ' environments s.t. $p \vdash \gamma(\text{this}) \Diamond_{ct}$, e an expression, t a type and ϕ an effect. If $p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$, then $\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e \rrbracket_{expr}(p, \gamma) : theType(t) \parallel \llbracket \gamma \rrbracket \parallel \{ \}$.*

Proof. See Appendix B.

Theorem 3. *Let p be a program s.t. $\vdash p \diamond$, and c a class name. If $p \vdash c \diamond$, then $\llbracket p \rrbracket_{prog} \vdash c \diamond$.*

Proof. See Appendix B.

Theorem 4. *Let p be a program. If $\vdash p \diamond$, then $\vdash \llbracket p \rrbracket_{prog} \diamond$.*

Proof. See Appendix B.

6.2 Preservation of dynamic semantics

In this section we show that the dynamic semantics of expressions is preserved by the translation.

We introduce a relation between stores $p, \gamma \vdash \sigma \approx \sigma'$ that expresses the fact that store σ' contains the “translation” of the objects in store σ . More precisely, an object o of class c in σ is translated in σ' into an object of class **Identity** whose **imp** field points to an implementor object o' which is an instance of the translation of the class c . Values of identifiers in σ are preserved in σ' , except for **this**, whose value in σ' is (the address of) an implementor object whose **id** field points to the address which is the value of **this** in σ . The store σ and σ' (except for **this**), are assumed to agree with the environments γ and $\llbracket \gamma \rrbracket$, that is, they contain values which agree, w.r.t. typing, with their definitions (see Fig. 5 for the formal definition of $p, \gamma \vdash \sigma \diamond$). Regarding the agreement of **this** in store σ' , observe that the store σ'' resulting from the evaluation of the translation of the re-classification of **this**, from class c to class d , is such that $\sigma''(\mathbf{this}) = \iota$ and $\sigma''(\iota) = \llbracket [\mathbf{id} : \iota' \dots] \rrbracket^c$, (the translation of the expression does not contain re-classifications, whereas the original expression did) and $\sigma''(\sigma''(\iota'))(\mathbf{imp}) = \llbracket [\mathbf{id} : \iota' \dots] \rrbracket^d$. So $\sigma''(\mathbf{this})$ does not agree with d but agrees with c which is the type of **this** before re-classification.

Definition 2. *Let p be a program, γ an environment, σ and σ' stores such that $p, \gamma \vdash \sigma \diamond$ and $\llbracket p \rrbracket, \llbracket \gamma \rrbracket[\mathbf{this} \mapsto c] \vdash \sigma' \diamond$, for some c . We say that σ' is the translation of σ , and write $p, \gamma \vdash \sigma \approx \sigma'$, if*

1. $\sigma(\mathbf{this}) = \sigma'(\sigma'(\mathbf{this}))(\mathbf{id})$
2. for all x , $\gamma(x) \neq \mathbf{Udf}$ implies $\sigma(x) = \sigma'(x)$, and
3. for all ι , if $\sigma(\iota) = \llbracket [f_1 : v_1, \dots, f_n : v_n] \rrbracket^c$ then
 - $\sigma'(\iota) = \llbracket [\mathbf{imp} : \iota'] \rrbracket^{\mathbf{id}}$,
 - $\sigma'(\iota') = \llbracket [\mathbf{id} : \iota, f_1 : v_1, \dots, f_n : v_n] \rrbracket^{\mathbf{theName}(c)}$.

for a suitable renaming of the addresses in σ' (or σ).

We can now state the theorem that asserts that our translation is adequate. The proof of the theorem is in Appendix C.

Theorem 5. *Let e be an expression such that: $p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$, and σ , and σ_1 , be stores such that $p, \gamma \vdash \sigma \approx \sigma_1$. Then*

$$e, \sigma \rightsquigarrow_p w, \sigma' \quad \text{if and only if} \quad \llbracket e \rrbracket, \sigma_1 \rightsquigarrow_{\llbracket p \rrbracket} w, \sigma'_1$$

where either

- $w = v$, and $p, \gamma' \vdash \sigma' \approx \sigma'_1$, or
- $w \in \{\mathbf{castExc}, \mathbf{nullPtrExc}\}$.

Proof. See Appendix C.

7 Four design alternatives

The design of the translation of *Fickle* is the outcome of several iterations. In this section we outline and compare these.

We first developed *Version_1*, which we implemented through *Carmela*, a Java program mapping *Fickle*st onto Java [4]. *Fickle*st is a statement oriented version of *Fickle*_{IT}. The development of *Carmela* proved more complex than anticipated, and we thus started a formal treatment, which we continued after the development of the software. This work led to *Version_2* [1]. We had then some further ideas for improvement, which lead to *Version_3* [2]. Finally, we developed *Version_4*, the approach described in this paper. *Version_4* has been implemented through *Isabella* [5], a Java program which maps *Fickle*st onto Java.

All translations are based on the idea of pairs of $\langle id, imp \rangle$ objects, which represent the identity and the implementor of the corresponding *Fickle* object. Starting from that basic idea, the following questions needed to be assessed:

- Would one translated object play both the role of the identity and implementor? Originally we allowed translated objects to take on the role of both the identity and implementor. This requires the object to have both `id` and `imp` fields. Later we separated the roles and each object was either an implementor or identity. Hence, each object contained either an `id` or `imp` field.
- Does the translation of a reference to a *Fickle* object refer to the identity or implementor part of the pair? Having the representation of the *Fickle* object point to the implementor allows for a type preserving translation. However, it also allows references to objects that are “outdated”. This reduces the garbage collection possibilities and increases the length of expressions required to reach the “active” implementation object. Referencing the identity simplifies expressions and allows for garbage collection of “outdated” objects.
- How are variables of state class type translated? In the original definition of *Fickle* only `this` could have state class type. Therefore, the early designs did not cater for that, and only the *Version_4* does.

Figures 8, 9, 10 and 11 show the representation of the *Fickle* objects before and after re-classification in *Version_1*, *Version_2*, *Version_3*, and *Version_4*, respectively. The bold arrows represent references that exist between *Fickle* objects, whereas the normal arrows represent references introduced by the translation. Figures 12 and 13 summarize the differences between the various translations.

7.1 *Version_1*

Fickle objects may be represented through one or through a pair of objects, depending on their history. Namely, objects that have not been re-classified are represented by a single object. Implementor and identity objects belong to state subclasses of the root class. This follows because an object that has not yet been re-classified is both the implementor and identity. Both the identity and implementor objects contain a field `id` and `imp`.

Access to members of all classes goes through the indirection of `id` and `imp`. References to *Fickle* objects are represented through references to the implementor object,

e.g., the variable `account` in Fig. 8. Therefore, during run-time of the translated program references to “outdated” objects are possible: again in Fig. 8, `account` refers to an outdated object after the re-classification.

The translation preserves types up to roots, *i.e.*, the translation of a *Fickle* expression of type t has type $\mathcal{R}(p, t)$. The translation is optimized in terms of type casts and number of objects created at run-time.

7.2 Version_2

Version_2 is a simplification over *Version_1*, in that *all Fickle* objects are represented through a pair of $\langle \text{id}, \text{imp} \rangle$ objects. In contrast to *Carmela*, implementor objects belong to state classes, and identity objects belong to root classes. As in *Version_1*, identity objects contain a field `id` and implementor objects contain a field `imp`.

Access to members of all classes goes through the indirection of `id` and `imp`. Type casts are required when accessing members because the `imp` field has type `FickleObject`. `FickleObject` is the superclass of all translated *Fickle* classes. References to *Fickle* objects are represented through references to the identity object. References to “outdated” objects are not possible. This opens more possibilities for garbage collection.

This translation preserves types up to roots. Overall the translation is simpler, but less efficient than *Version_1*.

Fig. 8: Objects in *Version_1*

7.3 Version_3

In *Version_3* we realized that we could achieve a significant simplification over *Version_2*, by representing identities through objects of the same class, regardless of the root class of the *Fickle* object. Thus, we introduced the class `Identity`, which has the field `imp`, pointing to the implementor of class `FickleObject`. The translated *Fickle* classes contain a field `id`, pointing to objects of type `Identity`.

Access to members of all classes goes through the indirection of `id` and `imp`. Type casts are required when accessing members from all classes. References to *Fickle* objects

Fig. 9: Objects in *Version_2*

are represented through references to the implementor object, and therefore references to “outdated” objects are possible.

The translation preserves types, *i.e.*, the translation of a *Fickle* expression of type $\mathbf{t}$ has type $\mathbf{t}$. Compared with *Version_2*, *Version_3* requires the same number of objects, but the identity objects are smaller than the corresponding objects of root class type in *Version_2*, as they only contain the field **imp**. Thus, the translation is simpler, and more efficient than *Version_2*.

7.4 *Version_4*

Finally, we realized that we could achieve a further simplification, by adopting the identity objects as they are in *Version_3*, and representing references to *Fickle* objects through references to identity objects.

Access to members of all classes goes through the indirection of **imp** only, thus, requiring fewer access than any other translation. Type casts are required for all field accesses and method calls. Access to members of the receiver (**this**) is different because **this** is the only *Fickle* entity that is represented by an implementor rather than an identity object. Because **this** may be re-classified during a method activation, access to its members goes through the indirection of both **id** and **imp**. As in *Version_2*, because references to *Fickle* objects are represented through references to the identity, references to “outdated” objects are not possible, and thus more possibilities for garbage collection are open.

The translation does not preserve types, *i.e.*, the translation of any *Fickle* expression of class type c has type **Identity**.

Comparing *Version_4* with *Version_3*, it requires the same number of objects, but fewer intermediate steps to represent field access and method call, and allows more opportunities to garbage collection. Therefore, *Version_4* combines simplicity with efficiency.

Fig. 10: Objects in *Version_3*

Fig. 11: Objects in *Version_4*

	<i>Version_1</i>	<i>Version_2</i>	<i>Version_3</i>	<i>Version_4</i>
Variables of type c are represented by variables of type:	$\mathcal{R}(p, c)$	$\mathcal{R}(p, c)$	$\mathcal{R}(p, c)$	Identity
References represented through reference to:	implementor	identity	implementor	identity
The id field has type:	$\mathcal{R}(p, c)$	FickleObject	Identity	Identity
The imp field has type:	$\mathcal{R}(p, c)$	FickleObject	FickleObject	FickleObject
Expression of type c translates to expression of type:	$\mathcal{R}(p, c)$	$\mathcal{R}(p, c)$	c	Identity
References to outdated objects	Possible	Impossible	Possible	Impossible

Fig. 12: Comparison of translation approaches

<i>Fickle</i> _{II}	<i>Version_1</i>	<i>Version_2</i>	<i>Version_3</i>	<i>Version_4</i>
A a	A a	A a	A a	Identity a
P p	P p	P p	P p	Identity p
a.i	a.i	((A)a.imp).i	((A)((A)a.id.imp)).i	((A)a.imp).i
this.o	((SA)this.id.imp).o	((SA)((A)this.id).imp).o	((P)((SA)this.id.imp)).o.id.imp	((SA)this.id.id.imp).o
a1 = a2	a1 = a2	a1 = a2	a1 = (A)(a2.id.imp)	a1 = a2
SA sa	--	--	--	Identity sa
sa.o	--	--	--	sa.imp.o

Fig. 13: Differences between the translation of expressions, with Account, SavingsAccount, Person, Account::interest, Account::owner represented by A, SA, P, i, o respectively. a1, a2 have type Account and this has type SavingsAccount.

8 *Isabella*, a prototype implementation

Our prototype implementation, *Isabella*, follows *Version_4* to map *Fickle*st onto Java. It is an extension of *Carmela* [4], which followed *Version_1*. *Isabella* is written in Java and follows a design based on the current Sun Java compiler (version 1.4). *Isabella* consists of a type checker and code generator, both implemented using the visitor pattern [14] as in the Java compiler. The whole compiler consists of approximately 6000 lines of code and can be found at WEBREF. *Isabella* extends *Fickle*st in order to make testing easier, the extensions include:

- Output via `System.out`
- Integers and booleans with relevant operations such as `++`, `--` etc.

One of the challenges in *Isabella* was representing the blocks of code that arise in the translation *Version_4*. Noting that Java does not allow blocks as expressions, any entity in the source that requires a block of code in the translation must be represented as a flattened statement and a fresh local variable to contain the result. The variable can then be used where the value of the block is required.

For example, consider the translation of `a = new DailyAccount (); a.transact ( 1200 ); a.interest();`. For expression, `a = new DailyAccount ()` we first translate `new DailyAccount` to get:

```
DailyAccount s1;
Identity s2;
s2= new Identity();
s1= new DailyAccount();
s1.id = s2;
s2.imp = s1;
```

Note the use of new temporary variables `s1` and `s2`, with the result being in variable `s2`. The assignment to variable `a` is represented as `a = s2`. For method call `a.transact(1200)` we have:

```

Identity s3 = a;
if (s3 == null) {
    throw new NullPointerException();
} else {
    int s5 = 1200;
    ((Account)(s3.imp)).transact (s5);
}

```

Note the use of temporary variables `s3` for the receiver and (assuming no `NullPointerException`) `s5` for the argument. As method `transact` is `void` we have no temporary variable for the return value. Finally for method call `a.interest()`; we have:

```

Identity s6 = a;int s7;
    if (s6 == null) {
        throw new NullPointerException();
    } else {
        s7 = ((Account)(s6.imp)).interest ();
    }

```

As with the previous method call we have a temporary variable for the receiver and in this case a temporary variable for the result `s7`.

Thus, the code produced by *Isabella* is identical to that produced by the translation given in this paper, apart from erasure of blocks and the extra features mentioned above.

9 Conclusions and Further Work

We have defined a translation from $Fickle_{II}$ into $Fickle_{II}^-$ (the subset of $Fickle_{II}$ without re-classification), and have proven that this translation is well-behaved in the sense that it preserves static and dynamic semantics. The translation described in this paper is the basis of *Isabella*, an implementation which maps $Fickle^{st}$ onto Java.

The proofs give us confidence in the correctness of *Isabella*, but they also gave us guidance in the design of the translation. Namely, we needed to define a correspondence between $Fickle_{II}$ objects and $Fickle_{II}^-$ objects, and also between $Fickle_{II}$ heaps and $Fickle_{II}^-$ heaps. These correspondences were necessary for the formulation of the theorems, but also expressed concepts that arose naturally as properties of the translation. We had already defined these concepts informally at the earlier versions, and in the later versions, their complete and formal definition guided the design of the translation of $Fickle_{II}$ class definitions and $Fickle_{II}$ expressions.

This work started when we decided to implement *Fickle*, through a translation into a Java. We expected the translation to be straightforward, and were surprised how many subtle points needed consideration, and how many alternative designs were possible. In fact, we now realize that the formalization of the translation and the formulation of the correspondences were indispensable. We conclude that any such translation tasks should not be attempted before a prior formalization of the languages, the translation and its invariants.

Except for the prototype implementations, we are interested in investigating the possibility of implementing an extension of Java with re-classification. From this point

of view, our translation is a good basis since it exhibits the following additional properties:

- It is fully compatible with Java separate compilation, since each $\mathcal{Fickle}_{\text{II}}$ class can be translated without having other class bodies. Therefore, it would be sufficient to have the other classes in binary form, as done by current Java compilers.
- The dependencies across classes are exactly those of standard Java compilation, in the sense that a $\mathcal{Fickle}_{\text{II}}$ class can be translated only if the type information from all the ancestor and all used classes is available.

Further work includes the extension of $\mathcal{Fickle}_{\text{II}}$ onto the full Java language. On the one hand, such an extension should take into account other Java features (like overloading) which, though in principle orthogonal to re-classification, should be carefully analyzed in order to be sure that the interaction behaves correctly. On the other hand, as mentioned above, an extended compiler should be able to work even in a context where only binary files are available, while our prototype implementation works on source files.

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A Operational semantics, definitions of lookup, subtypes, acyclic programs, and agreements

Figures 14, 15, 16, and 17 list all the rewrite rules of $\mathcal{Fickle}_{II}$.

Fig. 18 defines the judgment $\vdash p \diamond_u$, which guarantees that a program has unique definitions. In the judgment $defs$ is defined by

$$defs ::= (field \mid meth)^*$$

The first requirement says that there should be no more than one class definition for any identifier c – note that it implicitly guarantees $c' = c''$ and that the class bodies are identical. The second requirement says that there should be no more than one field definition in c for any identifier f – note that it implicitly guarantees $t = t'$. The third requirement says that there should be a unique method definition in c for any identifier m – note that it implicitly guarantees $t = t'$, $t_1 = t'_1, \dots, t_q = t'_q$, $x_1 = x'_1, \dots, x_q = x'_q$, $\phi = \phi'$, and $block = block'$.

For program p with $\vdash p \diamond_u$, class name $c \neq \mathbf{Object}$, and qualifier $qual = \mathbf{root}$, or $qual = \mathbf{state}$, or $qual = \epsilon$, we define the lookup of the class declaration for c :

$$\mathcal{C}(p, c) = \begin{cases} qual \text{ class } c \text{ extends } c'\{defs\} & \text{if } p = p' \text{ qual class } c \text{ extends } c'\{cBody\} p'', \\ Udf & \text{otherwise} \end{cases}$$

The assertion $p \vdash c \sqsubseteq c'$, defined in Fig. 19, means that the class c is a subclass of c' . The class hierarchy in a program p is well-formed, *i.e.*, $\vdash p \diamond_h$, if the subclass relationship is acyclic, root classes extend only non-root and non-state classes, and state classes extend either root classes or state classes. Notice that $\vdash p \diamond_u$ whenever $\vdash p \diamond_h$.

It is straightforward to prove the following properties of programs with well-formed inheritance hierarchies: Two types that are in the subclass relationship are classes, the relation $\sqsubseteq$ is reflexive, transitive and antisymmetric, and the subclass hierarchy forms a tree with $\mathbf{Object}$ at its root.

The following judgments, also defined in Fig. 19, distinguish the kinds of classes: $p \vdash c \diamond_{ct}$ means that c is any class, $p \vdash c \diamond_{rt}$ means that c is a re-classifiable type *i.e.*, either a root or a state class. The judgment $p \vdash t \diamond_{ft}$ means that t is a non-state type, *i.e.*, either a primitive type or a non-state class.

Widening, the extension of the subclass relationship to types, is expressed by the assertion $p \vdash t \leq t'$, and is also defined by the rules in Fig. 19.

Environment lookup and update, and the least upper bound operation on types and environments are defined in Fig. 20.

For program p with $\vdash p \diamond_h$, class name c such that

$$\mathcal{C}(p, c) = [\mathbf{root} \mid \mathbf{state}] \text{ class } c \text{ extends } c'\{defs\},$$

field name f and method name m we define:

$\frac{}{sv, \sigma \rightsquigarrow_p sv, \sigma} \quad (val)$	$\frac{e, \sigma \rightsquigarrow_p \text{null}, \sigma'}{(c)e, \sigma \rightsquigarrow_p \text{null}, \sigma'} \quad (n\text{-cast})$
$\frac{e, \sigma \rightsquigarrow_p \iota, \sigma' \quad \sigma'(\iota) = [[\dots]]^{c'} \quad p \vdash c' \leq c}{(c)e, \sigma \rightsquigarrow_p \iota, \sigma'} \quad (cast)$	$\frac{e, \sigma \rightsquigarrow_p \iota, \sigma' \quad \sigma'(\iota) = [[\dots]]^{c'} \quad p \not\vdash c' \leq c}{(c)e, \sigma \rightsquigarrow_p \text{castExc}, \sigma'} \quad (e\text{-cast})$
$\frac{\sigma(id) \neq \mathcal{U}df}{id, \sigma \rightsquigarrow_p \sigma(id), \sigma} \quad (id)$	$\frac{\sigma'(\iota)(f) \neq \mathcal{U}df \quad e, \sigma \rightsquigarrow_p \iota, \sigma' \quad \sigma', \sigma \rightsquigarrow_p \sigma'(\iota)(f), \sigma'}{e.f, \sigma \rightsquigarrow_p \sigma'(\iota)(f), \sigma'} \quad (field)$
$\frac{e, \sigma \rightsquigarrow_p \text{null}, \sigma'}{\text{isnull}(e), \sigma \rightsquigarrow_p \text{true}, \sigma'} \quad (t\text{-isnull})$	$\frac{e, \sigma \rightsquigarrow_p \iota, \sigma' \quad \sigma'(\iota) \neq \mathcal{U}df}{\text{isnull}(e), \sigma \rightsquigarrow_p \text{false}, \sigma'} \quad (f\text{-isnull})$
$\frac{e, \sigma \rightsquigarrow_p v, \sigma'}{x = e, \sigma \rightsquigarrow_p v, \sigma'[x \mapsto v]} \quad (a\text{-var})$	$\frac{e, \sigma \rightsquigarrow_p \iota, \sigma'' \quad e_1, \sigma'' \rightsquigarrow_p v, \sigma''' \quad \sigma'''(\iota)(f) \neq \mathcal{U}df \quad \sigma' = \sigma'''[\iota \mapsto \sigma'''(\iota)[f \mapsto v]]}{e.f = e_1, \sigma \rightsquigarrow_p v, \sigma'} \quad (a\text{-field})$
$\frac{e, \sigma \rightsquigarrow_p \text{true}, \sigma'' \quad e_1, \sigma'' \rightsquigarrow_p v, \sigma'}{\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma \rightsquigarrow_p v, \sigma'} \quad (t\text{-cond})$	$\frac{e, \sigma \rightsquigarrow_p \text{false}, \sigma'' \quad e_2, \sigma'' \rightsquigarrow_p v, \sigma'}{\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma \rightsquigarrow_p v, \sigma'} \quad (f\text{-cond})$
$\frac{v_l \text{ initial for } t_l \ (l \in \{1, \dots, s\}) \quad \sigma_0 = \sigma[x_1 \mapsto v_1, \dots, x_s \mapsto v_s] \quad e_i, \sigma_{i-1} \rightsquigarrow_p v_i, \sigma_i \ (i \in \{1, \dots, n\})}{\{t_1 x_1; \dots t_s x_s; e_1; \dots e_n\}, \sigma \rightsquigarrow_p v_n, \sigma_n[x_1 \mapsto \sigma(x_1), \dots, x_s \mapsto \sigma(x_s)]} \quad (block)$	
$\frac{\mathcal{F}s(p, c) = \{f_1, \dots, f_r\} \quad v_l \text{ initial for } \mathcal{F}(p, c, f_l) \ (l \in \{1, \dots, r\}) \quad \iota \text{ is new in } \sigma}{\text{new } c, \sigma \rightsquigarrow_p \iota, \sigma[\iota \mapsto [[f_1 : v_1, \dots, f_r : v_r]]^c]} \quad (new)$	
$\frac{e, \sigma \rightsquigarrow_p \iota, \sigma_0 \quad e_i, \sigma_{i-1} \rightsquigarrow_p v_i, \sigma_i \ (i \in \{1, \dots, n\}) \quad \sigma_n(\iota) = [[\dots]]^c \quad \mathcal{M}(p, c, m) = t \ m(t_1 x_1, \dots, t_n x_n) \ \phi \ block \quad \sigma' = \sigma_n[\text{this} \mapsto \iota, x_1 \mapsto v_1, \dots, x_n \mapsto v_n] \quad block, \sigma' \rightsquigarrow_p v, \sigma''}{e.m(e_1, \dots, e_n), \sigma \rightsquigarrow_p v, \sigma''[\text{this} \mapsto \sigma_n(\text{this}), x_1 \mapsto \sigma_n(x_1), \dots, x_n \mapsto \sigma_n(x_n)]} \quad (meth)$	
$\frac{e, \sigma \rightsquigarrow_p \text{null}, \sigma' \quad e.f = e_1, \sigma \rightsquigarrow_p \text{nullPtrExc}, \sigma' \quad e.f, \sigma \rightsquigarrow_p \text{nullPtrExc}, \sigma' \quad e.m(e_1, \dots, e_n), \sigma \rightsquigarrow_p \text{nullPtrExc}, \sigma'}{e.f = e_1, \sigma \rightsquigarrow_p \text{nullPtrExc}, \sigma'} \quad (e\text{-null})$	

Fig. 14: *Fickle*_{II} expression evaluation – without generation and propagation of errors

$\sigma(id) = \iota$		
$\sigma(\iota) = \llbracket \dots \rrbracket^c$		
$\mathcal{F}s(p, \mathcal{R}(p, c)) = \{f_1, \dots, f_r\}$		
$v_l = \sigma(\iota)(f_l) \quad (l \in \{1, \dots, r\})$	(recl)	(n-recl)
$\mathcal{F}s(p, d) \setminus \{f_1, \dots, f_r\} = \{f_{r+1}, \dots, f_{r+q}\}$		
$v_l \text{ initial for } \mathcal{F}(p, d, f_l) \quad (l \in \{r+1, \dots, r+q\})$		
$id !! d, \sigma \xrightarrow{\gamma} \iota, \sigma[\iota \mapsto \llbracket f_1 : v_1, \dots, f_{r+q} : v_{r+q} \rrbracket^d]$		$\frac{\sigma(id) = \text{null}}{id !! d, \sigma \xrightarrow{\gamma} \text{null}, \sigma'}$

Fig. 15: *Fickle*_{II} expression evaluation – without generation and propagation of errors

$$\begin{aligned}
\mathcal{FD}(p, c, f) &= \begin{cases} t & \text{if } \text{defs} = \dots t f \dots \\ \text{Udf} & \text{otherwise} \end{cases} \\
\mathcal{F}(p, c, f) &= \begin{cases} \mathcal{FD}(p, c, f) & \text{if } \mathcal{FD}(p, c, f) \neq \text{Udf}, \\ \mathcal{F}(p, c', f) & \text{otherwise} \end{cases} \\
\mathcal{F}(p, \text{Object}, f) &= \text{Udf} \\
\mathcal{F}s(p, c) &= \{f \mid \mathcal{F}(p, c, f) \neq \text{Udf}\} \\
\mathcal{MD}(p, c, m) &= \begin{cases} t m(t_1 \ x_1, \dots, t_n \ x_n) \phi \text{ body} & \text{if } \text{defs} = \dots t m(t_1 \ x_1 \dots t_n \ x_n) \phi \text{ body} \dots \\ \text{Udf} & \text{otherwise} \end{cases} \\
\mathcal{M}(p, c, m) &= \begin{cases} \mathcal{MD}(p, c, m) & \text{if } \mathcal{MD}(p, c, m) \neq \text{Udf}, \\ \mathcal{M}(p, c', m) & \text{otherwise} \end{cases} \\
\mathcal{M}(p, \text{Object}, m) &= \text{Udf}
\end{aligned}$$

B Proof of preservation of static semantics

We write $\gamma \subseteq \gamma'$ if for all id $\gamma(id) \neq \text{Udf} \Rightarrow \gamma(id) = \gamma'(id)$.

Lemma 1. *Let p be a program, γ, γ' two environments s.t. $\gamma \subseteq \gamma'$, e an expression, and t a type.*

If $p, \gamma \vdash e : t \parallel \gamma \parallel \{ \}$, then $p, \gamma' \vdash e : t \parallel \gamma' \parallel \{ \}$.

Proof. First, by induction on the typing rules for expressions the following claim can be proved:

$$(*) \text{ if } p, \gamma \vdash e : t \parallel \gamma' \parallel \{ \}, \text{ then } \gamma = \gamma'.$$

Then, the lemma is proved by induction on the typing rules using claim (*).

Lemma 2. *Let p be a program, γ, γ' two environments, e an expression, t a type, and ϕ an effect.*

If $p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$ is provable, then $\llbracket \gamma \rrbracket = \llbracket \gamma' \rrbracket$.

Proof. By induction on the typing rules for expressions.

Lemma 3. *Let p be a program s.t. $\vdash p \Diamond$, c, c' two class names s.t. $p \vdash c \leq c'$, f a field name, m a method name, $t, t_1, \dots, t_n$ types ($n \geq 0$), $x_1, \dots, x_n$ variables, and ϕ an effect. Then*

$\frac{\sigma(id) = \mathcal{U}df}{id, \sigma \xrightarrow{p} \text{stuckErr}, \sigma}$	$\frac{e, \sigma \xrightarrow{p} v, \sigma' \quad v \neq \text{null} \quad \sigma'(v) = \mathcal{U}df}{e.f, \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}$ $\frac{\text{isnull}(e), \sigma \xrightarrow{p} \text{stuckErr}, \sigma' \quad (c) e, \sigma \xrightarrow{p} \text{stuckErr}, \sigma' \quad e.f = e_1, \sigma \xrightarrow{p} \text{stuckErr}, \sigma' \quad e.m(e_1, \dots, e_n), \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}{\text{isnull}(e), \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}$
$\frac{e, \sigma \xrightarrow{p} \iota, \sigma' \quad \sigma'(\iota)(f) = \mathcal{U}df}{e.f, \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}$	$\frac{e, \sigma \xrightarrow{p} \iota, \sigma'' \quad e', \sigma'' \xrightarrow{p} v, \sigma' \quad \sigma'(\iota)(f) = \mathcal{U}df}{e.f = e_1, \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}$
$\frac{e, \sigma \xrightarrow{p} \iota, \sigma_0 \quad e_i, \sigma_{i-1} \xrightarrow{p} v_i, \sigma_i \quad (i \in \{1, \dots, q\}) \quad \sigma_1(\iota) = [[\dots]]^c \quad \mathcal{M}(p, c, m) = \mathcal{U}df \quad \text{or} \quad (\mathcal{M}(p, c, m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ \text{block} \ \text{and} \ n \neq q)}{e.m(e_1, \dots, e_q), \sigma \xrightarrow{p} \text{stuckErr}, \sigma_n}$	
$\frac{e, \sigma \xrightarrow{p} v, \sigma' \quad v \neq \text{true} \ \text{and} \ v \neq \text{false} \quad \text{if } e \text{ then } e_1 \text{ else } e_2, \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}{\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma \xrightarrow{p} \text{stuckErr}, \sigma'}$	$\frac{\mathcal{C}(p, c) = \mathcal{U}df}{\text{new } c, \sigma \xrightarrow{p} \text{stuckErr}, \sigma}$
$\frac{\sigma(id) = \mathcal{U}df \quad \text{or} \quad (\sigma(id) \neq \text{null} \ \text{and} \ \sigma(id) \notin \text{addr})}{id !! d, \sigma \xrightarrow{p} \text{stuckErr}, \sigma}$	

Fig. 16: *Fickle*_{II} expression evaluation – generation of errors

$$\begin{array}{c}
\frac{e, \sigma \rightsquigarrow_p dv, \sigma'}{(c)e, \sigma \rightsquigarrow_p dv, \sigma'} \\
x = e, \sigma \rightsquigarrow_p dv, \sigma' \\
e.f, \sigma \rightsquigarrow_p dv, \sigma' \\
\text{isnull}(e), \sigma \rightsquigarrow_p dv, \sigma' \\
e.m(e_1, \dots, e_n), \sigma \rightsquigarrow_p dv, \sigma' \\
e.f = e_1, \sigma \rightsquigarrow_p dv, \sigma'
\end{array}
\qquad
\begin{array}{c}
\frac{e, \sigma \rightsquigarrow_p \iota, \sigma''}{e_1, \sigma'' \rightsquigarrow_p dv, \sigma'} \\
e.f = e_1, \sigma \rightsquigarrow_p dv, \sigma'
\end{array}$$

$$\begin{array}{c}
e, \sigma \rightsquigarrow_p \iota, \sigma_0 \\
\sigma_0(\iota) \neq \text{Udf} \\
e_i, \sigma_{i-1} \rightsquigarrow_p v_i, \sigma_i \quad (i \in \{1, \dots, q\}, q < n) \\
\frac{e_{q+1}, \sigma_q \rightsquigarrow_p dv, \sigma_{q+1}}{e.m(e_1, \dots, e_n), \sigma \rightsquigarrow_p dv, \sigma_{q+1}}
\end{array}$$

$$\begin{array}{c}
e, \sigma \rightsquigarrow_p \iota, \sigma_0 \\
e_i, \sigma_{i-1} \rightsquigarrow_p v_i, \sigma_i \quad (i \in \{1, \dots, n\}) \\
\sigma_n(\iota) = [[\dots]]^c \\
\mathcal{M}(p, c, m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ \text{block} \\
\sigma' = \sigma_n[\text{this} \mapsto \iota, x_1 \mapsto v_1, \dots, x_n \mapsto v_n] \\
\frac{\text{block}, \sigma' \rightsquigarrow_p dv, \sigma''}{e.m(e_1, \dots, e_n), \sigma \rightsquigarrow_p dv, \sigma''[\text{this} \mapsto \sigma_n(\text{this}), x_1 \mapsto \sigma_n(x_1), \dots, x_n \mapsto \sigma_n(x_n)]}
\end{array}$$

$$\begin{array}{c}
e, \sigma \rightsquigarrow_p dv, \sigma' \\
\text{or} \quad (e, \sigma \rightsquigarrow_p \text{true}, \sigma'' \text{ and } e_1, \sigma'' \rightsquigarrow_p dv, \sigma') \\
\text{or} \quad (e, \sigma \rightsquigarrow_p \text{false}, \sigma'' \text{ and } e_2, \sigma'' \rightsquigarrow_p dv, \sigma') \\
\frac{}{\text{if } e \text{ then } e_1 \text{ else } e_2, \sigma \rightsquigarrow_p dv, \sigma'}
\end{array}$$

$$\begin{array}{c}
v_l \text{ initial for } t_l \quad (l \in \{1, \dots, s\}) \\
\sigma_0 = \sigma[x_1 \mapsto v_1, \dots, x_s \mapsto v_s] \\
e_i, \sigma_{i-1} \rightsquigarrow_p v_i, \sigma_i \quad (i \in \{1, \dots, q\}, q < n) \\
\frac{e_{q+1}, \sigma_q \rightsquigarrow_p dv, \sigma_{q+1}}{\{t_1 \ x_1; \dots t_s \ x_s; e_1; \dots e_n\}, \sigma \rightsquigarrow_p dv, \sigma_{q+1}[x_1 \mapsto \sigma(x_1), \dots, x_s \mapsto \sigma(x_s)]}
\end{array}$$

Fig. 17: *Fickle*_{II} expression evaluation – propagation of exceptions and errors

$$\begin{array}{l}
\forall c : \quad p = p_1 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \dots \} p_2, \\
\quad \quad p = p_3 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c'' \{ \dots \} p_4 \\
\quad \quad \quad \implies p_1 = p_3, p_2 = p_4 \\
\forall f : \quad p = p_1 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \text{defs}_1 \ t \ f \ \text{defs}_2 \} p_2, \\
\quad \quad p = p_1 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \text{defs}_3 \ t' \ f \ \text{defs}_4 \} p_2 \\
\quad \quad \quad \implies \text{defs}_1 = \text{defs}_3, \text{defs}_2 = \text{defs}_4; \\
\forall m : \quad p = p_1 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \text{defs}_1 \ t \ m \ (t_1 \ x_1, \dots, t_q \ x_q) \ \phi \ \text{block} \ \text{defs}_2 \} p_2, \\
\quad \quad p = p_1 [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \text{defs}_3 \ t' \ m \ (t'_1 \ x'_1 \ \dots, t'_n \ x'_n) \ \phi' \ \text{block}' \ \text{defs}_4 \} p_2 \\
\quad \quad \quad \implies \text{defs}_1 = \text{defs}_3, \text{defs}_2 = \text{defs}_4 \\
\hline
\vdash p \diamond_u
\end{array}$$

Fig. 18: Programs with unique definitions

$$\begin{array}{c}
\frac{\vdash p \diamond_u}{p \vdash \text{Object} \sqsubseteq \text{Object}} \quad \frac{\frac{\vdash p \diamond_u}{p = \dots [\text{root} \mid \text{state}] \text{ class } c \text{ extends } c' \{ \dots \} \dots} \quad \frac{p \vdash c \sqsubseteq c'}{p \vdash c \sqsubseteq c''}}{p \vdash c \sqsubseteq c'}
\end{array}$$

$$\begin{array}{l}
\forall c, c' : \\
p \vdash c \sqsubseteq c' \text{ and } p \vdash c' \sqsubseteq c \implies c = c' \\
\mathcal{C}(p, c) = \text{class } c \text{ extends } c' \{ \dots \} \implies \mathcal{C}(p, c') = \text{class } c' \dots \\
\mathcal{C}(p, c) = \text{root class } c \text{ extends } c' \{ \dots \} \implies \mathcal{C}(p, c') = \text{class } c' \dots \\
\mathcal{C}(p, c) = \text{state class } c \text{ extends } c' \{ \dots \} \implies \\
((\mathcal{C}(p, c') = \text{root class } c' \dots) \text{ or } (\mathcal{C}(p, c') = \text{state class } c' \dots)) \\
\hline
\vdash p \diamond_h
\end{array}$$

$$\begin{array}{ccc}
\frac{\vdash p \diamond_h}{\mathcal{C}(p, c) = \text{class } c \dots} & \frac{\vdash p \diamond_h}{\mathcal{C}(p, c) = \text{root class } c \dots} & \frac{\vdash p \diamond_h}{\mathcal{C}(p, c) = \text{state class } c \dots} \\
\frac{p \vdash c \diamond_{ft}}{p \vdash c \diamond_{ct}} & \frac{p \vdash c \diamond_{ft}}{p \vdash c \diamond_{rt}} & \frac{p \vdash c \diamond_{rt}}{p \vdash c \diamond_{ct}} \\
p \vdash c \diamond_{ct} & p \vdash c \diamond_{ct} &
\end{array}$$

$$\begin{array}{ccc}
\frac{}{p \vdash \text{bool} \diamond_{ft}} & \frac{}{p \vdash \text{bool} \leq \text{bool}} & \frac{p \vdash c \sqsubseteq c'}{p \vdash c \leq c'}
\end{array}$$

Fig. 19: Subclasses, well-formed inheritance hierarchy, subtypes

$$\frac{\gamma = x_1 : t_1, \dots, x_n : t_n, \mathbf{this} : c}{\gamma(\mathbf{id}) = \begin{cases} t_i & \text{if } \mathbf{id} = x_i \\ c & \text{if } \mathbf{id} = \mathbf{this} \\ \mathcal{Udf} & \text{otherwise} \end{cases}} \quad \gamma[\mathbf{id} \mapsto t](\mathbf{id}') = \begin{cases} t & \text{if } \mathbf{id}' = \mathbf{id} \\ \gamma(\mathbf{id}') & \text{otherwise} \end{cases}$$

$$t_1 \sqcup_p t_2 = \begin{cases} t & \text{if } p \vdash t_1 \leq t \quad p \vdash t_2 \leq t \quad \forall t'. (p \vdash t_1 \leq t' \text{ and } p \vdash t_2 \leq t') \Rightarrow p \vdash t \leq t' \\ \mathcal{Udf} & \text{otherwise} \end{cases}$$

$$\gamma \sqcup_p \gamma' = \{id : (t \sqcup_p t') \mid \gamma(id) = t \text{ and } \gamma'(id) = t'\}$$

Fig. 20: Environment lookup and update, lub on types and environments

1. $\mathcal{F}(p, c', f) = t \Rightarrow \mathcal{F}(p, c, f) = t$
2. $\mathcal{M}(p, c', m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ \{ \dots \} \Rightarrow$
 $\mathcal{M}(p, c, m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi' \ \{ \dots \}$ for some effect ϕ' .

Proof. By induction on the height of the inheritance tree.

Lemma 4. Let p be a program s.t. $\vdash p \diamond$, c a class name, f a field name, m a method name, $t, t_1, \dots, t_n$ types ($n \geq 0$), $x_1, \dots, x_n$ variables, and ϕ an effect. Then

1. $\mathcal{F}(p, c, f) = \mathcal{Udf} \Rightarrow \mathcal{F}(\llbracket p \rrbracket_{prog}, \mathit{theName}(c), f) = \mathcal{Udf}$;
2. $\mathcal{F}(p, c, f) = t \Rightarrow \mathcal{F}(\llbracket p \rrbracket_{prog}, \mathit{theName}(c), f) = \mathit{theType}(t)$;
3. $\mathcal{M}(p, c, m) = \mathcal{Udf} \Rightarrow \mathcal{M}(\llbracket p \rrbracket_{prog}, \mathit{theName}(c), m) = \mathcal{Udf}$;
4. $\mathcal{M}(p, c, m) = t \ m(t_1 \ x_1, \dots, t_n \ x_n) \ \phi \ \{ \dots \} \Rightarrow \mathcal{M}(\llbracket p \rrbracket_{prog}, \mathit{theName}(c), m) =$
 $\mathit{theType}(t) \ m(\mathit{theType}(t_1) \ x_1, \dots, \mathit{theType}(t_n) \ x_n) \ \{ \} \ \{ \dots \}$.

Proof. By induction on the height of the inheritance tree.

Lemma 5. Let p be a program s.t. $\vdash p \diamond$, and c a class name s.t. $p \vdash c \diamond_{ct}$. Then $\mathcal{F}(\llbracket p \rrbracket_{prog}, \mathit{theName}(c), \mathbf{id}) = \mathbf{Identity}$.

Proof. By induction on the height of the inheritance tree.

Lemma 6. Let p be a program s.t. $\vdash p \diamond$, and c a class name s.t. $p \vdash c \diamond_{ct}$. Then $\llbracket p \rrbracket_{prog} \vdash \mathit{theName}(c) \leq \mathbf{FickleObject}$.

Proof. By induction on the height of the inheritance tree.

Lemma 7. Let p be a program.
If $\vdash p \diamond$, then $\vdash \llbracket p \rrbracket_{prog} \diamond_h$.

Proof. By induction on the height of the inheritance tree.

Lemma 8. Let p be a program, and t, t' two types.
Then $p \vdash t \leq t'$ implies $\llbracket p \rrbracket_{prog} \vdash \mathit{theType}(t) \leq \mathit{theType}(t')$.

Proof. Trivial.

Lemma 9. *Let p be a program s.t. $\vdash p \Diamond$. Then $\llbracket p \rrbracket_{prog} \vdash \text{Identity} \Diamond_{ct}$.*

Proof. Trivial.

Lemma 10. *Let p be a program s.t. $\vdash p \Diamond$. Then $\mathcal{F}(\llbracket p \rrbracket_{prog}, \text{Identity}, \text{imp}) = \text{FickleObject}$.*

Proof. Trivial.

Lemma 11. *Let p be a program s.t. $\vdash p \Diamond$, t a type, and ϕ an effect. Then $p \vdash t \leq \phi @_p t$.*

Proof. Trivial.

Lemma 12. *Let p be a program s.t. $\vdash p \Diamond$, and t, t' two types. Then $\text{theType}(t \sqcup_p t') = \text{theType}(t) \sqcup_{\llbracket p \rrbracket_{prog}} \text{theType}(t')$.*

Proof. Trivial.

Proof of Theorem 2

Proof. The proof proceeds by induction on the typing rules (or, equivalently, on the structure of expressions) and by case analysis on the kinds of expressions and relies on the fact that the generation (or inversion) lemma for the typing relation, see [16], is trivial since there is a one-to-one relation between typing rules and kinds of expressions.

Cases $e \equiv \text{true}, \text{false}, n$ are trivial.

Case $e \equiv \text{id}$.

If $\text{id} \equiv x$, then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv x$ and by hypothesis $\gamma(x) = t$, therefore $\llbracket \gamma \rrbracket(x) = \text{theType}(t)$ and we can conclude by applying the suitable typing rule.

If $\text{id} \equiv \text{this}$, then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \text{this.id}$ and by hypothesis $\gamma(\text{this}) = t$, with $p \vdash t \Diamond_{ct}$ and $t \neq \text{Object}$; therefore $\text{theType}(t) = \text{Identity}$, $\text{theName}(t) = t$, $\llbracket \gamma \rrbracket(\text{this}) = t$, and Lemma 5 is applicable hence $\mathcal{F}(\llbracket p \rrbracket_{prog}, t, \text{id}) = \text{Identity}$; we conclude by applying the suitable typing rules.

Case $e \equiv \text{null}$.

Then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \text{null}$ and by hypothesis $p \vdash t \Diamond_{ct}$, therefore $\text{theType}(t) = \text{Identity}$ and by Lemma 9 $\llbracket p \rrbracket_{prog} \vdash \text{Identity} \Diamond_{ct}$, hence we can conclude by applying the suitable typing rule.

Case $e \equiv \text{isnull}(e')$.

Then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \text{isnull}(\llbracket e' \rrbracket_{expr}(p, \gamma))$ and by hypothesis $p, \gamma \vdash e' : c \parallel \gamma' \parallel \phi$, therefore by inductive hypothesis $\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e' \rrbracket_{expr}(p, \gamma) : \text{Identity} \parallel \llbracket \gamma' \rrbracket \parallel \{ \}$, hence we can conclude by applying the suitable typing rule.

Case $e \equiv \text{new } c$.

Then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \{$
 $\quad \text{theName}(c) \text{ theImp};$
 $\quad \text{Identity theId};$
 $\quad \text{theId} = \text{new Identity};$
 $\quad \text{theImp} = \text{new theName}(c);$
 $\quad \text{theImp.id} = \text{theId};$
 $\quad \text{theId.imp} = \text{theImp};$
 $\quad \text{theId}$
 $\quad \}$

where $theImp$ and $theId$ are chosen s.t. $\gamma(theImp) = \mathcal{U}df$, $\gamma(theId) = \mathcal{U}df$. By hypothesis $p \vdash t \diamond_{ct}$, therefore $theType(t) = \mathbf{Identity}$; furthermore by Lemmas 5, 10 and 6 it is possible to apply the suitable typing rules to derive the conclusion.

Case $e \equiv (c) e'$.

Then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \{ \mathbf{Identity} \ x;$
 $\quad \text{if } (\text{isnull}(x = \llbracket e' \rrbracket_{expr}(p, \gamma)))$
 $\quad \quad \text{then null}$
 $\quad \quad \text{else } ((theName(c))(x.\text{imp})).\text{id}$
 $\quad \}$

where x is chosen s.t. $\gamma(x) = \mathcal{U}df$. By hypothesis $p, \gamma \vdash e' : c' \parallel \gamma' \parallel \phi$ and $t = c$, therefore $theType(t) = \mathbf{Identity}$ and by inductive hypothesis

$\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e' \rrbracket_{expr}(p, \gamma) : \mathbf{Identity} \parallel \llbracket \gamma \rrbracket \parallel \{ \}$ and by Lemma 1,
 $\llbracket p \rrbracket_{prog}, \llbracket \gamma_0 \rrbracket \vdash \llbracket e' \rrbracket_{expr}(p, \gamma) : \mathbf{Identity} \parallel \llbracket \gamma_0 \rrbracket \parallel \{ \}$ where $\gamma_0 = \llbracket \gamma \rrbracket[x \mapsto \mathbf{Identity}]$;
 finally, by Lemmas 5, 9, 10 and 6, it is possible to apply the suitable typing rules in order to conclude.

Case $e \equiv e'.f$.

By hypothesis $p, \gamma \vdash e' : c \parallel \gamma' \parallel \phi$, then the translation is well-defined:

$\llbracket e \rrbracket_{expr}(p, \gamma) \equiv ((theName(c)) (\llbracket e' \rrbracket_{expr}(p, \gamma).\text{imp})).f$
 By inductive hypothesis $\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e' \rrbracket_{expr}(p, \gamma) : \mathbf{Identity} \parallel \llbracket \gamma \rrbracket \parallel \{ \}$, furthermore, by hypothesis $\mathcal{F}(p, c, f) = t$, therefore by Lemma 4, $\mathcal{F}(\llbracket p \rrbracket_{prog}, theName(c), f) = theType(t)$; finally, by Lemmas 10 and 6, it is possible to apply the suitable typing rules in order to conclude.

Case $e \equiv e_1.f = e_2$.

By hypothesis $p, \gamma \vdash e_1 : c \parallel \gamma_1 \parallel \phi_1$ and $p, \gamma_1 \vdash e_2 : t \parallel \gamma_2 \parallel \phi_2$, then the translation is well-defined:

$\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \{ \mathbf{Identity} \ x_1;$
 $\quad theType(t) \ x_2;$
 $\quad \text{if } (\text{isnull}(x_1 = \llbracket e_1 \rrbracket_{expr}(p, \gamma)))$
 $\quad \quad \text{then null.f} = x_2$
 $\quad \quad \text{else } \{ x_2 = \llbracket e_2 \rrbracket_{expr}(p, \gamma_1);$
 $\quad \quad \quad ((\phi_2 @_p theName(c))(x_1.\text{imp})).f = x_2$
 $\quad \quad \}$
 $\quad \}$

with x_1 and x_2 chosen s.t. $\gamma(x_1) = \gamma(x_2) = \mathcal{U}df$. By inductive hypothesis

$\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e_1 \rrbracket_{expr}(p, \gamma) : \mathbf{Identity} \parallel \llbracket \gamma \rrbracket \parallel \{ \}$ and
 $\llbracket p \rrbracket_{prog}, \llbracket \gamma_1 \rrbracket \vdash \llbracket e_2 \rrbracket_{expr}(p, \gamma_1) : theType(t) \parallel \llbracket \gamma_1 \rrbracket \parallel \{ \}$
 furthermore by Lemma 2, $\llbracket \gamma \rrbracket = \llbracket \gamma_1 \rrbracket$, and by Lemma 1,
 $\llbracket p \rrbracket_{prog}, \gamma_0 \vdash \llbracket e_1 \rrbracket_{expr}(p, \gamma) : \mathbf{Identity} \parallel \gamma_0 \parallel \{ \}$ and
 $\llbracket p \rrbracket_{prog}, \gamma_0 \vdash \llbracket e_2 \rrbracket_{expr}(p, \gamma_1) : theType(t) \parallel \gamma_0 \parallel \{ \}$
 with $\gamma_0 = \llbracket \gamma \rrbracket[x_1 \mapsto \mathbf{Identity}, x_2 \mapsto theType(t)]$.

Again, by hypothesis $\mathcal{F}(p, \phi_2 @_p c, f) = t'$ and $p \vdash t \leq t'$, and by definition $theName(\phi_2 @_p c) = \phi_2 @_p theName(c)$, therefore by Lemmas 4 and 8, $\mathcal{F}(\llbracket p \rrbracket_{prog}, \phi_2 @_p theName(c), f) = theType(t')$ and $\llbracket p \rrbracket_{prog} \vdash theType(t) \leq theType(t')$. Finally, by Lemmas 10 and 6, it is possible to apply the suitable typing rules in order to conclude.

Case $e \equiv x = e_1$.

Then $\llbracket e \rrbracket_{expr}(p, \gamma) \equiv x = \llbracket e_1 \rrbracket_{expr}(p, \gamma)$
 By hypothesis $p, \gamma \vdash e_1 : t \parallel \gamma' \parallel \phi$, $\gamma'(x) = t'$ and $p \vdash t \leq t'$, therefore by inductive hypothesis and Lemmas 2 and 8, $\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e_1 \rrbracket_{expr}(p, \gamma) : theType(t) \parallel \llbracket \gamma \rrbracket \parallel \{ \}$,

$\llbracket \gamma \rrbracket = \llbracket \gamma' \rrbracket$ hence $\llbracket \gamma \rrbracket(x) = \text{theType}(t')$, and $\vdash \text{theType}(t) \leq \text{theType}(t')$. Finally, it is possible to apply the suitable typing rule in order to conclude.

Case $e \equiv e_0.m(e_1, \dots, e_n)$.

By hypothesis, $p, \gamma \vdash e_0 : c \parallel \gamma_0 \parallel \phi_0$ and $p, \gamma_{i-1} \vdash e_i : t_i \parallel \gamma_i \parallel \phi_i$ for all $i = 1, \dots, n$, then the translation is well-defined:

```

 $\llbracket e \rrbracket_{\text{expr}}(p, \gamma) \equiv \{$ 
  Identity  $x$ ;
   $\text{theType}(t_1) \ x_1$ ;
  ...
   $\text{theType}(t_n) \ x_n$ ;
  if (isnull( $x = \llbracket e_0 \rrbracket_{\text{expr}}(p, \gamma)$ ))
  then null.m( $x_1, \dots, x_n$ )
  else  $\{ x_1 = \llbracket e_1 \rrbracket_{\text{expr}}(p, \gamma_0)$ ;
    ...
     $x_n = \llbracket e_n \rrbracket_{\text{expr}}(p, \gamma_{n-1})$ ;
     $((\phi_1 \cup \dots \cup \phi_n)@_p \text{theName}(c))(x.\text{imp})) . m(x_1, \dots, x_n)$ 
   $\}$ 
 $\}$ 

```

with $x, x_1, \dots, x_n$ chosen s.t. $\gamma(x) = \gamma(x_1) = \dots = \gamma(x_n) = \text{Udf}$. By Lemma 2,

$\llbracket \gamma \rrbracket = \llbracket \gamma_0 \rrbracket = \dots = \llbracket \gamma_n \rrbracket$, and by inductive hypothesis and Lemma 1,

$\llbracket p \rrbracket_{\text{prog}}, \gamma' \vdash \llbracket e_0 \rrbracket_{\text{expr}}(p, \gamma) : \text{Identity} \parallel \gamma' \parallel \{ \}$ and

$\llbracket p \rrbracket_{\text{prog}}, \gamma' \vdash \llbracket e_i \rrbracket_{\text{expr}}(p, \gamma_{i-1}) : \text{theType}(t_i) \parallel \gamma' \parallel \{ \}$ for all $i = 1, \dots, n$, with $\gamma' = \llbracket \gamma \rrbracket[x \mapsto \text{Identity}, x_1 \mapsto \text{theType}(t_1), \dots, x_n \mapsto \text{theType}(t_n)]$.

Again by hypothesis, $\mathcal{M}(p, (\phi_1 \cup \dots \cup \phi_n)@_p c, m) = t \ m(t'_1 \ x_1, \dots, t'_n \ x_n) \ \phi \{ \dots \}$ and $p \vdash (\phi_{i+1} \cup \dots \cup \phi_n)@_p t_i \leq t'_i$ for all $i = 1, \dots, n$, and by definition $\text{theName}((\phi_1 \cup \dots \cup \phi_n)@_p c) = (\phi_1 \cup \dots \cup \phi_n)@_p \text{theName}(c)$ and $\{ \} @_{\llbracket p \rrbracket_{\text{prog}}} \text{theType}(t_i) = \text{theType}(t'_i)$ for all $i = 1, \dots, n$, therefore by Lemmas 4, 8 and 11 and transitivity of the subtyping relation,

$\mathcal{M}(\llbracket p \rrbracket_{\text{prog}}, (\phi_1 \cup \dots \cup \phi_n)@_p \text{theName}(c), m) =$
 $\text{theType}(t) \ m(\text{theType}(t'_1) \ x_1, \dots, \text{theType}(t'_n) \ x_n) \ \{ \} \{ \dots \}$ and

$\llbracket p \rrbracket_{\text{prog}} \vdash \{ \} @_{\llbracket p \rrbracket_{\text{prog}}} \text{theType}(t_i) \leq \text{theType}(t'_i)$ for all $i = 1, \dots, n$. Finally, by Lemmas 10 and 6 it is possible to apply the suitable typing rules in order to conclude.

Case $e \equiv \text{id}!!c$.

Let us consider the case $\text{id} \equiv x$ (the case $\text{id} \equiv \text{this}$ is analogous); the translation is defined by

```

 $\llbracket e \rrbracket_{\text{expr}}(p, \gamma) \equiv \{$ 
  Identity  $\text{theId}$ ;
   $\text{theName}(c) \ \text{theImp}$ ;
   $\mathcal{R}(p, c) \ \text{theLastImp}$ ;
  if (isnull( $\text{theId} = x$ ))
  then null
  else  $\{$ 
     $\text{theImp} = \text{new } \text{theName}(c)$ ;
     $\text{theLastImp} = (\mathcal{R}(p, c))(\text{theId}.\text{imp})$ ;
     $\text{theId}.\text{imp} = \text{theImp}$ ;
     $\text{theImp}.\text{id} = \text{theId}$ ;
     $\text{theImp}.f_1 = \text{theLastImp}.f_1$ ;
    ...
     $\text{theImp}.f_r = \text{theLastImp}.f_r$ 
   $\}$ ;
   $\text{theId}$ 
 $\}$ 

```

with theId , theImp and theLastImp chosen s.t. $\gamma(\text{theImp}) = \gamma(\text{theId}) = \gamma(\text{theLastImp}) = \text{Udf}$ and $\{f_1, \dots, f_r\} = \mathcal{Fs}(p, \mathcal{R}(p, c))$.

By hypothesis, $p \vdash c \diamond_{rt} \mathcal{R}(p, c) = \mathcal{R}(p, \gamma(x))$. By definition, $f_i \in \mathcal{F}s(p, \mathcal{R}(p, c)) \Rightarrow \exists t \mathcal{F}(p, \mathcal{R}(p, c), f_i) = t$ for all $i = 1, \dots, r$; furthermore, since by definition $p \vdash c \leq \mathcal{R}(p, c)$, by Lemma 3, $\mathcal{F}(p, c, f_i) = t$ for all $i = 1, \dots, r$. Since by definition, $theName(\mathcal{R}(p, c)) = \mathcal{R}(p, c)$, by Lemma 4, $\mathcal{F}(\llbracket p \rrbracket_{prog}, \mathcal{R}(p, c), f_i) = \mathcal{F}(\llbracket p \rrbracket_{prog}, theName(c), f_i) = theType(t)$ for all $i = 1, \dots, r$. Finally, by Lemmas 5, 9, 10 and 6 the suitable typing rules can be applied in order to conclude.

Case $e \equiv \text{if } e_0 \text{ then } e_1 \text{ else } e_2$.

By hypothesis,

$p, \gamma \vdash e_0 : \text{bool} \parallel \gamma_0 \parallel \phi_0$

$p, \gamma_0 \vdash e_1 : t_1 \parallel \gamma_1 \parallel \phi_1$

$p, \gamma_0 \vdash e_2 : t_2 \parallel \gamma_2 \parallel \phi_2$

Then the translation is well-defined:

$\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \text{if } \llbracket e_0 \rrbracket_{expr}(p, \gamma) \text{ then } \llbracket e_1 \rrbracket_{expr}(p, \gamma_0) \text{ else } \llbracket e_2 \rrbracket_{expr}(p, \gamma_0)$

By Lemma 2, $\llbracket \gamma \rrbracket = \llbracket \gamma_0 \rrbracket = \llbracket \gamma_1 \rrbracket = \llbracket \gamma_2 \rrbracket$ and by inductive hypothesis,

$\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e_0 \rrbracket_{expr}(p, \gamma) : \text{bool} \parallel \llbracket \gamma \rrbracket \parallel \{ \}$

$\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e_1 \rrbracket_{expr}(p, \gamma_0) : theType(t_1) \parallel \llbracket \gamma \rrbracket \parallel \{ \}$

$\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash \llbracket e_2 \rrbracket_{expr}(p, \gamma_0) : theType(t_2) \parallel \llbracket \gamma \rrbracket \parallel \{ \}$

Finally, it is possible to conclude by applying the suitable typing rule and Lemma 12.

Case $e \equiv \{t_1 x_1; \dots t_s x_s; e_1; \dots e_n\}$.

By hypothesis,

$\gamma_0 = \gamma[x_1 \mapsto t_1, \dots, x_s \mapsto t_s]$

$p, \gamma_{i-1} \vdash e_i : t'_i \parallel \gamma_i \parallel \phi_i$ for all $i = 1, \dots, n$

Then the translation is well-defined:

$\llbracket e \rrbracket_{expr}(p, \gamma) \equiv \{ \llbracket t_1 x_1 \rrbracket_{var}; \dots \llbracket t_s x_s \rrbracket_{var}; \llbracket e_1 \rrbracket_{expr}(p, \gamma_0); \dots \llbracket e_n \rrbracket_{expr}(p, \gamma_{n-1}) \}$

By Lemma 2, $\llbracket \gamma_0 \rrbracket = \dots \llbracket \gamma_n \rrbracket$ and by translation of variable declarations and by inductive hypothesis,

$\llbracket p \rrbracket_{prog}, \llbracket \gamma_0 \rrbracket \vdash \llbracket e_i \rrbracket_{expr}(p, \gamma_{i-1}) : theType(t'_i) \parallel \llbracket \gamma_0 \rrbracket \parallel \{ \}$ for all $i = 1, \dots, n$

where, by definition, $\llbracket \gamma_0 \rrbracket = \llbracket \gamma \rrbracket[x_1 \mapsto theType(t_1), \dots, x_s \mapsto theType(t_s)]$. Finally, it is possible to conclude by applying the suitable typing rule.

Proof of Theorem 3

Proof. Let us take an arbitrary class named c and defined in p (recall that in the translation of programs - see comments in Section 5.1 - we assume no name conflicts, so that $c \neq \text{Identity}, \text{FickleObject}$ and $f \neq \text{id}$ for any field f declared in p); by translation of class and field declarations, for any type t'_f , and field name $f \neq \text{id}$, $\mathcal{FD}(\llbracket p \rrbracket_{prog}, c, f) = t'_f$ implies $\mathcal{FD}(p, c, f) = t_f$, for a type t_f s.t. $theType(t_f) = t'_f$. By definition of $theType$, $\llbracket p \rrbracket_{prog} \vdash t'_f \diamond_{ft}$; furthermore, by hypothesis, $\mathcal{F}(p, c', f) = \mathcal{Udf}$, therefore, by Lemma 4, $\mathcal{F}(\llbracket p \rrbracket_{prog}, theName(c'), f) = \mathcal{Udf}$.

By translation of class and method declarations, for all types $t'_0, \dots, t'_n$ ($n \geq 0$), method name m , variables $x_1, \dots, x_n$, effect ϕ_0 , and block $block'$,

$\mathcal{MD}(\llbracket p \rrbracket_{prog}, c, m) = t'_0 \ m(t'_1 x_1, \dots, t'_n x_n) \ \phi_0 \ block' \Rightarrow$

$\phi_0 = \{ \}$ and $\mathcal{MD}(p, c, m) = t \ m(t_1 x_1, \dots, t_n x_n) \ \phi \ block$,

for types $t, t_1, \dots, t_n$, s.t. $theType(t) = t'_0$, $theType(t_i) = t'_i$ for $i = 1, \dots, n$, an effect ϕ , and a block $block$ s.t. $\llbracket block \rrbracket_{expr}(p, \gamma) = block'$, with $\gamma = t_1 x_1, \dots, t_n x_n, c \ \text{this}$.

Therefore $p \vdash \phi_0 \diamond$ trivially holds, $\llbracket p \rrbracket_{prog}, \llbracket \gamma \rrbracket \vdash block' : theType(t') \parallel \llbracket \gamma \rrbracket \parallel \{ \}$ is provable by hypothesis and Theorem 2, $\llbracket p \rrbracket_{prog} \vdash theType(t') \leq t'_0$ holds by hypothesis and Lemma 8 and trivially $\{ \} \subseteq \phi_0$, therefore we can conclude by applying the typing rule for class declarations.

Proof of Theorem 4

Proof. By Lemma 7, $\vdash \llbracket p \rrbracket_{prog} \Diamond_h$; by definition of **Identity** and **FickleObject**, and by the typing rules, $\llbracket p \rrbracket_{prog} \vdash \mathbf{Identity} \Diamond$ and $\llbracket p \rrbracket_{prog} \vdash \mathbf{FickleObject} \Diamond$. Moreover, every class $c \neq \mathbf{Identity}, \mathbf{FickleObject}$ defined in $\llbracket p \rrbracket_{prog}$ is defined also in p (by definition of translation of programs and classes), and by Theorem 3 we have $\llbracket p \rrbracket_{prog} \vdash c \Diamond$. Hence we can conclude the result from the typing rule for programs.

C Proof of preservation of dynamic semantics

Lemma 13. *Let p be a program s.t. $\vdash p \Diamond$, c, c' two class names. If $p \vdash c \leq c'$, then $p \vdash \mathit{theName}(c) \leq \mathit{theName}(c')$.*

Proof. If both c and c' are different from **Object** then it is obvious since $\mathit{theName}(c) = c$ and $\mathit{theName}(c') = c'$. If either one is equal to **Object** then since $p \vdash c \leq c'$ it must be $c' = \mathbf{Object}$. Therefore, $\mathit{theName}(c') = \mathbf{FickleObject}$ and for all classes d we have that $p \vdash d \leq \mathbf{FickleObject}$.

Lemma 14. *Let t be a type. If v is the initial value of type t then v is also the initial value of type $\mathit{theType}(t)$.*

Proof. If t is a class type then $v = \mathbf{null}$ which is initial also for **Identity**, otherwise t is a primitive type and $\mathit{theType}(t) = t$.

Lemma 15. *Let e be an expression such that: $p, \gamma \vdash e : t \parallel \gamma' \parallel \phi$, and σ be a store such that $\llbracket p \rrbracket, \llbracket \gamma \rrbracket[\mathbf{this} \mapsto c] \vdash \sigma \Diamond$, for some c . If*

$$\llbracket e \rrbracket, \sigma \sim_{\llbracket p \rrbracket} w, \sigma'$$

then for all x , $\gamma(x) = \mathbf{Udf}$ implies $\sigma'(x) = \sigma(x)$.

Proof. By induction on the depth of the derivation tree of $\llbracket e \rrbracket, \sigma \sim_{\llbracket p \rrbracket} w, \sigma'$. For the translation of the expressions corresponding to values, variables, **this**, null test, and field selection $\llbracket e \rrbracket$ does not contain assignments, therefore for all x , $\sigma(x) = \sigma'(x)$.

For the translation of assignment, note that for $x = e'$ to be well-typed, it must be $\gamma(x) \neq \mathbf{Udf}$. So since $\llbracket e \rrbracket \stackrel{\Delta}{=} x = \llbracket e' \rrbracket$, the result is by induction hypothesis on $\llbracket e' \rrbracket$.

For all the other expressions note that the translation of the expression is a block in which the local variables y are such that $\gamma(y) = \mathbf{Udf}$. From the operational semantics of blocks we have that for all the local variables y , $\sigma'(y) = \sigma(y)$ (for all the local variables the value before execution of the block is restored). So the result is by induction hypotheses on the subexpressions.

Proof of Theorem 5

We will prove the two implications separately.

Proof of: $e, \sigma \xrightarrow[p]{\sim} w, \sigma'$ *implies* $\llbracket e \rrbracket, \sigma_1 \xrightarrow[p]{\sim} w, \sigma'_1$ By induction on the depth of the derivation tree of the judgement: $e, \sigma \xrightarrow[p]{\sim} w, \sigma'$. Note that, since e is well-typed from Theorem 1 its evaluation cannot produce **stuckErr** so the result is either a value or $w \in \{\text{castExc}, \text{nullPtrExc}\}$, and the rules that may have been used are the rules in Fig. 14, Fig. 15, and Fig. 17 where $dv \in \{\text{castExc}, \text{nullPtrExc}\}$.

We will consider a subset of the rules applied. Namely: (id) , (new) , $(recl)$, $(meth)$, $(cast)$, $(n-cast)$, $(e-null)$, and $(e-cast)$. The other cases are similar, and for the propagation rules the result is derived directly from the inductive hypothesis.

Consider rule (id) : $\text{id}, \sigma \xrightarrow[p]{\sim} \sigma(\text{id}), \sigma$. Since $\llbracket \text{id} \rrbracket_{\text{expr}}(p, \gamma) \triangleq \text{id}$ we derive that $\llbracket \text{id} \rrbracket, \sigma_1 \xrightarrow[p]{\sim} \sigma_1(\text{id}), \sigma_1$. Moreover, $p, \gamma \vdash \sigma \approx \sigma_1$ implies that $\sigma(\text{id}) = \sigma_1(\text{id})$.

Consider rule (new) :

$$\text{new } c, \sigma \xrightarrow[p]{\sim} \iota, \sigma[\iota \mapsto \llbracket f_1 : v_1, \dots, f_r : v_r \rrbracket^c]$$

where ι is new in σ and v_l is initial for $\mathcal{F}(p, c, f_l)$, $1 \leq l \leq r$. Assume that $c \neq \text{Object}$, so $\text{theName}(c) = c$. (The proof for the case $c = \text{Object}$ is simpler since $\mathcal{F}s(p, c) = \emptyset$.) From the definition of the translation $\llbracket \text{new } c \rrbracket(p, \gamma) \triangleq \{c \text{ theImp}; \text{Identity theId}; e_1; e_2; e_3; e_4; \text{theId}\}$ where

- e_1 is $\text{theId} = \text{new Identity}$,
- e_2 is $\text{theImp} = \text{new } c$,
- e_3 is $\text{theImp.id} = \text{theId}$, and
- e_4 is $\text{theId.imp} = \text{theImp}$.

and $\gamma(\text{theId}) = \gamma(\text{theImp}) = \text{Udf}$.

Let $\sigma_1'' = \sigma_1[\text{theImp} \mapsto \text{null}, \text{theId} \mapsto \text{null}]$. From the definition of the operational semantics of blocks, and the fact that **null** is the initial value for an object, we have that

- $e_1, \sigma_1'' \xrightarrow[p]{\sim} \iota, \sigma_2$ where we assume that ι is new also in σ_1 (otherwise we could rename all its occurrences in σ_1 with a new address), and $\sigma_2 = \sigma_1''[\iota \mapsto \llbracket \text{imp} : \text{null} \rrbracket^{\text{Id}}, \text{theId} \mapsto \iota]$
- $e_2, \sigma_2 \xrightarrow[p]{\sim} \iota', \sigma_3$ where $\iota' \neq \iota$ is new in σ_1 (we also assume that ι' does not appear in σ), and, since c extends **FickleObject**, $\sigma_3 = \sigma_2[\iota' \mapsto \llbracket \text{id} : \text{null}, f_1 : v'_1, \dots, f_r : v'_r \rrbracket^c, \text{theImp} \mapsto \iota']$ and v'_l is initial for $\text{theType}(\mathcal{F}(p, c, f_l))$, $1 \leq l \leq r$,
- $e_3; e_4, \sigma_3 \xrightarrow[p]{\sim} \iota, \sigma_1'''$, where σ_1''' is $\sigma_1[\iota' \mapsto \llbracket \text{id} : \iota, f_1 : v'_1, \dots, f_r : v'_r \rrbracket^c, \text{theImp} \mapsto \iota', \iota \mapsto \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}}, \text{theId} \mapsto \iota]$.

Let $\sigma'_1 = \sigma_1'''[\text{theImp} \mapsto \sigma_1(\text{theImp}), \text{theId} \mapsto \sigma_1(\text{theId})]$

$$\llbracket \text{new } c \rrbracket, \sigma_1 \xrightarrow[p]{\sim} \iota, \sigma'_1$$

Let $\sigma' = \sigma[\iota \mapsto \llbracket f_1 : v_1, \dots, f_r : v_r \rrbracket^c]$. For all x , $\sigma'(x) = \sigma(x)$, and $\sigma'_1(x) = \sigma_1(x)$ (by definition of σ'_1). Therefore, since $p, \gamma \vdash \sigma \approx \sigma_1$ clauses 1. and 2. of Definition 2 hold for σ' and σ'_1 . Moreover, for all $\iota'', \iota'' \neq \iota$, $\sigma'(\iota'') = \sigma(\iota'')$ and $\iota'' \neq \iota'$, implies also $\sigma'_1(\iota'') = \sigma_1(\iota'')$. So we have to consider only ι .

- $\sigma'(\iota) = \llbracket f_1 : v_1, \dots, f_r : v_r \rrbracket^c$,
- $\sigma'_1(\iota) = \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}}$, $\sigma'_1(\iota') = \llbracket \text{id} : \iota, f_1 : v'_1, \dots, f_r : v'_r \rrbracket^c$

Since we assumed that $\sigma(\iota') = Udf$, and from Lemma 14, we derive that $v'_i = v_i$, $1 \leq i \leq r$, then also clause 3. of Definition 2 is verified, and we conclude that $p, \gamma' \vdash \sigma' \approx \sigma'_1$.

Consider rule (*recl*), and assume that the expression is **this**!! d , therefore:

$$\begin{array}{l}
\sigma(\mathbf{this}) = \iota \\
\sigma(\iota) = [[\dots]]^c \\
\mathcal{Fs}(p, \mathcal{R}(p, c)) = \{f_1, \dots, f_r\} \\
v_l = \sigma(\iota)(f_l) \quad (l \in \{1, \dots, r\}) \\
\mathcal{Fs}(p, d) \setminus \{f_1, \dots, f_r\} = \{f_{r+1}, \dots, f_{r+q}\} \\
v_l \text{ initial for } \mathcal{F}(p, d, f_l) \quad (l \in \{r+1, \dots, r+q\}) \\
\hline
\mathbf{this}!! d, \sigma \rightsquigarrow_{\mathcal{F}} \iota, \sigma[\iota \mapsto [[f_1 : v_1, \dots, f_{r+q} : v_{r+q}]]^d]
\end{array} \tag{1}$$

Since the expression is well-typed, and **Object** is not a root or state class, we have that $d \neq \mathbf{Object}$, and so $theName(d) = d$. From the definition of the translation we have:

$$[[\mathbf{this}!! d]](p, \gamma) \triangleq \{d \text{ theImp}; \mathbf{Identity} \text{ theId}; \mathcal{R}(p, d) \text{ theLastThis}; e_1; e_2; e_3; e_4; e'_1; \dots; e'_r; \text{theId}\}$$

where

- e_1 is $theId = \mathbf{this.id}$,
- e_2 is $theLastThis = (\mathcal{R}(p, d))theId.\mathbf{imp}$,
- e_3 is $theImp = \mathbf{new} d$,
- e_4 is $theImp.id = theId$,
- e_5 is $theId.\mathbf{imp} = theImp$,
- e'_i is $theImp.f_i = theLastThis.f_i$ for $1 \leq i \leq r$.

and $\gamma(theId) = \gamma(theLastThis) = \gamma(theImp) = Udf$. From the fact that $p, \gamma \vdash \sigma \approx \sigma_1$, and rule (1) we have that:

$$\begin{array}{l}
\sigma_1(\mathbf{this}) = \iota' \quad \sigma_1(\iota') = [[\mathbf{id} : \iota, \dots]]^{c'} \quad \sigma_1(\iota) = [[\mathbf{imp} : \iota'']]^{\mathbf{Id}} \\
\sigma_1(\iota'') = [[\mathbf{id} : \iota, f_1 : v_1, \dots, f_r : v_r, f_{r+1} : v'_1, \dots, f_{r+p} : v'_p]]^c
\end{array} \tag{2}$$

and $v'_i = \sigma(\mathbf{this})(f_{r+i})$ for $1 \leq i \leq p$.

Let $\sigma''_1 = \sigma_1[theImp \mapsto \mathbf{null}, theId \mapsto \mathbf{null}, theLastThis \mapsto \mathbf{null}]$. From the definition of the operational semantics of blocks, the fact that **null** is the initial value for an object, we have that:

- $e_1, \sigma''_1 \rightsquigarrow_{[p]} \iota, \sigma'''_1$ where $\sigma'''_1 = \sigma''_1[theId \mapsto \iota]$
- $e_2, \sigma''_1 \rightsquigarrow_{[p]} \iota'', \sigma_2$ where $\sigma_2 = \sigma'''_1[theLastThis \mapsto \iota'']$ since $theId.\mathbf{imp}, \sigma'''_1 \rightsquigarrow_{[p]} \iota'', \sigma'''_1$ and from the hypothesis that **this**!! d is well-typed we have that $c \leq \mathcal{R}(p, d)$, so also $(\mathcal{R}(p, d))theId.\mathbf{imp}, \sigma'''_1 \rightsquigarrow_{[p]} \iota'', \sigma'''_1$ (no cast exception may occur).
- $e_3, \sigma_2 \rightsquigarrow_{[p]} \iota''', \sigma_3$ where ι''' is new in σ_1 (we also assume that it does not appear in σ), and, since d extends **FickleObject**, $\sigma_3 = \sigma_2[\iota''' \mapsto [[\mathbf{id} : \mathbf{null}, f_1 : v''_1, \dots, f_r : v''_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q}]]^d, theImp \mapsto \iota''']$ where v''_l is initial for $theType(\mathcal{F}(p, c, f_l))$, $l \in \{1, \dots, r+q\}$,
- $e_4; e_5, \sigma_3 \rightsquigarrow_{[p]} \iota, \sigma_4$, where σ_4 is $\sigma_3[\iota \mapsto [[\mathbf{imp} : \iota''']]^{\mathbf{Id}}, \iota''' \mapsto [[\mathbf{id} : \iota, f_1 : v''_1, \dots, f_r : v''_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q}]]^d]$, and

- $e'_1; \dots; e'_r, \sigma_4 \rightsquigarrow_{[p]} \iota, \sigma_1'''$ where from (2)
- $\sigma_1''' = \sigma_4[\iota \mapsto \llbracket \text{id} : \iota, f_1 : v_1, \dots, f_r : v_r, f_{r+1} : v_{r+1}', \dots, f_{r+q} : v_{r+q}' \rrbracket^d]$.

Let $\sigma'_1 = \sigma_1'''[\text{theImp} \mapsto \sigma_1(\text{theImp}), \text{theId} \mapsto \sigma_1(\text{theId}), \text{theLastThis} \mapsto \sigma_1(\text{theLastThis})]$.

$$\llbracket \text{this} !! d \rrbracket, \sigma_1 \rightsquigarrow_{[p]} \iota, \sigma'_1$$

Let $\sigma' = \sigma[\iota \mapsto \llbracket f_1 : v_1, \dots, f_{r+q} : v_{r+q} \rrbracket^c]$. We have to show that, $p, \gamma' \vdash \sigma' \approx \sigma'_1$. First notice that from Theorem 1, $p, \gamma' \vdash \sigma' \diamond$, and since $\llbracket p \rrbracket, \llbracket \gamma \rrbracket[\text{this} \mapsto c'] \vdash \sigma_1 \diamond$, see (2), also $\llbracket p \rrbracket, \llbracket \gamma' \rrbracket[\text{this} \mapsto c'] \vdash \sigma'_1 \diamond$. So we have to show that the three clauses of Definition 2 are satisfied. From the definition σ' and σ'_1 , we have that for all x , $\sigma'(x) = \sigma(x)$, and $\sigma'_1(x) = \sigma_1(x)$. Therefore, clause 1. of Definition 2 holds for σ' and σ'_1 , and since $\sigma'_1(\text{this})(\text{id}) = \sigma_1(\text{this})(\text{id})$, then also clause 2. holds. To conclude the proof, we have to verify clause 3. for the address ι .

- $\sigma'(\iota) = \llbracket f_1 : v_1, \dots, f_{r+q} : v_{r+q} \rrbracket^c$, $\sigma'_1(\iota) = \llbracket \text{imp} : \iota'' \rrbracket^{\text{Id}}$,
- $\sigma'_1(\iota'') = \llbracket \text{id} : \iota, f_1 : v_1, \dots, f_r : v_r, f_{r+1} : v_{r+1}', \dots, f_{r+q} : v_{r+q}' \rrbracket^d$

where for $1 \leq i \leq q$, v_{r+i} is an initial value for $\mathcal{F}(p, c, f_{r+i})$ and v_{r+i}' is an initial value for $\text{theType}(\mathcal{F}(p, c, f_{r+i}))$. Therefore, from Lemma 14, $v_{r+i} = v_{r+i}'$, and clause 3. holds. This concludes the proof that $p, \gamma' \vdash \sigma' \approx \sigma'_1$.

The proof for the case in which the rule (*recl*) is applied to $x !! d$ is similar.

Consider rule (*n-recl*). In this case the expression must be $x !! d$, since $p, \gamma \vdash \sigma \diamond$, see Fig. 5, implies that $\sigma(\text{this}) \neq \text{null}$. So $\sigma(x) = \text{null}$ and

$$x !! d, \sigma \rightsquigarrow_{[p]} \text{null}, \sigma.$$

From the definition of the translation (as for the case (*recl*), we have that $\text{theName}(d) = d$)

$$\llbracket x !! d \rrbracket(p, \gamma) \stackrel{\Delta}{=} \{d \text{ theImp}; \text{Identity theId}; \mathcal{R}(p, d) \text{ theLastImp}; \text{if } e_1 \text{ then } e_2; \text{theId}\}$$

where

- e_1 is $((\text{theId} = x) \neq \text{null})$,
- e_2 is

$$\begin{aligned} &\{ \text{theImp} = \text{new } d; \text{theLastImp} = (\mathcal{R}(p, d)) \text{theId. imp}; \\ &\text{theImp.id} = \text{theId}; \text{theId. imp} = \text{theImp}; \\ &e'_1; \dots e'_r \\ &\} \end{aligned}$$

where e'_i is $\text{theImp}.f_i = \text{theLastImp}.f_i$ for $1 \leq i \leq r$.

and $\gamma(\text{theId}) = \gamma(\text{theLastImp}) = \gamma(\text{theImp}) = \text{Udf}$.

Let $\sigma''_1 = \sigma_1[\text{theImp} \mapsto \text{null}, \text{theId} \mapsto \text{null}, \text{theLastImp} \mapsto \text{null}]$. Observe that, from $p, \gamma \vdash \sigma \approx \sigma_1$ we derive that $\sigma_1(x) = \text{null}$, and so also $\sigma''_1(x) = \text{null}$. From the definition of the operational semantics of blocks, and the fact that null is the initial value for an object, we have that:

$$e_1, \sigma''_1 \rightsquigarrow_{[p]} \text{false}, \sigma''_1$$

Therefore,

- if e_1 then e_2 ; $theId, \sigma_1'' \xrightarrow{\gamma_{[p]}} \text{null}, \sigma_1''$, and
- $\llbracket x !! d \rrbracket, \sigma_1 \xrightarrow{\gamma_{[p]}} \text{null}, \sigma_1$ since
 $\sigma_1' = \sigma_1''[theImp \mapsto \sigma_1(theImp), theId \mapsto \sigma_1(theId), theLastImp \mapsto \sigma_1(theLastImp)] = \sigma_1$, implies $\sigma_1' = \sigma_1$.

Consider rule (*meth*): We will consider methods with a single parameter, $e.m(e')$. So

$$\begin{array}{l}
\alpha. e, \sigma \xrightarrow{\gamma} \iota, \sigma'' \\
\beta. e', \sigma'' \xrightarrow{\gamma} v', \sigma''' \\
\chi. \sigma'''(\iota) = \llbracket \dots \rrbracket^{c'} \quad \mathcal{MD}(p, c', m) = t \ m(t_1 \ y_1) \ \phi \ block \\
\alpha'. \bar{\sigma} = \sigma'''[\text{this} \mapsto \iota, y_1 \mapsto v'] \\
\beta'. block, \bar{\sigma} \xrightarrow{\gamma} v, \bar{\sigma}' \\
\chi'. \sigma' = \bar{\sigma}'[\text{this} \mapsto \sigma'''(\text{this}), y_1 \mapsto \sigma'''(y_1)] \\
\hline
e.m(e'), \sigma \xrightarrow{\gamma} v, \sigma'
\end{array} \tag{3}$$

From the fact that the expression is well-typed, we have

$$\begin{array}{l}
a. p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0 \\
b. p, \gamma_0 \vdash e' : t_1 \parallel \gamma_1 \parallel \phi_1 \\
c. \mathcal{MD}(p, \phi_1 @_p c, m) = t \ m(t_1' \ y_1) \ \phi' \ block \\
d. p \vdash t_1 \leq t_1' \quad \phi = \phi_0 \cup \phi_1 \cup \phi' \quad \gamma' = \phi' @_p \gamma_1 \\
\hline
p, \gamma \vdash e.m(e') : t \parallel \gamma' \parallel \phi
\end{array} \tag{4}$$

Note that, since $\mathcal{MD}(p, \phi_1 @_p c, m) \neq \mathcal{Udf}$, and $p \vdash c \leq \phi_1 @_p c$, we have that $c \neq \text{Object}$. Therefore $theName(c) = c$. Consider the translation of the expression, from (4) we have

$$\llbracket e.m(e') \rrbracket_{expr}(p, \gamma) \triangleq \{\text{Identity } x; theType(t_1) \ x_1; \text{if } e_1 \text{ then } e_2 \text{ else } \{e_3; e_4\}\}$$

where

- e_1 is $\text{isnull}(x = \llbracket e \rrbracket_{expr}(p, \gamma))$
- e_2 is $\text{null}.m(x_1)$
- e_3 is $x_1 = \llbracket e' \rrbracket_{expr}(p, \gamma_0)$
- e_4 is $((\phi_1 @_p c)(x.\text{imp})).m(x_1)$

and $\gamma(x_1) = \gamma(x) = \mathcal{Udf}$. Since γ, γ_0 and γ' are defined on the same set of identifiers, we have that also $\gamma_0(x_1) = \gamma_0(x) = \mathcal{Udf}$ and $\gamma'(x_1) = \gamma'(x) = \mathcal{Udf}$.

Let $\sigma_1' = \sigma_1[x \mapsto \text{null}, x_1 \mapsto v']$, where v' is an initial value of type $theType(t)$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $p, \gamma \vdash \sigma \approx \sigma_1'$ (x_1 and x are not defined in γ). Since (4).a, and (3). α , we can apply the inductive hypothesis to e , and derive that $\llbracket e \rrbracket, \sigma_1' \xrightarrow{\gamma_{[p]}} \iota, \sigma_1'''$ where $p, \gamma_0 \vdash \sigma'' \approx \sigma_1'''$. Therefore, $e_2, \sigma_1' \xrightarrow{\gamma_{[p]}} \text{false}, \sigma_2$, where $\sigma_2 = \sigma_1'''[x \mapsto \iota]$, and $\{e_3; e_4\}$ is evaluated.

From $p, \gamma_0 \vdash \sigma'' \approx \sigma_1'''$, as before, we can derive that also $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$ (x is not defined in γ_0). Since (4).b, and (3). β , we can apply the inductive hypothesis to e' , and derive that $\llbracket e' \rrbracket, \sigma_2 \xrightarrow{\gamma_{[p]}} \iota, \sigma_2'$ where $p, \gamma' \vdash \sigma''' \approx \sigma_2'$ and from Lemma 15, $\sigma_2'(x_1) = \sigma_2(x_1) = \iota$. Therefore, $e_3, \sigma_2 \xrightarrow{\gamma_{[p]}} v, \sigma_3$, where $\sigma_3 = \sigma_2'[x_1 \mapsto v]$, $p, \gamma' \vdash \sigma''' \approx \sigma_3$, and $\sigma_3(x) = \sigma_2(x) = \iota$.

Now we have to evaluate $\llbracket e_4 \rrbracket$ in the store σ_3 . From the rule for method call (*meth*): we evaluate first $(\phi_1 @_p c)(x.\text{imp})$ in the store σ_3 . Since $p, \gamma' \vdash \sigma''' \approx \sigma_2'$, $\sigma_2'(\iota) = \sigma_3(\iota)$, and

- $\sigma_3(x) = \iota$,

- $\sigma_3(\iota) = \llbracket \text{imp} : \iota' \dots \rrbracket^{\text{Identity}}$, and
- $\sigma_3(\iota') = \llbracket \text{id} : \iota \dots \rrbracket^{c'}$.

Moreover, from (4).a, (3). α and Theorem 1 we have that $c' \leq c$. Again from (4).b, (3). β and Theorem 1 (in particular $p, \phi \vdash \sigma'' \triangleleft \sigma'''$) we have that $\phi_1 @_p c' \leq \phi_1 @_p c$. Therefore, the cast succeeds and we have that:

$$(\phi_1 @_p c)(x.\text{imp}), \sigma_3 \xrightarrow{\gamma_p} \iota', \sigma_3 \text{ and } x_1, \sigma_3 \xrightarrow{\gamma_p} v', \sigma_3.$$

From (3). χ , $\mathcal{MD}(p, c', m) = t \ m(t_1 \ y_1) \ \phi \ \text{block}$. Since the program is well formed from Figure 4, we have that

$$(*) \ p, \gamma'' \vdash \text{block} : t' \parallel \gamma''' \parallel \phi''$$

where $\gamma'' = t_1 \ y_1, c' \ \text{this}, t' \leq t$, and $\phi'' \subseteq \phi'$. From the definition of the translation of methods

$$\llbracket t \ m(t_1 \ y_1 \ \phi \ \text{block}) \rrbracket_{\text{meth}}(p, c) \triangleq \frac{\text{theType}(t) \ m(\llbracket t_1 \ y_1 \rrbracket_{\text{var}}) \{ \} \llbracket \text{block} \rrbracket_{\text{expr}}(p, \gamma'')}{\text{theType}(t) \ m(\llbracket t_1 \ y_1 \rrbracket_{\text{var}}) \{ \} \llbracket \text{block} \rrbracket_{\text{expr}}(p, \gamma'')}$$

Consider the store $\sigma_4 = \sigma_3[\text{this} \mapsto \iota', y_1 \mapsto v']$. It is immediate to see that $p, \gamma'' \vdash \bar{\sigma} \approx \sigma_4$. Applying the inductive hypothesis to (3). β' , $(*)$, we get:

$$\llbracket \text{block} \rrbracket_{\text{expr}}(p, \gamma''), \sigma_4 \xrightarrow{\gamma_p} v, \sigma_5$$

where $p, \gamma''' \vdash \bar{\sigma}' \approx \sigma_5$ and, since $\gamma''(x) = \gamma''(x_1) = \mathcal{Udf}$, from Lemma 15 and definition of σ_4 we have that $\sigma_4(x) = \sigma_3(x) = \sigma_5(x)$ and $\sigma_4(x_1) = \sigma_3(x_1) = \sigma_5(x_1)$. Let $\sigma_6 = \sigma_5[\text{this} \mapsto \sigma_3(\text{this}), y_1 \mapsto \sigma_3(y_1)]$ we have that

$$\llbracket e_4 \rrbracket, \sigma_3 \xrightarrow{\gamma_p} v, \sigma_6.$$

Let σ' be defined in (3). χ' . Since $p, \gamma' \vdash \sigma''' \approx \sigma_3$ we have that $p, \gamma' \vdash \sigma' \approx \sigma_6$. Let $\sigma'_1 = \sigma_6[x \mapsto \sigma_1(x), x_1 \mapsto \sigma_1(x_1)]$, it is immediate to show that $p, \gamma' \vdash \sigma' \approx \sigma'_1$. This concludes the proof.

Consider rule (cast):

$$\frac{e, \sigma \xrightarrow{\gamma_p} \iota, \sigma' \quad \sigma'(\iota) = \llbracket \dots \rrbracket^{c'} \quad p \vdash c' \leq c}{(c)e, \sigma \xrightarrow{\gamma_p} \iota, \sigma'}$$

Since the expression is well-typed we have that

$$\frac{a. p, \gamma \vdash e : c' \parallel \gamma' \parallel \phi \quad b. (p \vdash c' \leq c \text{ or } p \vdash c \leq c')}{p, \gamma \vdash (c)e : c \parallel \gamma' \parallel \phi} \quad (5)$$

Let $\llbracket (c)e \rrbracket_{\text{expr}}(p, \gamma) \triangleq \{ \text{Identity } x; \text{ if } e_1 \text{ then } e_2 \text{ else null } \}$, where

- e_1 is $(x = \llbracket e \rrbracket_{\text{expr}}(p, \gamma)) \neq \text{null}$, and
- e_2 is $(\text{theName}(c)x.\text{imp}).\text{id}$

Let $\sigma_1'' = \sigma_1[x \mapsto \text{null}]$. From $p, \gamma \vdash \sigma \approx \sigma_1$, we have $p, \gamma \vdash \sigma \approx \sigma_1''$. From (5).a and $e, \sigma \xrightarrow{p} \iota, \sigma'$ we can apply the inductive hypothesis to e , and derive that $\llbracket e \rrbracket, \sigma_1'' \xrightarrow{p} \iota, \sigma_1'''$ and $p, \gamma' \vdash \sigma' \approx \sigma_1'''$. So $\sigma_1'''(\iota) = \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}}$, and $\sigma_1'''(\iota') = \llbracket \dots, \text{id} : \iota \rrbracket^{\text{theName}(c')}$. Therefore, $\llbracket e_1 \rrbracket, \sigma_1'' \xrightarrow{p} \text{true}, \sigma_2$, where $\sigma_2 = \sigma_1'''[x \mapsto \iota]$, and the expression e_2 is evaluated.

Since $\sigma_2(x)(\text{imp}) = \iota'$, $\sigma_2(\iota')(\text{id}) = \iota$, (5).b, and Lemma 13, so $p \vdash \text{theName}(c') \leq \text{theName}(c)$, we have that $\llbracket e_2 \rrbracket, \sigma_2 \xrightarrow{p} \iota, \sigma_2$.

Let $\sigma_1' = \sigma_2[x \mapsto \sigma_1(x)]$, we have that $\llbracket (c)e \rrbracket, \sigma_1 \xrightarrow{p} \iota, \sigma_1'$, and from $p, \gamma' \vdash \sigma' \approx \sigma_1'''$, we get that $p, \gamma' \vdash \sigma' \approx \sigma_1'$.

Consider rule $(n\text{-cast})$, that is assume that $e, \sigma \xrightarrow{p} \text{null}, \sigma'$. So from rule $(n\text{-cast})$, $(c)e, \sigma \xrightarrow{p} \text{null}, \sigma'$. Consider $\llbracket (c)e \rrbracket$ as for rule $(cast)$. We can apply the inductive hypothesis to e , and derive that $\llbracket e \rrbracket, \sigma_1'' \xrightarrow{p} \text{null}, \sigma_1'''$ and $p, \gamma' \vdash \sigma' \approx \sigma_1'''$. Therefore, $\llbracket e_1 \rrbracket, \sigma_1'' \xrightarrow{p} \text{false}, \sigma_2$, where $\sigma_2 = \sigma_1'''[x \mapsto \text{null}]$, and the expression e_2 is not evaluated. Let $\sigma_1' = \sigma_1''[x \mapsto \sigma_1(x)]$, we have that $\llbracket (c)e \rrbracket, \sigma_1 \xrightarrow{p} \text{null}, \sigma_1'$. From $p, \gamma' \vdash \sigma' \approx \sigma_1'''$, we get that $p, \gamma' \vdash \sigma' \approx \sigma_1'$.

Let us now assume that the result of the evaluation is an exception. We will consider **nullPtrExc** raised during the evaluation of a field update and **castExc** raised by a cast expression.

Consider rule $(e\text{-null})$, and assume that the expression is an assignment to a field:

$$\frac{e, \sigma \xrightarrow{p} \text{null}, \sigma'}{e.f = e', \sigma \xrightarrow{p} \text{nullPtrExc}, \sigma'}$$

Consider the translation of the expression as for the case of rule $(a\text{-field})$. Let $\sigma_1'' = \sigma_1[x_1 \mapsto \text{null}, x_2 \mapsto v']$, where v' is an initial value of type $\text{theType}(t)$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $p, \gamma \vdash \sigma \approx \sigma_1''$ (x_1 and x_2 are not defined in γ). Since the expression is well typed $p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0$ for some c, γ_0 , and ϕ_0 . So, from $e, \sigma \xrightarrow{p} \text{null}, \sigma'$, we can apply the inductive hypothesis to e , and derive that $\llbracket e \rrbracket, \sigma_1'' \xrightarrow{p} \text{null}, \sigma_1'''$ where $p, \gamma_0 \vdash \sigma' \approx \sigma_1'''$. Therefore, $e_1, \sigma_1'' \xrightarrow{p} \text{false}, \sigma_2$, where $\sigma_2 = \sigma_1'''[x_1 \mapsto \text{null}]$, and e_2 is not evaluated.

Since $\sigma_2(x_1) = \text{null}$, we have that $e_3, \sigma_2 \xrightarrow{p} \text{nullPtrExc}, \sigma_2$ (we apply the rule that generate the exception to $x_1.\text{imp}$ and then the rules for propagation of exceptions of Fig. 17).

Let $\sigma_1' = \sigma_2[x_1 \mapsto \sigma_1(x_1), x_2 \mapsto \sigma_1(x_2)]$, then $\llbracket e.f = e' \rrbracket, \sigma_1 \xrightarrow{p} \text{nullPtrExc}, \sigma_1'$. Note that in this case it is important in the translation to avoid the evaluation of e_2 , since it could raise a different exception or not terminate.

Consider rule $(e\text{-cast})$:

$$\frac{\begin{array}{l} e, \sigma \xrightarrow{p} \iota, \sigma' \\ \sigma'(\iota) = \llbracket \dots \rrbracket^{c'} \\ p \not\vdash c' \leq c \end{array}}{(c)e, \sigma \xrightarrow{p} \text{castExc}, \sigma'}$$

Let $\llbracket (c)e \rrbracket_{\text{expr}}(p, \gamma)$, and $\sigma_1'' = \sigma_1[x \mapsto \text{null}]$ be as for the case of rule $(cast)$. From the inductive hypothesis applied to e ,

$\llbracket e \rrbracket, \sigma_1'' \xrightarrow{p} \iota, \sigma_1'''$ and $p, \gamma' \vdash \sigma' \approx \sigma_1'''$. So $\sigma_1'''(\iota) = \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}}$, and $\sigma_1'''(\iota') =$

$[[\dots]]^{theName(c')}$. Therefore, $\llbracket e_1 \rrbracket, \sigma_1'' \xrightarrow[p]{\gamma} \mathbf{true}, \sigma_2$, where $\sigma_2 = \sigma_1'''[x \mapsto \iota]$, and the expression e_2 is evaluated.

Since $\sigma_2(x)(\mathbf{imp}) = \iota'$, $\sigma_2(\iota') = [[\dots]]^{theName(c')}$, and $p \not\vdash c' \leq c$, so from Lemma 13 also $p \not\vdash theName(c') \leq theName(c)$ we have that

$\llbracket (c)x.\mathbf{imp} \rrbracket, \sigma_2 \xrightarrow[p]{\gamma} \mathbf{castExc}, \sigma_2$. Applying the rules for propagation of exceptions we get that $\llbracket e_2 \rrbracket, \sigma_2 \xrightarrow[p]{\gamma} \mathbf{castExc}, \sigma_2$.

Let $\sigma_1' = \sigma_2[x \mapsto \sigma_1(x)]$, we have that $\llbracket (c)e \rrbracket, \sigma_1 \xrightarrow[p]{\gamma} \mathbf{castExc}, \sigma_1'$. Note that the cast in the translation is essential to raise the exception, that otherwise would not occur since every **Identity** object has the field **imp** and every **FickleObject** has the field **id**.

Proof of: $\llbracket e \rrbracket, \sigma_1 \xrightarrow[p]{\gamma} w, \sigma_1' \text{ implies } e, \sigma \xrightarrow[p]{\gamma} w, \sigma'$ The result is proved by induction on the depth of the derivation tree of $\llbracket e \rrbracket, \sigma_1 \xrightarrow[p]{\gamma} w, \sigma_1'$. Note that, since e is well-typed, from Theorem 2 we have that also $\llbracket e \rrbracket$ is well-typed so from Theorem 1 its evaluation cannot produce **stuckErr** and w is either a value or $w \in \{\mathbf{castExc}, \mathbf{nullPtrExc}\}$. We will consider the translation of the following expressions: identifiers, object creation, re-classification, field update, and cast. The other cases are similar.

Consider $\llbracket \mathbf{id} \rrbracket \triangleq \mathbf{id}$. Since $w \neq \mathbf{stuckErr}$ the only rule applicable is $\llbracket \mathbf{id} \rrbracket, \sigma_1 \xrightarrow[p]{\gamma} \sigma_1(\mathbf{id}), \sigma_1$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $\sigma(\mathbf{id}) = \sigma_1(\mathbf{id})$. Therefore also $\mathbf{id}, \sigma \xrightarrow[p]{\gamma} \sigma(\mathbf{id}), \sigma$.

Consider $\llbracket \mathbf{new } c \rrbracket \triangleq \{theName(c) \text{ theImp}; \mathbf{Identity} \text{ theId}; e_1; e_2; e_3; e_4; theId\}$ where

- e_1 is $theId = \mathbf{new Identity}$,
- e_2 is $theImp = \mathbf{new } theName(c)$,
- e_3 is $theImp.\mathbf{id} = theId$, and
- e_4 is $theId.\mathbf{imp} = theImp$.

and $\gamma(theId) = \gamma(theImp) = Udf$. Assume that $c \neq \mathbf{Object}$, so $theName(c) = c$. (The case $c = \mathbf{Object}$ is simpler since $\mathcal{Fs}(p, \mathbf{Object}) = \emptyset$.) From the definition of the operational semantics, and the fact that: $c \leq \mathbf{FickleObject}$ has the field **id** and an object of class **Identity** has the field **imp** (so the evaluation of the expressions cannot produce an exception), as for the case of the corresponding “only if” proof, we have that:

$$\llbracket \mathbf{new } c \rrbracket, \sigma_1 \xrightarrow[p]{\gamma} \iota, \sigma_1'$$

where

- $e_1; e_2; e_3; e_4; theId, \sigma_1[theImp \mapsto \mathbf{null}, theId \mapsto \mathbf{null}] \xrightarrow[p]{\gamma} \iota, \sigma_1''$,
- $\sigma_1'' = \sigma_1[\iota' \mapsto [[\mathbf{id} : \iota, f_1 : v'_1, \dots, f_r : v'_r]]^c, theImp \mapsto \iota', \iota \mapsto [[\mathbf{imp} : \iota']]^{\mathbf{id}}, theId \mapsto \iota]$,
and
- $\sigma_1' = \sigma_1'''[theImp \mapsto \sigma_1(theImp), theId \mapsto \sigma_1(theId)]$.

We choose ι and ι' such that they are new both in σ_1 and σ . Since ι is new in σ we have

$$\mathbf{new } c, \sigma \xrightarrow[p]{\gamma} \iota, \sigma'$$

where $\sigma' = \sigma[\iota \mapsto [[f_1 : v_1, \dots, f_r : v_r]]^c]$, and v_l is initial for $\mathcal{F}(p, c, f_l)$, $1 \leq l \leq r$. As for the case of the corresponding “only if” proof, using Lemma 14 we can show that $p, \gamma' \vdash \sigma' \approx \sigma_1'$.

Consider $\llbracket x !! d \rrbracket \stackrel{\Delta}{=} \{theName(d) \ theImp; Identity \ theId; \mathcal{R}(p, d) \ theLastImp; \text{if } e_1 \text{ then } e_2; \ theId\}$ where

- e_1 is $((theId = x) \neq \text{null})$,
- e_2 is

$$\begin{aligned} &\{theImp = \text{new } theName(d); theLastImp = (\mathcal{R}(p, d))theId.\text{imp}; \\ &theImp.id = theId; theId.\text{imp} = theImp; \\ &e'_1; \dots e'_r \\ &\} \end{aligned}$$
- e'_i is $theImp.f_i = theLastImp.f_i$ for $1 \leq i \leq r$, and

$$\gamma(theId) = \gamma(theLastImp) = \gamma(theImp) = Udf.$$

If $\sigma_1(x) = \text{null}$, then $\llbracket x !! d \rrbracket, \sigma_1 \rightsquigarrow_{[p]} \text{null}, \sigma_1$ since

$$e_1, \sigma_1[theImp, theId, theLastImp \mapsto \text{null}] \rightsquigarrow_{[p]} \text{false}, \sigma_1[theImp, theId, theLastImp \mapsto \text{null}]$$

Since the expression is well-typed $d \neq \text{Object}$, and $theName(d) = d$. Applying rule (*n-recl*) also $x !! d, \sigma \rightsquigarrow_{[p]} \text{null}, \sigma$ and the result holds.

Consider now the case $\sigma_1(x) \neq \text{null}$. From $p, \gamma' \vdash \sigma \approx \sigma_1$ we have $\sigma(x) \neq \text{null}$. Moreover, since the expression is well-typed $\gamma(x) = c$, so from $p, \gamma \vdash \sigma \Diamond, \sigma(x) = \iota$ and $\sigma(\iota) = \llbracket \dots \rrbracket^{c'}$ for some $c' \leq c$. Again from $p, \gamma' \vdash \sigma \approx \sigma_1$, we derive that

$$\sigma_1(x) = \iota \quad \sigma_1(\iota) = \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}} \quad \sigma_1(\iota') = \llbracket \text{id} : \iota, \dots \rrbracket^{c'} \quad (6)$$

From the definition of the operational semantics of blocks, and the fact that null is the initial value for an object, we have that:

- $e_1, \sigma_1[theImp, theId, theLastImp \mapsto \text{null}] \rightsquigarrow_{[p]} \text{false}, \sigma'_1$ where $\sigma'_1 = \sigma_1[theImp, theLastImp \mapsto \text{null}, theId \mapsto \iota]$,
- $theImp = \text{new } d, \sigma'_1 \rightsquigarrow_{[p]} \iota'', \sigma_2$ where ι'' is new and $\sigma_2 = \sigma'_1[theImp \mapsto \iota'', \iota'' \mapsto \llbracket \text{id} : \text{null}, f_1 : v''_1, \dots, f_r : v''_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q} \rrbracket^d, theImp \mapsto \iota'']$ where v''_l is initial for $theType(\mathcal{F}(p, d, f_l))$, $l \in \{1, \dots, r+q\}$,
- from equation (6), the fact that the expression is well typed, therefore $\mathcal{R}(p, d) = \mathcal{R}(p, c')$, we have that

$$theLastImp = (\mathcal{R}(p, d))theId.\text{imp}, \sigma_2 \rightsquigarrow_{[p]} \iota', \sigma_3$$

where $\sigma_3 = \sigma_2[theLastImp \mapsto \iota']$ (neither the cast operation, nor the access to field imp produce an exception)

- $theImp.id = theId; theId.\text{imp} = theImp; \sigma_3 \rightsquigarrow_{[p]} \iota'', \sigma_4$ where $\sigma_4 = \sigma_3[\iota \mapsto \llbracket \text{imp} : \iota'' \rrbracket^{\text{Id}}, \iota'' \mapsto \llbracket \text{id} : \iota'', f_1 : v''_1, \dots, f_r : v''_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q} \rrbracket^d, theImp \mapsto \iota'']$, and finally
- $e'_1; \dots e'_r, \sigma_4 \rightsquigarrow_{[p]} \iota'', \sigma'_1$ where $\sigma'_1 = \sigma_4[\iota'' \mapsto \llbracket \text{id} : \iota'', f_1 : v_1, \dots, f_r : v_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q} \rrbracket^d]$ where $\{f_1, \dots, f_r\}$ are the fields of the class $\mathcal{R}(p, d)$ and $v_i = \sigma_1(x)(f_i)$ for $1 \leq i \leq r$.

Let $\sigma'_1 = \sigma'_1[theImp \mapsto \sigma_1(theImp), theId \mapsto \sigma_1(theId), theLastThis \mapsto \sigma_1(theLastThis)]$. We derive that

$$\llbracket x !! d \rrbracket, \sigma_1 \rightsquigarrow_{[p]} \iota, \sigma'_1.$$

On the other hand, since $\sigma(x) = \iota$, $\sigma(\iota) = \llbracket \dots \rrbracket^{c'}$, and $\{f_1, \dots, f_r\}$ are the fields of the class $\mathcal{R}(p, d)$ from rule (*recl*) we have that

$$x !! d, \sigma \rightsquigarrow_{[p]} \iota, \sigma'$$

where $\sigma' = \sigma[\iota \mapsto \llbracket f_1 : v'_1, \dots, f_r : v'_r, f_{r+1} : v'_{r+1}, \dots, f_{r+q} : v'_{r+q} \rrbracket^d$, $v'_i = \sigma(x)(f_i)$ for $1 \leq i \leq r$, and v'_i is the initial value for $\mathcal{F}(p, d, f_i)$, $r \leq i \leq r+q$.

To show that $p, \gamma' \vdash \sigma' \approx \sigma'_1$ we have to prove that the three clauses of Definition 2 are satisfied for σ' and σ'_1 . From the definition of σ' and σ'_1 and the fact that $p, \gamma \vdash \sigma \approx \sigma_1$ we have that for all x' , $\sigma'(x') = \sigma'_1(x')$. Therefore, clause 1. of Definition 2 holds, and since the only field **id** that is updated in σ'_1 is the one of a newly created location also $\sigma'_1(\mathbf{this})(\mathbf{id}) = \sigma_1(\mathbf{this})(\mathbf{id})$ and clause 2. holds. To conclude the proof, we have to verify clause 3. for the address ι . Let

$$\begin{aligned} - \sigma'(\iota) &= \llbracket f_1 : v'_1, \dots, f_r : v'_r \rrbracket^c, \sigma'_1(\iota) = \llbracket \mathbf{imp} : \iota'' \rrbracket^{\mathbf{id}}, \\ - \sigma'_1(\iota''') &= \llbracket \mathbf{id} : \iota, f_1 : v_1, \dots, f_r : v_r, f_{r+1} : v''_{r+1}, \dots, f_{r+q} : v''_{r+q} \rrbracket^d \end{aligned}$$

From $p, \gamma \vdash \sigma \approx \sigma_1$ for all i , $1 \leq i \leq r$, $v_i = v'_i$. Moreover, for all i , $r \leq i \leq r+q$, v'_i is an initial value for $\mathcal{F}(p, d, f_{r+i})$ and v''_{r+i} is an initial value for $\mathit{theType}(\mathcal{F}(p, d, f_{r+i}))$. Therefore, from Lemma 14, $v_{r+i} = v''_{r+i}$, and clause 3. holds. This concludes the proof that $p, \gamma' \vdash \sigma' \approx \sigma'_1$.

The case in which the expression is $\llbracket \mathbf{this} !! d \rrbracket$ is similar.

Consider $\llbracket e.f \rrbracket = \{ \mathbf{Identity} \ x_1; \mathit{theType}(c) \ x_2; \mathbf{if} \ e_1 \mathbf{then} \ e_2 \mathbf{else} \ \{ e_3; e_4 \} \}$ where

$$\begin{aligned} - e_1 \text{ is } \mathbf{isnull}(x_1 = \llbracket e \rrbracket_{\text{expr}}(p, \gamma)) \\ - e_2 \text{ is } \mathbf{null}.f = x_2 \\ - e_3 \text{ is } x_2 = \llbracket e' \rrbracket_{\text{expr}}(p, \gamma_0) \\ - e_4 \text{ is } ((\phi' @_p c)(x_1.\mathbf{imp})).f = x_2 \end{aligned}$$

and $\gamma(x_1) = \gamma(x_2) = \mathcal{Udf}$. From the fact that the expression is well-typed, we have

$$\begin{aligned} a. \ p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0 \\ b. \ p, \gamma_0 \vdash e' : t \parallel \gamma' \parallel \phi' \\ c. \ \mathcal{F}(p, \phi' @_p c, f) = t' \quad p \vdash t \leq t' \quad \phi = \phi \cup \phi' \\ \hline p, \gamma \vdash e.f = e' : t \parallel \gamma' \parallel \phi \end{aligned} \tag{7}$$

Note that, since $\mathcal{Fs}(p, \mathbf{Object}) = \emptyset$, $c \neq \mathbf{Object}$, and therefore $\mathit{theName}(c) = c$. Let $\sigma'_1 = \sigma_1[x_1 \mapsto \mathbf{null}, x_2 \mapsto v']$, where v' is an initial value of type $\mathit{theType}(t)$. Since $p, \gamma \vdash \sigma \approx \sigma_1$ and x_1 and x_2 are not defined in γ also $p, \gamma \vdash \sigma \approx \sigma'_1$. From $\llbracket e.f = e' \rrbracket, \sigma_1 \rightsquigarrow_{[p]} w, \sigma'_1$ we derive that:

$$\mathbf{if} \ e_1 \mathbf{then} \ e_2 \mathbf{else} \ \{ e_3; e_4 \}, \sigma'_1 \rightsquigarrow_{[p]} w, \sigma'_1 \tag{8}$$

where $\sigma'_1 = \sigma'_1'''[x_1 \mapsto \sigma_1(x_1), x_2 \mapsto \sigma_1(x_2)]$. Therefore,

1. $e_1, \sigma'_1 \rightsquigarrow_{[p]} w, \sigma'_1'''$, with $w \in \{\mathbf{castExc}, \mathbf{nullPtrExc}\}$, or
2. $e_1, \sigma'_1 \rightsquigarrow_{[p]} \mathbf{true}, \sigma_2$ for some σ_2 or
3. $e_1, \sigma'_1 \rightsquigarrow_{[p]} \mathbf{false}, \sigma_2$ for some σ_2 .

1. For the first case observe that, using the propagation rules for exceptions

$$\llbracket e.f = e' \rrbracket, \sigma_1 \sim_{\llbracket p \rrbracket} w, \sigma'_1$$

Moreover, since an assignment does not produce an exception it must be the case that

$$\llbracket e \rrbracket, \sigma''_1 \sim_{\llbracket p \rrbracket} w, \sigma'''_1$$

Since e is well-typed, then $p, \gamma \vdash e : c \parallel \gamma_0 \parallel \phi_0$ for some c , γ_0 , and ϕ_0 . So, from $p, \gamma \vdash \sigma \approx \sigma''_1$ we can apply the inductive hypothesis to e and derive that: $e, \sigma \sim_{\llbracket p \rrbracket} w, \sigma'$. From the propagation rules for exceptions we derive

$$e.f = e', \sigma \sim_{\llbracket p \rrbracket} w, \sigma'$$

that proves the result.

2. Assume that the evaluation of e_1 produces **false**. Therefore

$$\llbracket e \rrbracket, \sigma''_1 \sim_{\llbracket p \rrbracket} v, \sigma''_2 \quad (9)$$

where $\sigma_2 = \sigma''_2[x_1 \mapsto v]$ and $v \neq \text{null}$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $p, \gamma \vdash \sigma \approx \sigma''_1$ (x_1 and x_2 are not defined in γ). From (7).a, and (9), we can apply the inductive hypothesis to e , and derive that $e, \sigma \sim_{\llbracket p \rrbracket} v, \sigma''$ where $p, \gamma_0 \vdash \sigma'' \approx \sigma''_2$ and since x_1 is not defined in γ_0 also $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$. From (8) (since e_1 evaluates to **false**) we derive that

- $e_3, \sigma_2 \sim_{\llbracket p \rrbracket} w, \sigma'''_1$, with $w \in \{\text{castExc}, \text{nullPtrExc}\}$, or
- $e_3, \sigma_2 \sim_{\llbracket p \rrbracket} v', \sigma_3$ for some σ_3 and v' .

In the first case, as for the corresponding case of the evaluation of e_1 ,

$$\llbracket e.f = e' \rrbracket, \sigma_1 \sim_{\llbracket p \rrbracket} w, \sigma'_1.$$

From the inductive hypothesis on e' and the propagation rules for exceptions $e.f = e', \sigma \sim_{\llbracket p \rrbracket} w, \sigma'$.

In the second case (the evaluation of e_3 produces v') we have that

$$\llbracket e' \rrbracket, \sigma_2 \sim_{\llbracket p \rrbracket} v', \sigma'''_3 \quad (10)$$

where $\sigma_3 = \sigma'''_3[x_2 \mapsto v']$. (Note that since $\gamma_0(x_1) = \text{Udf}$ we also have that $\sigma_3(x_1) = \sigma_2(x_1) = \iota$.) From (7).b, (10), and $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$ we can apply the inductive hypothesis to e' and derive that $e', \sigma'' \sim_{\llbracket p \rrbracket} v', \sigma'''$ where $p, \gamma' \vdash \sigma''' \approx \sigma'''_3$, and since x_2 is not defined in γ' then also $p, \gamma' \vdash \sigma''' \approx \sigma_3$.

From (7).a, and Theorem 1, $p, \sigma'' \vdash v \triangleleft c$, therefore $v = \iota$ and $\sigma''(\iota) = \llbracket [\dots] \rrbracket^{c'}$ where $c' \leq c$. From $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$ also $\sigma_2(\iota) = \llbracket [\text{imp} : \iota'] \rrbracket^{\text{Id}}$, $\sigma_2(\iota') = \llbracket [\text{id} : \iota \dots] \rrbracket^{c'}$, and $\sigma_2(x_1) = \iota$. Moreover, (7).b, and Theorem 1, implies that $p, \phi' \vdash \sigma'' \triangleleft \sigma'''$ and $p, \gamma' \vdash \sigma''' \diamond$. So $\sigma'''(\iota) = \llbracket [\dots] \rrbracket^{c''}$, and $\phi' @_p c' = \phi' @_p c''$. So (7).c, $\phi' @_p c' \leq \phi' @_p c''$ implies that field f is defined for $\sigma'''(\iota)$. From $p, \gamma' \vdash \sigma''' \approx \sigma_3$ we have that $\sigma_3(\iota) = \llbracket [\text{imp} : \iota''] \rrbracket^{\text{Id}}$, $\sigma_3(\iota'') = \llbracket [\text{id} : \iota \dots] \rrbracket^{c''}$ (for some ι'') and field f is defined for $\sigma_3(\iota')$. Therefore,

$$e_4, \sigma_3 \sim_{\llbracket p \rrbracket} v', \sigma_3[\iota'' \mapsto \sigma_3(\iota'')[f \mapsto v']]$$

and $\llbracket e.f = e' \rrbracket, \sigma_1 \sim_{\llbracket p \rrbracket} v', \sigma'_1$ where $\sigma'_1 = \sigma_3[x_1 \mapsto \sigma_1(x_1), x_2 \mapsto \sigma_1(x_2), \iota'' \mapsto \sigma_3(\iota'')[f \mapsto v']]$.

On the other hand,

- $e, \sigma \rightsquigarrow_{\llbracket p \rrbracket}^* \iota, \sigma''$,
- $e', \sigma'' \rightsquigarrow_{\llbracket p \rrbracket}^* v', \sigma'''$, and
- field f defined for $\sigma'''(\iota)$,

implies that

$$e.f = e', \sigma \rightsquigarrow_{\llbracket p \rrbracket}^* \iota, \sigma'$$

where $\sigma' = \sigma'''[\iota' \mapsto \sigma'(\iota)[f \mapsto v']]$. From $p, \gamma' \vdash \sigma''' \approx \sigma_3$ it is easy to show that $p, \gamma' \vdash \sigma' \approx \sigma'_1$. This concludes the proof of the case in which the evaluation of e_1 was **false**.

3. Assume, now, that $\llbracket e_1 \rrbracket, \sigma'_1 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{true}, \sigma_2$. Therefore

$$\llbracket e \rrbracket, \sigma'_1 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{null}, \sigma_2 \quad (11)$$

and $\sigma_2(x_1) = \mathbf{null}$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $p, \gamma \vdash \sigma \approx \sigma'_1$ (x_1 and x_2 are not defined in γ). From (7).a, and (11), we can apply the inductive hypothesis to e , and derive that $e, \sigma \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{null}, \sigma''$ where $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$.

Therefore, $\mathbf{null}.f, \sigma_2 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{nullPtrExc}, \sigma_2$ and from the propagation rules

$$\llbracket e.f = e' \rrbracket, \sigma_1 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{nullPtrExc}, \sigma'_1$$

where $\sigma'_1 = \sigma_2[x_1 \mapsto \sigma_1(x_1), x_2 \mapsto \sigma_1(x_2)]$.

On the other hand, applying rule (*e-null*),

$$\frac{e, \sigma \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{null}, \sigma''}{e.f = e', \sigma \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{nullPtrExc}, \sigma''}$$

and $p, \gamma_0 \vdash \sigma'' \approx \sigma_2$ implies $p, \gamma_0 \vdash \sigma'' \approx \sigma'_1$

The proof for $e.m(e')$ is similar.

Consider $\llbracket (c)e \rrbracket_{\text{expr}} \stackrel{\Delta}{=} \{ \text{theType}(t) \ x; \text{ if } e_1 \text{ then } e_2 \text{ else null } \}$, where

- e_1 is $(x = \llbracket e \rrbracket_{\text{expr}}(p, \gamma)) \neq \mathbf{null}$, and
- e_2 is $(\text{theName}(c)x.\text{imp}).\text{id}$

Since the expression is well-typed

$$\frac{\begin{array}{l} a. p, \gamma \vdash e : c' \parallel \gamma' \parallel \phi \\ b. (p \vdash c' \leq c \text{ or } p \vdash c \leq c') \end{array}}{p, \gamma \vdash (c)e : c \parallel \gamma' \parallel \phi} \quad (12)$$

holds. From $\llbracket (c)e \rrbracket_{\text{expr}}, \sigma_1 \rightsquigarrow_p^* w, \sigma'_1$ and the definition of the operational semantics of blocks we have that

$$\text{if } e_1 \text{ then } e_2 \text{ else null}, \sigma'_1 \rightsquigarrow_p^* w, \sigma'_1''' \quad (13)$$

where $\sigma'_1 = \sigma_1[x \mapsto \mathbf{null}]$ and $\sigma'_1 = \sigma'_1'''[x \mapsto \sigma_1(x)]$. Therefore,

1. $e_1, \sigma'_1 \rightsquigarrow_{\llbracket p \rrbracket}^* w, \sigma'_1'''$, with $w \in \{\mathbf{castExc}, \mathbf{nullPtrExc}\}$, or
2. $e_1, \sigma'_1 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{true}, \sigma'_1'''$ or
3. $e_1, \sigma'_1 \rightsquigarrow_{\llbracket p \rrbracket}^* \mathbf{false}, \sigma'_1'''$.

1. For the first case observe that it must be that,

$$\llbracket e \rrbracket, \sigma_1'' \rightsquigarrow_{\llbracket p \rrbracket} w, \sigma_1'''$$

Using (12).a, and $p, \gamma \vdash \sigma \approx \sigma_1''$ we can apply the inductive hypothesis to e and derive that: $e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} w, \sigma'$ and $p, \gamma \vdash \sigma \approx \sigma'$. From the propagation rules for exceptions we derive that

$$\llbracket (c)e \rrbracket_{\text{expr}}, \sigma_1 \rightsquigarrow_{\llbracket p \rrbracket} w, \sigma'_1$$

and

$$(c)e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} w, \sigma'$$

and $p, \gamma \vdash \sigma \approx \sigma'$ implies $p, \gamma \vdash \sigma \approx \sigma'$ which prove the result.

2. Assume that the evaluation of e_1 produces **true**. Therefore

$$\llbracket e \rrbracket, \sigma_1'' \rightsquigarrow_{\llbracket p \rrbracket} v, \sigma_2 \quad (14)$$

where $\sigma_1''' = \sigma_2[x \mapsto v]$ and $v \neq \text{null}$. From $p, \gamma \vdash \sigma \approx \sigma_1$ we have that $p, \gamma \vdash \sigma \approx \sigma_1''$ (x is not defined in γ). From (12).a, and (14), we can apply the inductive hypothesis to e , and derive that $e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} v, \sigma'$ where $p, \gamma' \vdash \sigma' \approx \sigma_2$ and since x is not defined in γ' also $p, \gamma' \vdash \sigma' \approx \sigma_1'''$.

Observe that, (12).a, and Theorem 1, implies that $p, \sigma' \vdash v \triangleleft c'$, therefore $v = \iota$ and $\sigma'(\iota) = \llbracket \dots \rrbracket^{c''}$ where $c'' \leq c'$. From $p, \gamma' \vdash \sigma' \approx \sigma_1'''$ also $\sigma_1'''(\iota) = \llbracket \text{imp} : \iota' \rrbracket^{\text{Id}}$, $\sigma_1'''(\iota') = \llbracket \text{id} : \iota \dots \rrbracket^{\text{theName}(c'')}$, and $\sigma_1'''(x) = \iota$. For the evaluation of e_2 there are two cases:

- $e_2, \sigma_1''' \rightsquigarrow_{\llbracket p \rrbracket} \text{castExc}, \sigma_1'''$ if $c' \not\leq c$, or
- $e_2, \sigma_1''' \rightsquigarrow_{\llbracket p \rrbracket} \iota, \sigma_1'''$ if $c' \leq c$.

For the first case we have that from the propagation rules for exceptions

$$\llbracket (c)e \rrbracket, \sigma_1 \rightsquigarrow_{\llbracket p \rrbracket} \text{castExc}, \sigma'_1$$

and from the application of the rule ($e\text{-cast}$) also $(c)e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} \text{castExc}, \sigma'$.

For the second case

$$\llbracket (c)e \rrbracket, \sigma_1 \rightsquigarrow_{\llbracket p \rrbracket} \iota, \sigma'_1$$

On the other hand we can apply rule ($cast$) and get $(c)e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} \iota, \sigma'$. From $p, \gamma' \vdash \sigma' \approx \sigma_1'''$ the definition of σ'_1 , and the fact that $\gamma(x) = \text{Udf}$ we have $p, \gamma' \vdash \sigma' \approx \sigma'_1$.

3. If $e_1, \sigma_1'' \rightsquigarrow_{\llbracket p \rrbracket} \text{false}, \sigma_1'''$ then

$$\llbracket e \rrbracket, \sigma_1'' \rightsquigarrow_{\llbracket p \rrbracket} \text{null}, \sigma_1'''$$

Applying ($f\text{-cond}$) we have

$$\text{if } e_1 \text{ then } e_2 \text{ else null}, \sigma_1'' \rightsquigarrow_p \text{null}, \sigma_1'''$$

and finally

$$\llbracket (c)e \rrbracket, \sigma_1 \rightsquigarrow_p \text{null}, \sigma'_1$$

From (12).a, we can apply the inductive hypothesis to e , and derive that $e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} \text{null}, \sigma'$ where $p, \gamma' \vdash \sigma' \approx \sigma_1'''$. Therefore, applying rule ($n\text{-cast}$) we have

$$(c)e, \sigma \rightsquigarrow_{\llbracket p \rrbracket} \text{null}, \sigma'$$

From $p, \gamma' \vdash \sigma' \approx \sigma_1'''$ and the definition of σ'_1 we derive that $p, \gamma' \vdash \sigma' \approx \sigma'_1$. This concludes the proof.

□