Global typing of local agents

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References

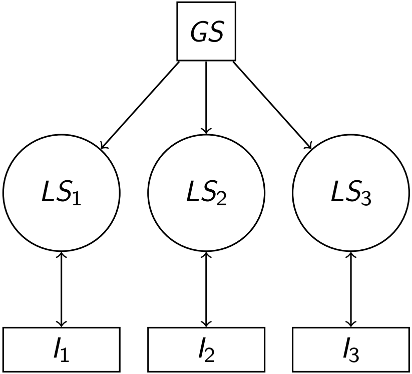
HYC08 Honda, Yoshida, Carbone:
Multiparty Asynchronous Session Types, POPL 2008

BLZ09 Bravetti, Lanese, Zavattaro:
Contract-Driven Implementation of Choreographies, TGC 2009

CDP12 Castagna, Dezani, Padovani:
On Global Types and Multiparty Sessions, LMCS 2012

Disclaim: references are arbitrary and incomplete even w.r.t. the work of the authors; this is not a comparative presentation of the contents of these papers, nor I will be faithful in defining concepts and reporting results.

Global-local scenario

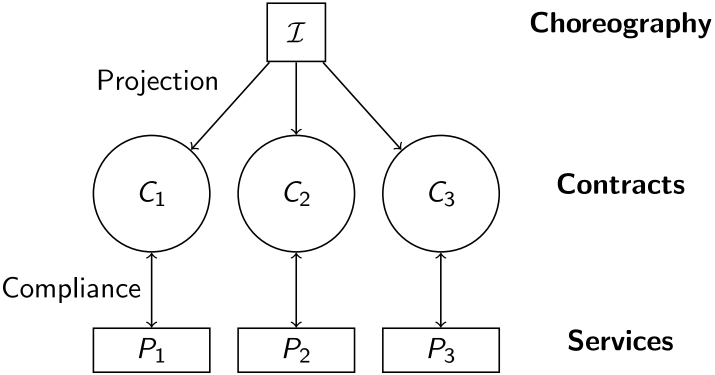


Global Specification

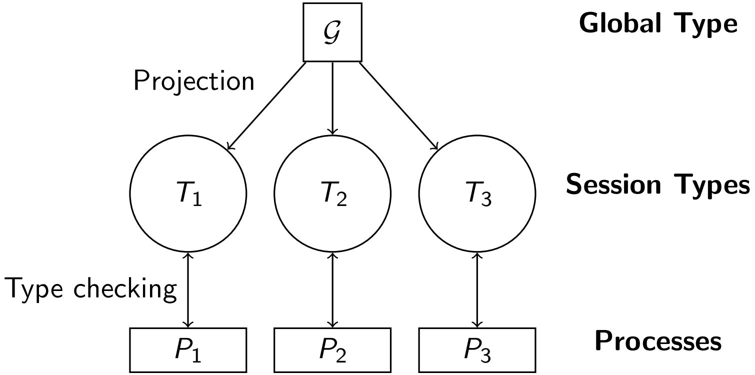
Local Specifications

Implementations

Global-local scenario: Choreographies and Contracts



Global type projection to local session types



Global types

$$
\begin{array}{rcll}
\mathcal{G} & ::= & \mathsf{end} & \text{(end)} \\
& | & p \xrightarrow{\alpha} q.\mathcal{G} & \text{(interaction)} \\
& | & \mathcal{G} + \mathcal{G} & \text{(branching)} \\
& | & \mathcal{G}\,;\,\mathcal{G} & \text{(sequence)} \\
& | & \mathcal{G} \parallel \mathcal{G} & \text{(shuffle)} \\
& | & X & \text{(variable)} \\
& | & \mu X.\mathcal{G} & \text{(recursion)}
\end{array}
$$

Here global types are instances of choreographies in [BLZ09], but for recursion.
In [HYC08] $\alpha = k\langle a \rangle$, shuffle is called parallel and there is no sequence.
Delegation has been omitted.
In [CDP12] $+$ is $\vee$ and $\parallel$ is $\wedge$. Recursion takes the restricted form

$$\mathcal{G}^* = \mu X.\,\mathcal{G}; X + \mathsf{end}$$

Lts associated to a global type (see [BLZ09])

Let $\pi = p \xrightarrow{\alpha} q$ or $\pi = \checkmark$

$$\pi.\mathcal{G} \xrightarrow{\pi} \mathcal{G} \qquad \frac{\mathcal{G}_i \xrightarrow{\pi} \mathcal{G}'_i \quad i = 1,2}{\mathcal{G}_1 + \mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_i}$$

$$\text{end} \xrightarrow{\checkmark} \text{end} \qquad \frac{\mathcal{G}_1 \xrightarrow{\checkmark} \mathcal{G}'_1 \quad \mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_2}{\mathcal{G}_1;\mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_2} \qquad \frac{\mathcal{G}_1 \xrightarrow{\pi} \mathcal{G}'_1 \quad \pi \neq \checkmark}{\mathcal{G}_1;\mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_1;\mathcal{G}_2}$$

$$\frac{\mathcal{G}_1 \xrightarrow{\pi} \mathcal{G}'_1 \quad \pi \neq \checkmark}{\mathcal{G}_1 \| \mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_1 \| \mathcal{G}_2} \qquad \frac{\mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}'_2 \quad \pi \neq \checkmark}{\mathcal{G}_1 \| \mathcal{G}_2 \xrightarrow{\pi} \mathcal{G}_1 \| \mathcal{G}'_2} \qquad \frac{\mathcal{G}_1 \xrightarrow{\checkmark} \mathcal{G}'_1 \quad \mathcal{G}_2 \xrightarrow{\checkmark} \mathcal{G}'_2}{\mathcal{G}_1 \| \mathcal{G}_2 \xrightarrow{\checkmark} \mathcal{G}'_1 \| \mathcal{G}'_2}$$

Rules for (guarded) recursion are standard.

Local session types

$$T ::= \mathsf{end} \mid \sum_{i\in I} p_i?a_i.T_i \mid \bigoplus_{i\in I} p_i!a_i.T_i \mid X \mid \mu X.T$$

A *multiparty session* is a finite set:

$$\Delta = \{p_1 : T_1, \ldots, p_k : T_k\}$$

Over sessions we define the lts:

- $\{\ldots, p : \bigoplus_{i\in I} p_i!a_i.T_i, \ldots\} \longrightarrow \{\ldots, p : p_j!a_j.T_j, \ldots\}$ if $j \in I$ and $|I| > 1$
- $\{\ldots p : q_j!a_j.T, q : \sum_{i\in I} q_i?a_i.T_i', \ldots\} \xrightarrow{\pi} \{\ldots, p : T, q : T_j', \ldots\}$ if $\pi = p \xrightarrow{a_j} q_j$ for some $j \in I$
- $\{p_1 : \mathsf{end}, \ldots, p_k : \mathsf{end}\} \xrightarrow{\checkmark} \{p_1 : \mathsf{end}, \ldots, p_k : \mathsf{end}\}$

Projection

A *trace* $\sigma = \pi_0 :: \pi_1 :: \cdots$ is a finite or infinite sequence of actions.

Traces($\mathcal{G}$) is the set of all traces σ of maximal length s.t. for any finite prefix τ of σ there exists $\mathcal{G}'$ s.t. $\mathcal{G} \stackrel{\tau}{\Longrightarrow} \mathcal{G}'$.

Traces(Δ) is defined in the same way.

Let $\Delta = \{p_1 : T_1, \ldots, p_k : T_k\}$ be a session and $p_1, \ldots, p_k$ be the parties of a global type $\mathcal{G}$:

- Δ is a *projection* of $\mathcal{G}$ if Traces(Δ) = Traces($\mathcal{G}$)

Projection: example

- $\mathcal{G}_1 = (p \xrightarrow{a} q \| p \xrightarrow{b} q); (q \xrightarrow{c} p + q \xrightarrow{d} p)$

  $\Delta_1 = \{p : T,\ q : T'\}$ where

$$
\begin{array}{rcl}
T & = & q!a.q!b.(q?c + q?d) \ \oplus q!b.q!a.(q?c + q?d) \\
T' & = & p?a.p?b.(p!c \oplus p!d) \ + p?b.p?a.(p!c \oplus p!d)
\end{array}
$$

  then $\text{Traces}(\Delta_1) = \text{Traces}(\mathcal{G}_1)$ is the set

$$
\left\{
\begin{array}{c}
p \xrightarrow{a} q :: p \xrightarrow{b} q :: q \xrightarrow{c} p :: \checkmark^{\infty}, \\
p \xrightarrow{a} q :: p \xrightarrow{b} q :: q \xrightarrow{d} p :: \checkmark^{\infty}, \\
p \xrightarrow{b} q :: p \xrightarrow{a} q :: q \xrightarrow{c} p :: \checkmark^{\infty}, \\
p \xrightarrow{b} q :: p \xrightarrow{a} q :: q \xrightarrow{d} p :: \checkmark^{\infty}
\end{array}
\right\}
$$

Projection: counterexample $\mathcal{G}_2$

- (no sequentiality)

$$\mathcal{G}_2 = p \xrightarrow{a} q; r \xrightarrow{b} s, \quad \Delta_2 = \{p : q!a,\ q : p?a,\ r : s!b,\ s : r?b\}$$

but $r \xrightarrow{b} s :: p \xrightarrow{a} q :: \checkmark^{\infty} \in \mathrm{Traces}(\Delta_2) \setminus \mathrm{Traces}(\mathcal{G}_2)$

Projection: counterexample $\mathcal{G}_3$

- (no knowledge for choice)

$$\mathcal{G}_3 = (p \xrightarrow{a} q; q \xrightarrow{a} r; r \xrightarrow{a} p) + (p \xrightarrow{b} q; q \xrightarrow{a} r; r \xrightarrow{b} p)$$

$$\Delta_3 = \left\{ \begin{array}{c} p : (q!a.r?a) \oplus (q!b.r?b), \\ q : (p?a.r!a) + (p?b.r!a), \\ r : q?a.(p!a \oplus p!b) \end{array} \right\}$$

now $\sigma = p \xrightarrow{a} q :: q \xrightarrow{a} r :: r \xrightarrow{b} p \in \mathrm{Traces}(\Delta_3) \setminus \mathrm{Traces}(\mathcal{G}_3)$
because

$$\Delta_3 \stackrel{\sigma}{\Longrightarrow} \{p : r?a,\ q : \mathrm{end},\ r : p!b\} \not\longrightarrow$$

so σ is finite and $\mathrm{Traces}(\mathcal{G})$ is made of infinite traces only for all $\mathcal{G}$.

Projection: counterexample $\mathcal{G}_4$

- (non local choice)

$$\mathcal{G}_4 = p \xrightarrow{a} q \;+\; q \xrightarrow{b} p$$

If $\Delta_4 = \{p : q!a,\; q : p?a\}$ then

$$q \xrightarrow{b} p :: \checkmark^{\infty} \in \mathrm{Traces}(\mathcal{G}_4) \setminus \mathrm{Traces}(\Delta_4)$$

If $\Delta_4' = \{p : q!a \odot q?b, \ldots\}$ then $\odot \neq \oplus, +$.

Issues

Let $\mathcal{P}$ be a property of sets of traces: find a condition $\mathcal{C}$ on global types s.t.

$$\mathcal{G} \text{ satisfies } \mathcal{C} \;\&\; \Delta \text{ is a projection of } \mathcal{G} \Rightarrow \text{Traces}(\Delta) \in \mathcal{P}$$

- if $\mathcal{C}$ is trivial one can ensure $\mathcal{P} =$ *session fidelity*
- if $\mathcal{C}$ is *linearity* [HYC08] then $\mathcal{P} =$ *progress*
- if $\mathcal{C}$ is *well formedness* [CDP12] (or *connectedness* [BLZ09]) then $\mathcal{P} =$ *liveness*