Prescriptive and Descriptive Obligations in Dynamic Epistemic Deontic Logic

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Abstract. Normative sentences can be used to change or to describe the normative system, known as prescriptive and descriptive obligations respectively. In applications of deontic logic it is important to distinguish these two uses of normative sentences. In this paper we show how they can be distinguished and analysed in a dynamic epistemic deontic logic.

1 Introduction

Alchourrón and Bulygin [1, 2] discuss the *possibility* of a logic of norms, which they distinguish from the logic of normative propositions. Roughly, the distinction between norms and normative propositions is that the former are prescriptive whereas the latter are descriptive. In the second sense, the sentence 'it is obligatory to keep right on the streets' is a description of the fact that a certain normative system contains an obligation to keep right on the streets. In the first sense this statement is the obligation of traffic law itself. This distinction goes back to an old philosophical problem discussed by Von Wright [17, 16], who was hesitant to call deontic formulas 'logical truths,' because "it seems to be a matter of extra-logical decision when we shall say that 'there are' or 'are not' such and such norms." Makinson [12] turns this fundamental problem into the central challenge in deontic logic, which led to new developments over the past decade such as deontic update semantics [15], input/output logic [13], imperative based deontic logic [10], and more.

The relevance of the distinction between prescriptive and descriptive obligations, and the related fundamental problem that norms do not have truth values, is not only theoretical and conceptual, but it has important practical implications. If one agent tells another agent that he is obliged to register before he can use a web service, but the second agent would like to disagree, then the agent should know whether the agent is creating a norm for him, or whether he is describing an existing norm of the normative system. In the first case he may disagree by responding that the agent is not authorized to create obligations for him, in the second case he may argue that the norm does not apply to him, or that the norm does not exist. To model such distinctions, we need to model not only the normative system, but also how norms can be changed over time. In this paper we therefore study the following question:

– How can we use dynamic epistemic deontic logic to analyze the distinction and relations between prescriptive and descriptive obligations?

We recently introduced a general and expressive Dynamic Epistemic Deontic Logic [4] combining a simplified version of Castañeda's deontic logic [8] with a dynamic epistemic logic. This logic can express the conditional character of norms, study the interaction between epistemic and deontic notions, and model norm dynamics. These three features were motivated by their use in multi-agent systems (MAS). First, in multi-agent systems it is necessary to express realistic regulations, which have a conditional character. Secondly, communication is an essential part of multi-agent systems, and this raises the issue of what it is permitted, prohibited or obliged to know by agents, for example, when modelling privacy regulations. A challenge here is that the interaction between epistemic and deontic notions is plagued by Åqvist's paradox of the knower [3]. Thirdly, normative multi-agent systems have a dynamic character, as witnessed by the second definition of normative multi-agent system provided in [7]. These last two issues, communication and dynamics are both useful for distinguishing when existing norms are communicated from the case where a norm is actually put into existence by a declaration, i.e., Alchourrón's distinction between the descriptive and prescriptive use of norms.

We use the following running example to exemplify the static and dynamic features of our logic.

Example 1. John is driving on a highway with speed limit 130 km/h. He does not know whether he is speeding ($\neg B\mathit{speed} \wedge \neg B\neg \mathit{speed}$), because his speed controller is defective. But it is obligatory by the law that he knows whether he is speeding: $O(B\mathit{speed} \vee B\neg\mathit{speed})$ (epistemic norm 1). Besides, if he drives too fast, he should slow down: $\mathit{speed} \rightarrow O\mathit{slow}$. He should also know that if he drives too fast he has to slow down: $OB'(\mathit{speed} \rightarrow O\mathit{slow})$ (epistemic norm 2). On the other hand, if he does not drive too fast, he is still permitted to speed up (and thus not to slow down), if he wants to overtake for example: $\neg\mathit{speed} \rightarrow P\neg\mathit{slow}$. Now we consider two kinds of normative events:

Prescriptive event. He comes upon road works and there is a sign announcing that he should slow down. This event can be modeled by the communicative act $[\mathit{slow}!!]$.

Descriptive event. A police car behind him tells him to slow down. This event can be modeled by the communicative act $[O\mathit{slow}!]$. As a result he learns that he is speeding.

The paper is structured as follow. In Section 2 we introduce an epistemic deontic logic which allows to express epistemic norms and which avoids Åqvist's paradox. In Section 3 we extend the logic by introducing update operators which change beliefs and norms, and show how the distinction between descriptive and prescriptive norms can be made in our logic if we map this distinction to the context of communication.

2 Epistemic Deontic Logic (EDL)

2.1 Propositions vs practitions

Because of its clear and natural distinction between propositions and practitions and its modal-like character, the well known deontic logic of Castañeda [8] lends itself very well to the introduction of an epistemic logic. Starting from a linguistic analysis, the insight of Castañeda's well known approach to deontic logic is to acknowledge the grammatical duality of expressions depending whether they are within or without the scope of deontic operators [8]. This leads him formally to introduce two sets of propositional letters: Φ^{ϕ} called propositions which cannot *alone* be the foci of deontic operators, unlike Φ^{α} called practitions. The former are usually expressed grammatically in the indicative form and the latter are usually expressed grammatically in the infinitive/subjunctive form. For example, "John is slowing down" in the indicative form is a proposition, but the same sentence in "*it is obligatory* that John slows down" in subjunctive/infinitive form is a practition. He then defines more general propositions $\mathcal{L}_{DL}^{\phi}$ and practitions $\mathcal{L}_{DL}^{\alpha}$ as follows.

$$\mathcal{L}_{DL}^{\phi} : \phi ::= p \mid \phi \wedge \phi \mid \neg\phi \mid O\alpha$$
$$\mathcal{L}_{DL}^{\alpha} : \alpha ::= \beta \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \wedge \phi \mid \phi \wedge \alpha$$

where β ranges over Φ^{α} and p over Φ^{ϕ}. We define the language $\mathcal{L}_{DL} = \mathcal{L}_{DL}^{\phi} \cup \mathcal{L}_{DL}^{\alpha}$, whose formulas are generally denoted ϕ^*. In the sequel, $P\alpha$ is an abbreviation for $\neg O \neg\alpha$. $O\alpha$ reads 'it is obligatory that α' and $P\alpha$ reads 'it is permitted that α'.

We now propose a semantics based on modal logic which is equivalent to the one of Castañeda, in the sense that any 'Castañeda'-model [8] can be transformed into a DL-model satisfying the same formulas, and vice versa.

Condition $(*)$ below ensures formally that conditional norms of the form "it is obligatory that if John knows that he drives too fast then he slows down" ($O(p \rightarrow \alpha)$) are equivalent to "if John knows that he drives too fast then it is obligatory that he slows down" ($p \rightarrow O\alpha$): $\models O(p \rightarrow \alpha) \leftrightarrow (p \rightarrow O\alpha)$.

Definition 1. *A DL-model M is a tuple $M = (W, D, V)$ where W is a non-empty set of possible worlds, D is a serial*[1] *accessibility relation on W and V is a valuation which assigns to each propositional letter $p^* \in \Phi^{\phi} \cup \Phi^{\alpha}$ a subset of W, such that for all $w \in W$, all $p \in \Phi^{\phi}$,*

$$V(p) \cap (D(w) \cup \{w\}) = D(w) \cup \{w\} \text{ or } \emptyset \qquad (*)$$

Let $M = (W, D, V)$ be a DL-model, $w \in W$ and $\phi^* \in \mathcal{L}_{DL}$. (M, w) is called a pointed DL-model. We define $M, w \models \phi^*$ inductively as follows.

$$\begin{array}{lll} M, w \models p^* & \textit{iff} & w \in V(p^*) \\ M, w \models \phi^* \wedge \psi^* & \textit{iff} & M, w \models \phi^* \textit{ and } M, w \models \psi^* \\ M, w \models \neg\phi^* & \textit{iff} & \textit{not } M, w \models \phi^* \\ M, w \models O\alpha & \textit{iff} & \textit{for all } v \in D(w),\ M, v \models \alpha. \end{array}$$

[1] A relation R is serial iff $R(w) \neq \emptyset$ for all $w \in W$.

2.2 Adding beliefs

Just as practitions are the foci of deontic operators, propositions are dually the foci of knowledge operators, as pointed out by Castañeda [9]. An expression ϕ in the scope of a belief operator $B\phi$ is always in the indicative form and never in the subjunctive/infinitive form, even if $B\phi$ is in the scope of a deontic operator O. We extend Castañeda [9]'s intuition to the context of epistemic permissions and obligations. In a deontic setting the reading of the term knowledge or belief can also be twofold: either as a proposition or as a practition. On the one hand, in the sentence "it is obligatory that John *knows* / for John *to know* that he is driving too fast" the verb 'to know' is the focus of a deontic operator and is in the subjunctive/infinitive form. On the other hand, the sentence "John *knows* that he is driving too fast' alone describes a circumstance and the interpretation of the verb 'to know' in the indicative form matches the one usually studied in epistemic logic. The former use of the term knowledge within the scope of a deontic operator is not studied in epistemic logic. For these reasons we enrich the language $\mathcal{L}_{DL}$ with two knowledge modalities, one for propositions and the other for practitions. This yields the following language $\mathcal{L}_{EDL} = \mathcal{L}^{\phi}_{EDL} \cup \mathcal{L}^{\alpha}_{EDL}$ whose formulas are generally denoted ϕ^*.

$$\mathcal{L}^{\phi}_{EDL} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid B\phi \mid O\alpha$$
$$\mathcal{L}^{\alpha}_{EDL} : \alpha ::= \beta \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \wedge \phi \mid \phi \wedge \alpha \mid B'\phi$$

where p ranges over Φ^{ϕ} and β over Φ^{α}. As argued above we do not allow formulas of the form $B\alpha$ or $B'\alpha$ because they are linguistically meaningless, which is actually in line with Castañeda [9]. $B\phi$ reads 'the agent believes ϕ'.

Definition 2. *An* EDL-model M *is a tuple* $M = (W, D, R, R', V)$ *where* W *is a non-empty set of possible worlds,* R, R' *and* D *are accessibility relations on* W, D *being serial, and* V *is a valuation such that:*

> *for all* $w \in W$, *all* $v, v' \in D(w) \cup \{w\}$, (M, v) *is RD-bisimilar to* (M, v').[2] $(**)$

The truth conditions for B *and* B' *are given by:*

$$M, w \models B\phi \quad \text{iff} \quad \text{for all } v \in R(w),\ M, v \models \phi$$
$$M, w \models B'\phi \quad \text{iff} \quad \text{for all } v \in R'(w),\ M, v \models \phi$$

$M \models \phi$ *if for all* $w \in W$, $M, w \models \phi$. (M, w) *is called a pointed EDL-model.*

Note that condition $(**)$ is a generalization of condition $(*)$ to the epistemic setting: the worlds of $D(w) \cup \{w\}$ are not only 'propositionally bisimilar' as in $(*)$, but also 'epistemically (and deontically) bisimilar'.

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[2] Two pointed models (M, v) and (M', v') are RD-bisimilar if there is a relation on $W \times W'$ satisfying the base condition for Φ^{ϕ} and the back and forth conditions for R and D (see Blackburn *et al.* [6] for details).

We do not assume any logical property for our notion of belief (such as consistency or introspection) because it is not really relevant for the topic of this paper. For the same reason, the operator B stands alternatively for knowledge or belief.

Theorem 1. *The semantics of* $\mathcal{L}_{EDL}$ *is* sound and complete *with respect to the logic* L_{EDL} *axiomatized as follows:*

A_1 *All propositional tautologies based on* $\Phi^\phi \cup \Phi^\alpha$
A_2 $\vdash \phi \wedge O\alpha \rightarrow O(\phi \wedge \alpha)$
A_3 $\vdash O\alpha \rightarrow \neg O \neg \alpha$
A_4 $\vdash O(\alpha \rightarrow \alpha') \rightarrow (O\alpha \rightarrow O\alpha')$
A_5 $\vdash B^*(\phi^* \rightarrow \psi^*) \rightarrow (B^*\phi^* \rightarrow B^*\psi^*)$
R_1 *If* $\vdash \alpha$ *then* $\vdash O\alpha$
R_2 *If* $\vdash \phi^*$ *then* $\vdash B^*\phi^*$
R_3 *If* $\vdash \phi^* \rightarrow \psi^*$ *and* $\vdash \phi^*$ *then* $\vdash \psi^*$

where B^* *stands for* B *or* B'.

Note that axioms A_1 to A_4 and rules R_1 and R_3 provide an alternative axiomatization of Castañeda's language $\mathcal{L}_{DL}$.

Proof. Soundness is routine. We only prove completeness by building the canonical model of our logic. Let W be the set of all maximal consistent subsets of $\mathcal{L}_{EDL}$. For all $\Gamma, \Gamma' \in W$, we set $\Gamma' \in R(\Gamma)$ iff for all $B\phi \in \Gamma, \phi \in \Gamma'$. We define O and R' similarly. Besides, for all $\Gamma \in W$, $\Gamma \in V(p)$ iff $p \in \Gamma$ and $\Gamma \in V(\beta)$ iff $\beta \in \Gamma$. We have therefore defined the canonical model $M = (W, D, R, R', V)$. We now show by induction on ϕ the 'truth lemma': for all $\Gamma \in W$ and $\phi \in \mathcal{L}_{EDL}$, $M, \Gamma \models \phi$ iff $\phi \in \Gamma$ $(*)$. If $\Gamma = p$ then $(*)$ clearly holds. The other boolean cases clearly work by induction hypothesis. Assume $\phi = B\phi'$. If $B\phi' \in \Gamma$ then for all $\Gamma' \in R(\Gamma)$, $\phi' \in \Gamma'$ by definition of R. So $M, \Gamma' \models \phi'$ for all $\Gamma' \in R(\Gamma)$ by induction hypothesis, i.e., $M, \Gamma \models B\phi'$. If $M, \Gamma \models B\phi'$ then assume that $S \subseteq \{\phi \in \mathcal{L}_{EDL} \mid B\phi \in \Gamma\} \cup \{\neg\phi'\}$ is consistent. It follows that there is $\Gamma^0 \in W$ such that $S \subseteq \Gamma^0$. So there is $\Gamma^0 \in R(\Gamma)$ such that $\neg\phi' \in \Gamma^0$. Therefore $M, \Gamma \models \neg B\phi'$ which is absurd. So S is inconsistent and so there must be $\phi^1, \ldots, \phi^n \in S$ such that $\vdash (\phi^1 \wedge \ldots \wedge \phi^n) \rightarrow \phi'$. By R_2 and A_5 we get $\vdash (B\phi^1 \wedge \ldots \wedge B\phi^n) \rightarrow B\phi'$ and because $B\phi^i \in \Gamma$, we finally have $B\phi' \in \Gamma$. The proof is similar for the operators O and B'. One can also easily show that D is serial.

Now we have to show that condition $(**)$ holds in our canonical model M. We first show that for all $\Gamma \in W$, all $\Gamma', \Gamma'' \in D(\Gamma) \cup \{\Gamma\}$, $\Gamma' \leftrightsquigarrow \Gamma''$,i.e., for all $\phi \in \mathcal{L}^\phi_{EDL}$, $\phi \in \Gamma'$ iff $\phi \in \Gamma''$. Let $\phi \in \mathcal{L}^\phi_{EDL}$ and assume $\phi \in \Gamma'$. If $\phi \notin \Gamma$ then $\neg\phi \in \Gamma$, and $O\alpha \in \Gamma$ for some $\alpha \in \mathcal{L}^\alpha_{EDL}$. So $M, \Gamma \models \neg\phi \wedge O\alpha$, therefore $M, \Gamma \models O(\neg\phi \wedge \alpha)$. Then $M, \Gamma' \models \neg\phi \wedge \alpha$, and so $\neg\phi \in \Gamma'$. This is impossible, so $\phi \in \Gamma$. By the same reasoning we get that $\phi \in \Gamma''$. Likewise vice versa. We now show that $\leftrightsquigarrow$ is a RD-bisimulation relation. Assume $\Gamma \leftrightsquigarrow \Gamma'$. The base case for Φ^ϕ clearly works. We prove the forth condition for R. Let $\Gamma_1 \in R(\Gamma)$

and let $\Gamma_1^* = \{\phi \in \mathcal{L}_{EDL}^{\phi} \mid \phi \in \Gamma_1\}$ and assume that for all $\Gamma_1' \in R(\Gamma')$ it is not the case that $\Gamma_1 \leftrightsquigarrow \Gamma_1'$, i.e., $\Gamma_1^* \nsubseteq \Gamma_1'$. Let $S_1 = \Gamma_1^* - \bigcup_{\Gamma_1' \in R(\Gamma')} \Gamma_1'$ and let us define $S = S_1 \cup S_2$ where $S_2 = \{\phi \in \mathcal{L}_{EDL}^{\phi} \mid B\phi \in \Gamma\}$. S is consistent because $S \subseteq \Gamma_1$. So there is $\Gamma_2 \in W$ such that $S \subseteq \Gamma_2$. But $\{\phi \in \mathcal{L}_{EDL} \mid B\phi \in \Gamma'\} = \{\phi \in \mathcal{L}_{EDL}^{\phi} \mid B\phi \in \Gamma'\} = \{\phi \in \mathcal{L}_{EDL}^{\phi} \mid B\phi \in \Gamma\}$ because $\Gamma \leftrightsquigarrow \Gamma'$. $\Gamma_2 \in R(\Gamma')$ and $S_1 \subseteq \Gamma_2$ which is impossible by assumption. So there is $\Gamma_1' \in R(\Gamma)$ such that $\Gamma^* \subseteq \Gamma_1'$, i.e., such that $\Gamma_1 \leftrightsquigarrow \Gamma_1'$. the same reasoning applies for the back condition. It also applies for the back and forth conditions for D by replacing S_2 by $S_2' = \{\alpha \in \mathcal{L}_{EDL}^{\alpha} \mid O\alpha \in \Gamma\}$.

Theorem 2. L_{EDL} *is* decidable

Proof (sketch). One can show that L_{EDL} has the finite model property by adapting the selection method [6].

2.3 Epistemic norms and Åqvist's paradox

Our logic can express conditional norms because our logic inherits from the properties of Castañeda's deontic logic (such as $\vdash \phi \rightarrow O\alpha \leftrightarrow O(\phi \rightarrow \alpha)$). Due to its combination of the deontic and epistemic notions, it can also express the knowledge-based obligations of [14]. But because our combination is quite general, we can also express *epistemic norms.* Formally, an *epistemic norm* n is a formula of the following form, where φ is an epistemic formula and ψ a propositional formula.

$$\begin{array}{ll} \varphi \rightarrow PB'\psi & \varphi \rightarrow OB'\psi \\ \varphi \rightarrow \neg PB'\psi & \varphi \rightarrow \neg OB'\psi \end{array}$$

Example 2 (driving example). Assume that John does not know whether he is speeding on a highway ($\neg B\neg speed \wedge \neg B speed$) because his speed controller is defective. As a matter of fact, by law, he should know whether he drives too fast (n_1). Besides, he should also know that if he speeds, then he has to slow down (n_2). These two epistemic norms are formalized as follows:

$$\begin{array}{c} n_1 = O(B' speed \vee B'\neg speed) \\ n_2 = OB'(speed \rightarrow O slow) \\ n_3 = B speed \rightarrow O slow \end{array}$$

where *speed* stands for 'John is speeding' and *slow* stands for 'slow down'. Note that n_3 is an epistemic norm with an epistemic condition.

This situation is depicted in the EDL-model M of Figure 1, where *speed* stands for 'John is driving too fast' and *slow* for the practition 'slow down'. The dotted arrows correspond to the deontic accessibility relation D and the plain arrows correspond to accessibility relations R and R'. w corresponds to the actual world. We therefore have $M, w \models (\neg B speed \wedge \neg B\neg speed) \wedge B(speed \leftrightarrow O slow) \wedge O(B' speed \vee B'\neg speed)$: John does not know whether he is driving too fast, but he knows that it is obligatory that he slows down if and only if he is speeding, and he is obliged to know whether he is driving too fast.

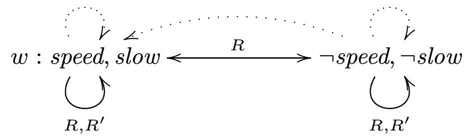

Fig. 1. Driving example

Epistemic norms should be free from paradoxes due to the interaction between epistemic and deontic modalities. Åqvist paradox of the knower is not possible in our framework, due to the distinction between propositions and practitions. In the following, we rephrase the paradox in our running example:

Example 3. Assume that if John is speeding, he should know this. Moreover, he is actually speeding. However, in our logic, even if knowing something implies that it is true, it does not follow paradoxically that he should speed:

$$\{\mathit{speed}, \mathit{speed} \rightarrow OB'\mathit{speed}\} \vdash OB'\mathit{speed}$$

but even if

$$B'\mathit{speed} \rightarrow \mathit{speed}$$

due to Axiom T for knowledge, one cannot derive $O\mathit{speed}$:

$$\nvdash OB'\mathit{speed} \rightarrow O\mathit{speed}$$

due to the fact that *speed* is a proposition and it cannot appear in an obligation as a practition.

3 Dynamic Epistemic Deontic Logic (DEDL)

3.1 Changing norms and beliefs

We now want to add dynamics to the picture by means of communicative acts made to the agent. The content of these communicative acts can affect the situation in two ways: either it affects the epistemic realm (represented in a EDL-model by the relation R) or it affects the normative realm (represented in a EDL-model by the relations R' and D). This leads us to enrich the language $\mathcal{L}_{EDL}$ with two dynamic operators $[\phi!]$ and $[\phi^*!!]$, yielding the language $\mathcal{L}_{DEDL}$, whose formulas are generally denoted ϕ^*:

$$\mathcal{L}^{\phi}_{DEDL} : \phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid B\phi \mid O\alpha \mid [\phi!]\phi \mid [\phi^*!!]\phi$$
$$\mathcal{L}^{\alpha}_{DEDL} : \alpha ::= \beta \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \wedge \phi \mid \phi \wedge \alpha \mid B'\phi \mid [\phi!]\alpha \mid [\phi^*!!]\alpha$$

where p ranges over Φ^{ϕ}, β over Φ^{α}.

$[\psi!]\phi$ reads 'after learning ψ, ϕ holds', and $[\psi^*!!]\phi$ reads 'after the promulgation/enforcement of ψ^*, ϕ holds'. Note that it is possible that $\psi^* \in \mathcal{L}^{\phi}_{EDL}$ because propositions can affect the normative realm via R'. The semantics of these dynamic operators is inspired by Kooi [11] and defined as follows.

Definition 3. *Let $M = (W, D, R, R', V)$ be an EDL-model, $\phi \in \mathcal{L}^{\phi}_{EDL}$ and $\psi^* \in \mathcal{L}_{EDL}$. We define the EDL-models $M * \psi!$ and $M * \psi^*!!$ as follows.*

- $M * \psi! = (W, D, R!, R', V)$ *where for all* $w \in W$, $R!(w) = R(w) \cap ||\psi||$.
- $M * \psi^*!! = (W, D!!, R, R'!!, V)$ *where for all* $w \in W$,
$R'!!(w) = \begin{cases} R'(w) \cap ||\psi^*|| & \text{if } \psi^* \in \mathcal{L}^{\phi}_{EDL} \\ R'(w) & \text{otherwise.} \end{cases}$
$D!!(w) = \begin{cases} D(w) \cap ||\psi^*|| & \text{if } \psi^* \in \mathcal{L}^{\alpha}_{EDL} \text{ and } M, w \models P\psi^* \\ D(w) & \text{otherwise.} \end{cases}$

where $||\phi^*|| = \{v \in M \mid M, v \models \phi^*\}$. The truth conditions:

$$\begin{array}{lll} M, w \models [\psi!]\phi^* & \text{iff} & M * \psi!, w \models \phi^* \\ M, w \models [\psi^*!!]\phi^* & \text{iff} & M * \psi^*!!, w \models \phi^*. \end{array}$$

Definition 4. *The logic L_{DEDL} is axiomatized as follows:*

L_{EDL}	*All the axiom schemes and inference rules of* L_{EDL}
A_6	$\vdash [\psi!]B\phi \leftrightarrow B(\psi \to [\psi!]\phi)$
A_7	$\vdash [\psi!]B'\phi \leftrightarrow B'[\psi!]\phi$
A_8	$\vdash [\psi!]O\alpha \leftrightarrow O[\psi!]\alpha$
A_9	$\vdash [\psi^*!!]B\phi \leftrightarrow B[\psi^*!!]\phi$
A_{10}	$\vdash [\psi!!]B'\phi \leftrightarrow B'(\psi \to [\psi!!]\phi)$
A_{11}	$\vdash [\alpha!!]B'\phi \leftrightarrow B'[\alpha!!]\phi$
A_{12}	$\vdash [\psi!!]O\alpha \leftrightarrow O[\psi!!]\alpha$
A_{13}	$\vdash [\alpha!!]O\alpha' \leftrightarrow (P\alpha \to O(\alpha \to [\alpha!!]\alpha')) \wedge$ $(\neg P\alpha \to O[\alpha!!]\alpha')$
A_{14}	$\vdash \Box p \leftrightarrow p$
A_{15}	$\vdash \Box\beta \leftrightarrow \beta$
A_{16}	$\vdash \Box\neg\phi^* \leftrightarrow \neg\Box\phi^*$
A_{17}	$\vdash \Box(\phi^* \to \psi^*) \to (\Box\phi^* \to \Box\psi^*)$
R_4	*If* $\vdash \phi^*$ *then* $\vdash \Box\phi^*$

where $\Box$ stands for $[\psi!]$ or $[\chi!!]$.

Proposition 1. *For all $\phi \in \mathcal{L}^{\phi}_{DEDL}$ there is $\phi' \in \mathcal{L}^{\phi}_{EDL}$ such that $\vdash \phi \leftrightarrow \phi'$. For all $\alpha \in \mathcal{L}^{\alpha}_{DEDL}$ there is $\alpha' \in \mathcal{L}^{\alpha}_{EDL}$ such that $\vdash \alpha \leftrightarrow \alpha'$.*

Proof (sketch). First, note that if ψ is a formula without dynamic operator then one shows by induction on ψ using A_6 to A_{16} that $\Box\psi$ is provably equivalent to a formula ψ' without dynamic operator. Now if ϕ is an arbitrary formula with n dynamic operators, it has a sub-formula of the form $\Box\psi$ where ψ is without dynamic operators which is equivalent to a formula ψ' without dynamic operators. So we just substitute $\Box\psi$ by ψ' in ϕ and we get a provably equivalent formula thanks to A_{17} and R_4 with $n-1$ dynamic operators. We then iterate the process.

As usual in dynamic epistemic logic, we use the previous key proposition to prove the following theorem.

Theorem 3. *The semantics of $\mathcal{L}_{DEDL}$ is* sound and complete *with respect to the logic L_{DEDL}. L_{DEDL} is also* decidable*.*

Proof. The soundness part is routine. Let $\phi \in \mathcal{L}_{DEDL}$ such that $\vdash \phi$. Then there is $\phi' \in \mathcal{L}_{EDL}$ such that $\vdash \phi \leftrightarrow \phi'$ by the previous proposition, and therefore $\models \phi \leftrightarrow \phi'$ by soundness. But $\models \phi'$ by Theorem 1, so $\models \phi$ as well. Decidability is proved similarly.

In dynamic epistemic logic, Balbiani *et al.* [5] is the closest work to ours. They focus in a multi-agent setting on the notion of permission to announce. They provide a sound, complete and decidable logic by enriching public announcement logic with the operator $P(\psi, \phi)$ which reads 'after ψ has been publicly announced, it is permitted to say ϕ'.

Changing beliefs: $[\phi!]$*.* Our logic is a dynamic epistemic logic, which allows to express communicative acts changing the beliefs of agents.

Example 4. Let us take up Example 2. John is driving too fast, and a policeman who detected this using a radar informs him:

$$M, w \models \mathit{speed} \wedge [\mathit{speed}!]B\,\mathit{speed}$$

After the communicative act John knows that he is driving too fast. The resulting situation is depicted in Figure 2.

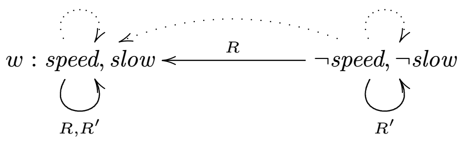

Fig. 2. Update by *speed!*

Changing norms: $[\phi!!]$*.* Our logic is a dynamic deontic logic, which allows to express communicative acts changing the norms.

Example 5. Let us take up Example 2 again. John comes upon road works and there is a sign announcing that he should slow down. This event can be modeled by the communicative act $[\mathit{slow}!!]$:

$$M, w \models [\mathit{slow}!!](O\mathit{slow} \wedge BO\mathit{slow})$$

After this communicative act, it is obligatory for John to slow down and he knows this. This resulting situation is depicted in Figure 3

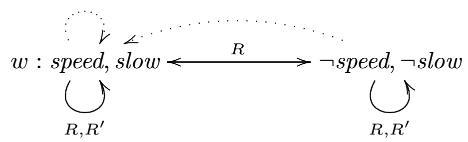

Fig. 3. Update by *slow*!!

3.2 Norms and Normative Propositions

Alchourrón and Bulygin [1, 2] discussed the *possibility* of a logic of norms, which they distinguish from the logic of normative propositions. Alchourrón explains the distinction with the following box metaphor.

> "We may depict the difference between the descriptive meaning (normative propositions) and the prescriptive meaning (norm) of deontic sentences by means of thinking the obligatory sets as well as the permitted sets as different boxes ready to be filled. When the authority α uses a deontic sentence prescriptively to norm an action, his activity belongs to the same category as *putting something into a box.* When α, or someone else, uses the deontic sentence descriptively his activity belongs to the same category as *making a picture of* α *putting something into a box.* A proposition is like a picture of reality, so to assert a proposition is like making a picture of reality. On the other hand to issue (enact) a norm is like putting something in a box. It is a way of creating something, of building a part of reality (the normative qualification of an action) with the purpose that the addressees have the option to perform the authorized actions while performing the commanded actions." [1]

In our logic we can distinguish Alchourrón's distinction between descriptive and prescriptive norms. We map this distinction to the context of communication. The descriptive communicative act of a police car behind John telling him to slow down can be modeled by the communicative act [O*slow*!]. Note that informing about the existence of a norm can enable the audience to know more information: for example, if it is obligatory to slow down when speeding, John learns that he is speeding. The prescriptive communicative act of John being informed by the sign that he should slow down can be modeled by the communicative act [*slow*!!].

This mapping allows to understand the role of agent systems in deontic logic, since a traditional problem can be solved by stating it in terms of interaction among agents.

Example 6. Let us take up Example 2. Concerning the descriptive character of norms, we model the action of communicating that there is a norm obliging John to slow down as the announcement of the obligation (O*slow*!). The resulting situation is the same as the one depicted in Figure 2. After such announcement to John, not only he believes that he is obliged to slow down but also that he is speeding:

$$M, w \models \neg B\mathit{speed} \wedge [O\mathit{slow}!](BO\mathit{slow} \wedge B\mathit{speed})$$

The inference $[O\mathit{slow}!]B\mathit{speed}$ is possible if speeding up obliges to slow down:

$$\mathit{speed} \rightarrow O\mathit{slow}$$

Note that *slow* is a practition, since it is in the scope of a deontic operator.

Concerning the prescriptive character of norms, we model the action of putting a norm into existence, for example, by the road works sign telling to slow down ($\mathit{slow}!!$). The resulting situation is depicted in Figure 3. Note that, in this case, even if $\mathit{speed} \rightarrow O\mathit{fine}$, we cannot derive that John knows that he is speeding too much:

$$M, w \models \neg B\mathit{speed} \wedge [\mathit{slow}!!](O\mathit{slow} \wedge \neg B\mathit{speed}).$$

4 Conclusions

Distinguishing the prescriptive and descriptive use of language is a classical challenge from deontic logic with practical consequences. If one agent tells another agent that he is obliged to do something, but the second agent would like to disagree, then the second agent should know whether the agent is creating a norm for him, or whether he is describing an existing normative system. In the first case he may disagree by responding that the agent is not authorized to create obligations for him, in the second case he may argue that the norm does not apply to him, or that the norm does not exist. Several formal systems therefore distinguish between prescriptive and descriptive obligations, but thus far the distinction was not analyzed in more detail, and the two kinds of obligations were not related to each other.

In this paper, we give a more detailed analysis by modeling besides the normative system also the epistemic states of the agents, and how norms can be changed over time. Few articles in deontic logic deal with the interaction among deontic and epistemic notions, though they often entertain a tight relationship. Citizens *must* often *know* their *obligations*, e.g., people should know that it is forbidden to drive too fast. Moreover, some obligations hold only in an epistemic context, e.g., John is *obliged* to slow down if he *knows* that he is driving fast [14]. To specify such examples of autonomous agents acting within a normative system, there is a need for the logical formalization of these relationships. To model the interaction between epistemic and normative notions in a dynamic setting we introduced a general Dynamic Epistemic Deontic Logic. The logic extends a simplified version of Castañeda's deontic logic of practitions and propositions with epistemic and dynamic update operators. It combines epistemic and deontic features to express the notions of permitted and obligatory beliefs.The paradox of the knower of Åqvist is analyzed by restricting the language.

Further research concerns making the logic multi-agent, to study the implications of our approach for contrary to duties and deontic detachment.

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