An Abstraction from Power to Coalition Structures

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The concept of power plays an important role in the social sciences and Castelfranchi [2, 3] emphasizes the importance of this concept for multiagent systems. In [1] we build upon this work by distinguishing four viewpoints on multiagent systems: a mind structure, a power structure, a dependence structure and a coalition structure. These viewpoints are increasingly abstract conceptualizations of systems as collections of autonomous cognitive agents. In this paper we formally define one kind of power and coalition structures, and an abstraction from this power to its coalition structure.

1 Power Structure

Power structures are a more abstract conceptualization of a multiagent system than the usual one, because it does not mention actions or capabilities of agents. They directly characterize the power of agents as the goals they can achieve. We do not discuss here how a power structure can be derived from the mind structure in which actions are explicitly represented, see [1].

A power structure is composed of a set of agents *Ag*, a set of all goals G, a function $\mathit{goals}\colon \mathit{Ag} \to 2^G$ that associates with each agent the subset of goals G it desires to achieve, and, finally, a function $\mathit{pow}\colon 2^{\mathit{Ag}} \to 2^{2^G}$ that associates with a set of agents A the sets of goals they have the power to achieve, provided that the set of agents A is minimal with respect to set inclusion among sets of agents that can do so. In this paper we only consider power structures of which the function *pow* satisfies the following conditions:

1. If a set of agents has the power to achieve goal g, then there is at least an agent in *Ag* that desires this goal g.
2. All agents A are necessary to achieve a set of goals in $\mathit{pow}(A)$. There are no superfluous agents.
3. The function *pow* is continuous, in the sense that if a set of agents A has the power to achieve a set of goals D, then for all subsets of goals $D' \subseteq D$ there is a subset of agents $A' \subseteq A$ that has the power to achieve this subset of goals D'.

In a compositional power structure, the function *pow* also satisfies the following condition:

4. If two sets of agents A_1 and A_2 have the power to achieve respectively the sets of goals D_1 and D_2, then there exists a subset of all the agents $A_1 \cup A_2$, with the power to achieve all goals $D_1 \cup D_2$.

In the last condition, note that the set of agents with the power to achieve $D_1 \cup D_2$ is not necessarily $A_1 \cup A_2$, because an agent can become superfluous in the union. Compositional power structures are abstractions of conflict free multiagent systems, such as, for example, systems in which all actions can be executed simultaneously.

Definition 1 *A power structure, PS, is a tuple*

$$\langle \mathit{Ag}, G, \mathit{goals}\colon \mathit{Ag} \to 2^G, \mathit{pow}\colon 2^{\mathit{Ag}} \to 2^{2^G} \rangle$$

where Ag is a set of agents, G a set of goals, goals is a function that associates with each agent in Ag the subset of goals G it desires to achieve, and pow is a function that associates with every set of agents a set of sets of goals that the agents can achieve, such that for all sets of agents $A \subseteq \mathit{Ag}$ and for all sets of goals $D \in \mathit{pow}(A)$ the following conditions hold:

1. $D \subseteq \bigcup_{a \in \mathit{Ag}} \mathit{goals}(a)$
2. $\forall D' \supseteq D\, \forall A' \subset A\, [D' \notin \mathit{pow}(A')]$
3. $\forall D' \subseteq D\, \exists A' \subseteq A\ [D' \in \mathit{pow}(A')]$

A compositional power structure is a power structure of which the function pow also satisfies the following condition for all sets of agents $A_1, A_2 \subseteq \mathit{Ag}$ and for all sets of goals $D_1 \in \mathit{pow}(A_1)$ and $D_2 \in \mathit{pow}(A_2)$:

4. $\exists A \subseteq A_1 \cup A_2\ [D_1 \cup D_2 \in \mathit{pow}(A)]$

The strongest assumption of power structures seems to be the second condition, because it cannot discriminate between cases in which the agents can achieve a set of goals only by themselves, or they can achieve them either by themselves or they can achieve them by working together, and maybe more efficiently or cheaply. We do not further consider this issue of costs of actions in this paper, but we turn to a notion of coalitions.

2 Coalition Structure

A coalition structure describes a set of agents that *can* decide to work together to achieve a set of goals, and it thus represents the *possible* coalitions in a multiagent structure. It captures two properties of a coalition not captured by a power structure. The first property states that coalition structures consider only agents that are not altruistic, i.e., agents that accept to be partners in a coalition only if they benefit from it. The second property states that a union of sub-coalitions is not called a coalition if the formation of one of them cannot interfere with the formation of the others.

Now we provide a formal definition. A coalition structure has the same elements as a power structure, but it satisfies other conditions. It is composed of a set of agents *Ag*, a set of all goals G, a function $\mathit{goals}\colon \mathit{Ag} \to 2^G$ and a function $\mathit{coal}\colon 2^{\mathit{Ag}} \to 2^{2^G}$ that associates with a set of agents the sets of goals they can achieve by forming a coalition. If $\mathit{coal}(A)$ is empty for a set of agents A, then this set of agents cannot form a coalition at all. The function *coal* satisfies the following conditions, where the first one is a specialization of the first condition of the function *pow*, the second condition is identical to the second condition of the function *pow*, and the third and fourth condition correspond to the new properties of coalition structures:

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1. If a goal g is achieved by forming a coalition, then at least an agent in the coalition desires this goal g.
2. All agents A are necessary to achieve a set of goals in $coal(A)$. Coalitions do not admit superfluous members.
3. A coalition can be formed only if all the members benefit from it.
4. A coalition cannot be decomposed in sub-coalitions unless for each sub-coalition there is an agent necessary to achieve a goal of another sub-coalition, and there is at least an agent that benefits from another sub-coalition.

A compositional coalition structure is a coalition structure in which the function *coal* also satisfies the following condition, which is implied by the condition of compositional power structures:

5. The achievement capability of coalitions is monotone with respect to the containment relation for sets of agents: if a coalition of agents A_2 can achieve the set of goals $D_2 \in coal(A_2)$ and a sub-coalition $A_1 \subseteq A_2$ can achieve the set of goals D_1, then goals of the sub-coalition D_1 can be transferred to the goals of coalition D_2.

Definition 2 *Let $\mathcal{P}(S)$ be the set of all partitions of the set S.*
A coalition structure, CS, is a tuple

$$\langle Ag, G, goals\colon Ag \rightarrow 2^G, coal\colon 2^{Ag} \rightarrow 2^{2^G} \rangle$$

where Ag is a set of agents, G a set of goals, goals is a function that associates to each agent a subset of goals G it desires to achieve, and coal is a function that associates with each set of agents a set of sets of goals such that for all sets of agents $A \subseteq Ag$ and for all sets of goals $D \in coal(A)$ the following conditions hold:

1. $D \subseteq goals(A)$
2. $\forall D' \supseteq D \, \forall A' \subset A \; [D' \notin coal(A')]$
3. $\forall a \in A \; [D \cap goals(a) \neq \emptyset]$
4. $\forall \{A_1, \ldots, A_n\} \in \mathcal{P}(A) \; \forall \{D_1, \ldots, D_n\} \in \mathcal{P}(D)$
   $[(\forall i \; [D_i \in coal(A_i)]) \Longrightarrow$
   $(\exists \pi(n) \in \Pi(n) \; \forall i \; [goals(A_{\pi(i)}) \cap D_{\pi((i+1) \mod n)} \neq \emptyset])]$

where for a set $A \subseteq Ag$, $goals(A) = \bigcup_{a \in A} goals(a)$ and $\Pi(n)$ for the set of the permutations of $\{1, \ldots, n\}$.

A compositional coalition structure is a coalition structure of which the function coal also satisfies the following condition: for all sets of agents $A_1, A_2 \subseteq Ag$ and for all sets of goals $D_1 \in coal(A_1)$ and $D_2 \in coal(A_2)$:

5. $A_1 \subseteq A_2 \Longrightarrow \forall D \subseteq D_1 : \; [D \cup D_2 \in coal(A_2)]$

We formalized a weak notion of decomposition. E.g., the fourth condition of a more sophisticated definition also has to hold for a decomposition with one agent overlap between sub-coalitions.

3 Abstraction from power to coalition

An abstraction of a coalition structure from a power structure is derived as follows.

Definition 3 *Given $\langle Ag, G, goals : Ag \rightarrow 2^G, pow : 2^{Ag} \rightarrow 2^{2^G} \rangle$, the reduction of the power structure is defined as follows.*

1. *For all sets $A \subseteq Ag$, remove from $pow(A)$ all sets of goals that violate the first or third condition of a coalition structure.*
2. *For $i := 1$ to $|Ag|$, for any $A \subseteq Ag$ with $|A| = i$, remove from $pow(A)$ all sets of goals that violate the fourth condition.*

Theorem 1 *A reduced power structure satisfies all conditions of a coalition structure. A reduced compositional power structure satisfies all conditions of a compositional coalition structure.*

Definition 4 *A coalition structure CS is relative to a power structure PS iff CS is the reduction of the power structure, renaming pow to coal. In such cases, we say that the coalition structure CS is an abstraction of the power structure PS, and that the power structure PS is a refinement of the coalition structure CS.*

Theorem 2 *Given two functions $f, h\colon 2^{Ag} \rightarrow 2^{2^G}$, we say that $f \subseteq h$ if for all $A \subseteq Ag$ we have $f(A) \subseteq h(A)$. Let a coalition structure CS be an abstraction of a power structure PS. Then $coal \subseteq pow$ and there does not exist $f \subseteq pow$ such that CS with f is a coalition structure, and $coal \subset f$. The converse not necessarily holds.*

Theorem 3 *For every power structure, there is a unique coalition structure that is an abstraction of it. For every compositional power structure, there is a unique compositional coalition structure that is an abstraction of it.*

Theorem 4 *A refinement does not have to be unique.*
Proof. Consider three agents $Ag = \{a_1, a_2, a_3\}$, three goals $G = \{g_1, g_2, g_3\}$, and $goals(a_i) = \{g_i\}$ for $i = 1, 2, 3$. Moreover, consider PS_1 with $pow(\{a_1\}) = \{\{g_2\}\}$, $pow(\{a_2\}) = \{\{g_3\}\}$, and $pow(\{a_3\}) = \{\{g_1\}\}$, and PS_2 with $pow(\{a_1\}) = \{\{g_3\}\}$, $pow(\{a_2\}) = \{\{g_1\}\}$, and $pow(\{a_3\}) = \{\{g_2\}\}$ (and $pow(A_1 \cup A_2) = pow(A_1) \cup pow(A_2)$). Both PS_1 and PS_2 have the same abstraction with $coal(Ag) = \{G\}$, and for all $A \subset Ag : coal(A) = \emptyset$.

A drawback of the present definitions is that we do not have that for every coalition structure, there is a power structure that refines it, and likewise, for every compositional coalition structure, there is a compositional power structure that refines it.

4 Conclusions and related work

We define power structure as sets of goals agents can achieve, and coalition structures as reductions of power structures in which agents are not altruistic and sets of agents cannot be decomposed in sub-coalitions. Moreover, we define an abstraction from a power to its coalition structure. In further research we consider power and coalition structures based, e.g., on stronger decomposition conditions.

Conte and Sichman [4] use graph theory to emphasize the topology of achievement dependencies, and the possibility to form coalitions. We focus on the composition problem of different coalitions and we constrain coalitions not to be decomposable in unrelated coalitions. Shehory and Kraus [6] define coalition in terms of tasks and capabilities, instead of in terms of goals and power. Finally, Pauly [5] only considers coalitions that can achieve states, regardless of whether these states satisfy goals of the agents in the coalition, while we consider only coalitions where each agent both contributes to and receives from being in a coalition.

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