Modelling a Car Safety Controller in Road Tunnels using Hybrid Petri Nets

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Abstract

In the last years, major accidents have occurred in road tunnels with human losses (e.g. Mont Blanc, March '99, St. Gotthard, Oct '01 and Frejus, June '05). The accidents have caused a large debate on how to increase road tunnel safety. One way to reduce risk is to minimize the undesired disturbances of the traffic flow by forcing vehicles to respect speed limits and safety distances. This paper investigates this specific issue and proposes a hybrid modeling approach based on fluid Petri nets (FPN). An FPN is used to represent a controller that keeps track of two consecutive vehicles, controls their speed and distance and issues alarms when the prescribed limits are violated. In the second part of the paper, it is shown that FPN is a valid paradigm to model the dynamics of a car in a detailed way.

1 Introduction

In the last years several proposals have been discussed on how to increase road tunnel safety. In 2002, the EU proposed a new Directive [1] to achieve a uniform and high level protection for all citizens driving through tunnels. Furthermore, several projects have been funded by EU with the aim of studying and improving road tunnel safety. The project Safetunnel [2] stated as a main objective "To contribute to reducing the number of accident inside road tunnels through preventive safety measurements". The Safe Tunnel SafeT network [3] is aimed at analysing existing literature and current practices on tunnel safety, and make harmonised guidelines for tunnel safety. The main facts about road tunnel accidents can be expressed in the following issues [10]. The number of accidents is higher outside than inside tunnels, but the accident severity is higher inside than outside tunnels. Collisions between vehicles are mainly rear end collisions; cruise control can help to reduce them dramatically. For what concerns long tunnels, collisions between cars are less than collisions with infrastructures.

As suggested in [6], a safety goal would be to minimize the disturbances of the traffic flow, and to avoid congestion and the presence of obstacles or obstructions. To achieve this goal, measures should be taken to prevent traffic speed to increase over a critical level, and to assure the preservation of safety distances between vehicles.

The safety requirements for road tunnels should take benefits from the current trend in the automotive industry to increase the power of programmable electronic systems [9]. This trend will lead to the emergence of safety related onboard electronic equipment that will provide a number of safety related functions and will improve the overall safety of the vehicle. This paper concentrates on an important aspect of the tunnel safety: the control of the speed limits and the safety distances. The control system under study is a typical case of hybrid system where a discrete state controller operates on continuous quantities like distance, speed and acceleration. The paper shows that a particular case of hybrid Petri nets, called fluid stochastic Petri net (FSPN) [4, 8, 7] constitutes a suitable formalism for modelling and analyzing this kind of hybrid systems. The main characteristics of FSPN is that its primitives (places, transitions and arcs) are partitioned in two groups: discrete and continuous (or fluid). Hence, in a single formalism, both discrete and continuous variables can be accommodated and their mutual interaction represented. A brief description of the FSPN paradigm is presented in Section 2.

In Section 3, we model a controller that keeps track of two consecutive vehicles whose behaviour is simplified for the sake of proving the feasibility of the approach. The controller checks the speed and distance of the two vehicles and issues alarms when the prescribed limits are violated. Section 4 provides some results based on this model. In Section 5, in order to improve the model trustworthiness, we show that FSPN is a valuable framework to model the dynamics of a vehicle in a highly detailed manner.

2 Fluid stochastic Petri nets

Fluid Petri nets were introduced both in non-stochastic [4] and stochastic [11] settings. Here only a brief descrip-

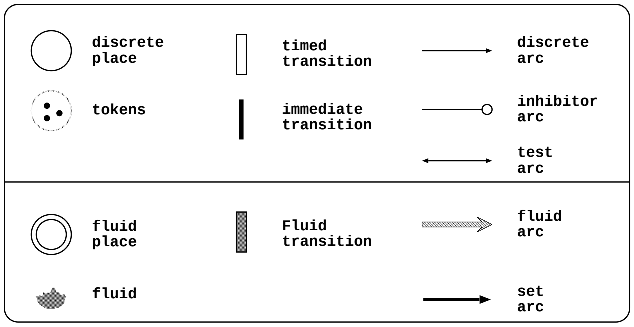

Figure 1. All the primitives of the formalism

tion of the applied formalism is given, for more details the reader is referred to [7]. A FSPN contains two kinds of places: discrete places that contain a discrete number of tokens and fluid places that contain a continuous amount of fluid. Marking of the net is given by the joint distribution of tokens in discrete places and fluid levels in fluid places. Graphical representation of all the primitives of the formalism is depicted in Figure 1. Discrete tokens are moved according to firings of immediate and timed transitions through discrete arcs. Firing times of timed transitions are defined by instantaneous firing rates that can depend on the marking of the net. Fluid levels are changed either by fluid transitions according to fluid rate that can depend on the marking of the net or they can be set directly by a set arc to a given value when a transition fires. Inhibitor arcs (test arcs) disable (enable) a transition when a given number of token or a given quantity of fluid is present in a place.

3 FSPN model of the controller

To analyze the safety controller, we model two vehicles that follow each other in the tunnel. To distinguish them, the vehicle proceeding ahead in the tunnel will be called the truck, while the one following is the car.

The submodel that describes the behavior of the truck is depicted in Figure 2. Three discrete places are used to distinguish the situations when the truck is braking (place $brake_t$ is marked), proceeding at constant speed (place con_t), accelerating (place acc_t). The token among these three places are moved by transitions bc_t, cb_t, ac_t and ca_t. Different numeric values for the firing rates of these four transitions are assigned for different experiments and will be given in the next section. The speed of this vehicle is represented by the level of fluid in place $speed_t$ which has a lower bound at zero. The speed is decreasing (increasing) by transition $decrease_t$ ($increase_t$) when this transition is enabled, i.e. when place $brake_t$ is marked (place acc_t is marked and place *standstill* is not). Places *move* and *standstill* together with transition *halt* are added, in order to model a sudden stop of the truck due to a front collision with a vehicle running in the opposite direction. When this happens the speed of the truck is set to 0 and cannot increase anymore.

The submodel representing the car is given in Figure 3. The behavior of the car is similar to that of the truck. There are two differences. First, sudden stop of this vehicle is not considered. Second, we model what happens when the alarm with which the tunnel is equipped sounds. Initially the alarm is off (place *off* is marked). If the distance between the two vehicles is too small, transition *off_on* gets enabled and the alarm turns on. Transition *off_on* is connected to fluid place *distance* (Figure 4) by an inhibitor arc, i.e. this transition is disabled until the distance is larger then a given limit. The alarm turns off when transition *on_off* gets enabled. This happens when the level of place *distance* is higher then a given limit since transition *on_off* is connected by a test arc to place *distance*. When the alarm is on, the car begins immediately to brake (since transitions $alarm_1$ and $alarm_2$ get enabled) and keeps braking while the alarm is on.

Figure 4 depicts the submodel representing the distance between the two vehicles and their position in the tunnel. The distance between the two vehicles is represented by the level of fluid place *distance*. The level of this place is increased by transition *increase* and decreased by transition *decrease*. Transition *increase* pumps fluid to place *distance* according to the level of fluid place $speed_t$, i.e. according to the speed of the truck. While transition *decrease* takes away fluid from place *distance* according to the speed of the car, i.e. according to the level of fluid place $speed_c$. The distance covered by the car is represented by the level of fluid place *position*. The level of this place is increased according to the speed of the car by transition $speed_c$. Discrete place *proceed* is marked until the vehicles are in the tunnel and no accident has happened. If the distance between the two cars becomes 0, transition *crash* becomes enabled and place *accident* becomes marked. Moreover, the system is stopped because a token is placed to place *stop* too. The level of fluid in place *position* provides the location where the accident happened in the tunnel. If the car leaves the tunnel, transition *end* becomes enabled and a token is put in place *passed* which means that the two vehicles passed the tunnel.

4 Results

The model described in Section 3 was analyzed by simulation under different assumptions. The following parameters were the same in all the experiments. Acceleration and braking change the speed of the vehicle according to a constant derivative. The vehicles accelerate from 0 to 100 km/h in 10 seconds. When traveling at 100 km/h, the vehicles need 100 meters to stop. Initially the distance between the

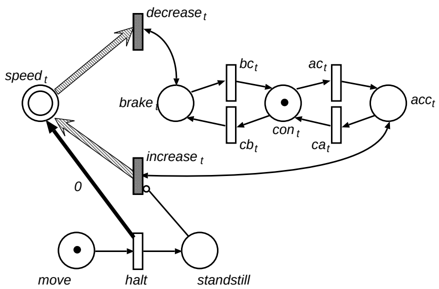

Figure 2. Truck submodel

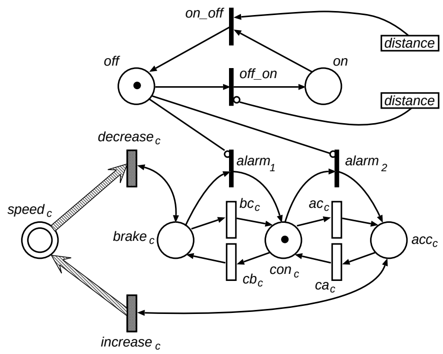

Figure 3. Car submodel

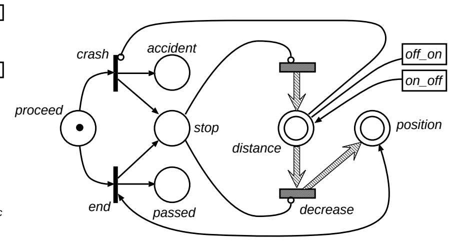

Figure 4. Distance and position submodel

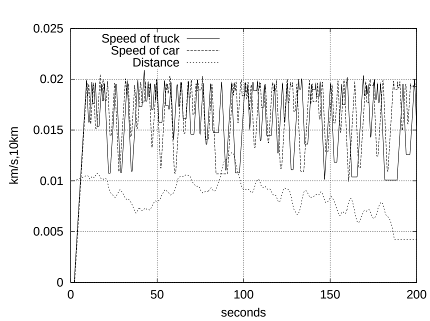

Figure 5. Simulation trace with completely random behavior

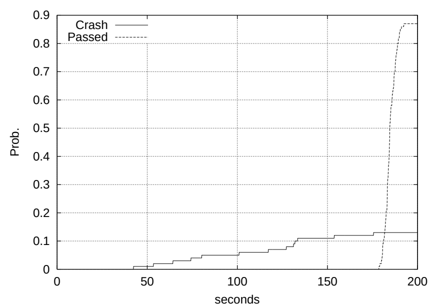

Figure 6. Probability of accident and passing the tunnel with completely random behavior

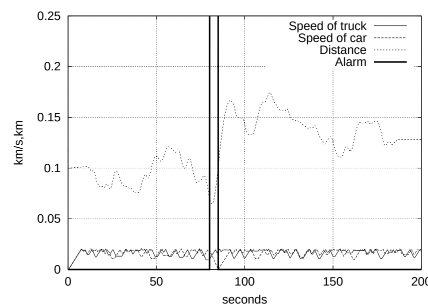

Figure 7. Simulation trace with reasonable drivers and alarm

truck and the vehicle is 100 meters. Both vehicles start at speed 0. The tunnel is 3 km long.

Completely random behavior During the first experiment we assume that the tunnel is not equipped with an alarm (upper part of Figure 3 is not present), sudden stop of the truck is not considered (lower part of Figure 2 is not present) and that the drivers decides to brake, drive at constant speed or accelerate in a completely random manner. In order to model random behavior, transitions *bc*$_\bullet$, *cb*$_\bullet$, *ac*$_\bullet$ and *ca*$_\bullet$ have exponential firing times with rate 1.0 ($\bullet$ stands either for t or c).

A simulation trace with speeds of the vehicles and the distance between them is given in Figure 5. Since we assume random behaviuor the speed has high fluctuating. Probability of crash and passing the tunnel as function of time is given in Figure 6.

As we go through the subsequent results we describe only what changes compared to the previous setting.

Reasonable drivers For the following experiments the tunnel is equipped with an alarm. The alarm switches on when the distance between the vehicles is less than 70 meters and switches off when it exceeds 100 meters.

Moreover, from now on, we assume that the drivers are trying to maintain a predefined speed. This is done by defining firing rates for transitions *bc*$_\bullet$, *cb*$_\bullet$, *ac*$_\bullet$ and *ca*$_\bullet$ that depend on the actual speeds of the vehicle. These firing rates for both the vehicles are

$$fr_{ca_\bullet}(speed_\bullet) = \begin{cases} 1.0 & speed_\bullet \leq s_t, \\ 0.0 & speed_\bullet > s_t, \end{cases}$$

$$fr_{cb_\bullet}(speed_\bullet) = fr_{ac_\bullet}(speed_\bullet) = \begin{cases} 0.01 & speed_\bullet \leq s_t, \\ 10.0 & speed_\bullet > s_t, \end{cases}$$

$$fr_{bc_\bullet}(speed_\bullet) = \begin{cases} 10.0 & speed_\bullet \leq s_m, \\ 1.0 & s_m < speed_\bullet \leq s_t, \\ 0.0 & speed_\bullet > s_t, \end{cases}$$

where $s_t = 70$km/h ($s_m = 40$km/h) denotes the target (minimal) speed for the vehicles. Figure 7 and 8 gives the results of these experiments. In the next experiment we analyze the effect of having the alarm broken down. Results are reported in Figure 9 and 10.

5 A detailed physical model of the dynamics of a vehicle

In this section we show how to implement detailed models of the vehicle dynamics by means of FSPN. We assume

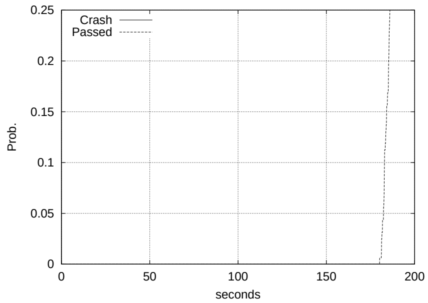

Figure 8. Probability of accident and passing the tunnel with reasonable drivers and alarm

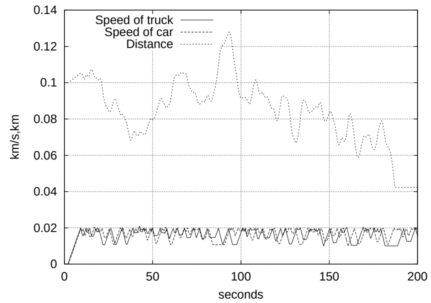

Figure 9. Simulation trace with reasonable drivers and alarm broken down

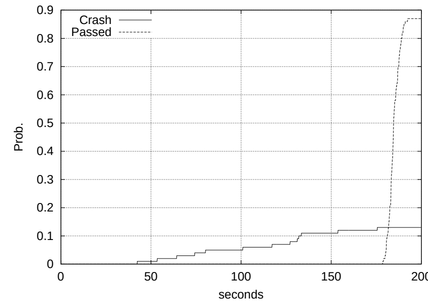

Figure 10. Probability of accident and passing the tunnel with reasonable drivers and alarm broken down

that the driver's behaviour is completely captured through the position of the gear (modeled through the current gear ratio x_g), the position of the throttle, denoted by $0 \leq h \leq 1$, and the position of the break, $0 \leq b \leq 1$, where $h, b = 0$ means no action and $h, b = 1$ means maximum action. (Parameters and variables of the car physics models are summarised in Table 1 and 2.)

In the simplest case the throttle position and the break position together with the maximum acceleration and the maximum braking force (F_{ACC} and F_{BR}) determine the total force experienced by the car. The acceleration can be written as

$$\frac{dv}{dt} = \frac{F_{ACC}h - F_{BR}b}{M_C} \tag{1}$$

where M_C is the mass of the car.

Adding rolling and air resistance the acceleration becomes

$$\frac{dv}{dt} = \frac{F_{ACC}h - C_{DR}v^2 - C_{RR}v - F_{BR}b}{M_C} \tag{2}$$

where C_{DR} is the air drag resistance coefficient and C_{RR} is the rolling resistance coefficient.

Another step toward more realistic modeling of the car is to consider the gears and the fact that the force of the engine depends on the angular speed of the engine. The model becomes

$$\omega = \frac{v}{r_W} x_g x_d \tag{3}$$

$$\frac{dv}{dt} = \frac{1}{M_C} \left(\frac{T_E(\omega) h x_g x_d \eta}{r_W} - C_{DR}v^2 - C_{RR}v - F_{BR}b \right) \tag{4}$$

where (3) provides the connection between the angular speed of the engine ω, the speed of the car v, the wheel radius r_W, the current gear ratio x_g and the differential ratio x_d. Then the acceleration of the car can be written as in (4) where $T_E(\omega)$ is the engine torque function and η is the transmission efficiency.

In the most complex car physics model, also the sliding friction of the tire on the road is considered. The model becomes

$$\sigma = \frac{\omega_W r_W - v}{|v|} \tag{5}$$

$$\frac{dv}{dt} = \frac{f_P(\sigma)}{M_c} \tag{6}$$

$$\omega = \omega_W x_g x_d \tag{7}$$

$$\frac{d\omega_W}{dt} = \frac{1}{I_w} \left(T_E(\omega) h x_g x_d \eta - \left(C_{DR}v^2 + C_{RR}v + F_{BR}b + f_P(\omega) \right) r_W \right) \tag{8}$$

where, in (5), the longitudinal slip of the tire, σ, is calculated based on the angular speed of the wheel, ω_W, the wheel radius, r_W, and the speed of the car, v. Note that for the previous models we had $\omega_W r_W = v$ while in this case $\omega_W r_W \neq v$ since the tyre is sliding. The acceleration of the car in (6) is determined by the force given by Pacejka's magic formula, $f_P(\sigma)$, [5]. The connection between the angular speed of the engine and that of the wheel is provided by (7); this equation is equivalent to (3). The angular acceleration of the wheel is given by (8) where I_w is the inertia moment of the wheels. The set of equations (5-8) causes numerical instability (oscillation) around $v = 0$. This can be alleviated by substituting (5) with

$$\frac{d\sigma}{dt} = \frac{\omega_W r_W - v}{B} - \frac{|v|\sigma}{B}$$

where B is the damping constant.

FSPN model of the dynamics of a vehicle In this section we describe how to implement the simplest and the most complex model by hybrid Petri nets.

The hybrid Petri net model of the simplest car physics model is depicted in Figure 11. The model contains three

F_{ACC}	maximum acceleration force
F_{BR}	maximum braking force
M_C	car mass
C_{DR}	air drag resistance coefficient
C_{RR}	rolling resistance coefficient
r_W	wheel radius
x_g	gear ratio
x_d	differential ratio
η	transmission efficiency
$T_E(\omega)$	engine torque function
$f_P(\sigma)$	Pacejka's magic formula
I_w	inertia moment of the wheels
B	damping constant

Table 1. Parameters used in the car model

h	throttle position ($0 \leq h \leq 1$)
b	brake position ($0 \leq b \leq 1$)
v	car speed
ω	engine angular speed
ω_W	wheel angular speed
σ	longitudinal slip

Table 2. Variables used in the car model

fluid places. Fluid places $throttle$ and $brake$ model the behaviour of the driver. The speed of the car is modeled by fluid place $speed$ and a fluid transition connected to it which changes the level of fluid place $speed$ according to (1). The level of fluid places $throttle$ and $brake$ are modified by other submodels of the system which can take into account road conditions or drivers' decision.

The net modeling the most complex car physics model is shown in Figure 12. We have two additional fluid places with respect to the simple model: fluid place $slip$ models the longitudinal slip of the wheel and fluid place $wheel$ the angular speed of the wheel. The level of fluid places $slip$, $wheel$ and $speed$ changes according to equations (5), (8) and (6). The position of the gear is modeled by the discrete place g. The number of tokens x present in place g represents the fact that the car is in gear x (in Figure 12, the car is in gear 1). The gear change is modeled by transitions up and $down$. These transitions fire when the r.p.m. of the car increases over or decreases under a certain limit. These limits depend on the number of tokens in place g.

The controller model We now exploit the most detailed car physics model to represent the situation, already considered in Section 3, where a car follows a truck and a controller issues signals in order to keep the distance between the two vehicles among reasonable limits.

The FSPN model is depicted in Figure 13. We assume that the truck has a constant speed, $80km/h$, and stops suddenly after 2750 meters (it happens after 123.75 seconds). At the beginning of the tunnel the speed of the car is 0 and it is 250 meters behind the truck.

The controller switches between two discrete states: $Brake$ and $Accel$. In the $Accel$ state, the controller tries to reach and maintain a target speed, $v_t = 90km/h$. To accomplish this goal, it varies the throttle position h with a pressure depending on the difference between the target speed , v_t, and the actual speed, v. The variation of the throttle position is given by

$$\frac{dh}{dt} = \frac{v_t - v}{v_t} P_t \tag{9}$$

where P_t represents the speed at which the position of the pedals is changed (in our example $P_t = 2$). When the place $Brake$ is marked, the car controller tries to increase the distance from the truck by applying a pressure on the brake. The pressure is inversely proportional to the distance from the truck with respect to the given minimum safety distance, $d_m = 200$:

$$\frac{db}{dt} = -\left(b - \frac{d_m - d}{d_m}\right) P_t, \tag{10}$$

where d represents the distance between the car and the truck.

The transition from the $Brake$ to the $Accel$ states occurs as soon as the distance between the car and the truck becomes greater than a given value $d_M = 300$. It is expressed defining its transition rate fr_{ba_c} using a Dirac's delta on the distance between the car and the truck:

$$fr_{ba_c}(distance) = \delta(distance - d_M) \tag{11}$$

The transition between the $Accel$ and the $Brake$ state occurs after an exponentially distributed amount of time (with parameter $\lambda = 1.0$), starting when the distance between the car and the truck becomes less than d_m. It's transition rate fr_{ab_c} is defined as:

$$fr_{ab_c}(distance) = \begin{cases} 0.0 & distance \leq d_m, \\ \lambda & distance > d_m. \end{cases} \tag{12}$$

Having 8 fluid places, the only possibility to solve the model is simulation. Figure 14, 15 depict a possible execution paths of the model. Figure 14 gives the position of the vehicles as function of time. Figure 15 describes the drivers' behaviour through the position of the gear, the throttle and the break.

Figure 16 shows the probability density function of the distance of the two vehicles at different time instants. After 25 seconds the distance is deterministic and it is around 210 meters. After 100 seconds the distance is not deterministic anymore because the break was already used and it is pressed after an exponential delay. The trucks stops after 123.75 seconds. After 150 also the car is stopped and it can happen that their distance is zero which means that an accident happened.

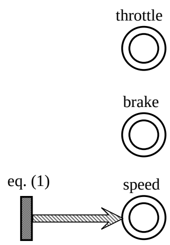

Figure 11. The simplest car physics model

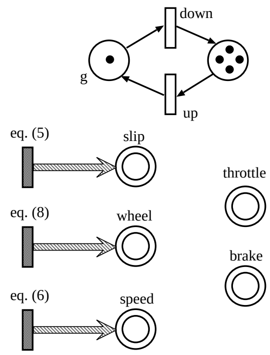

Figure 12. The most complex car physics model

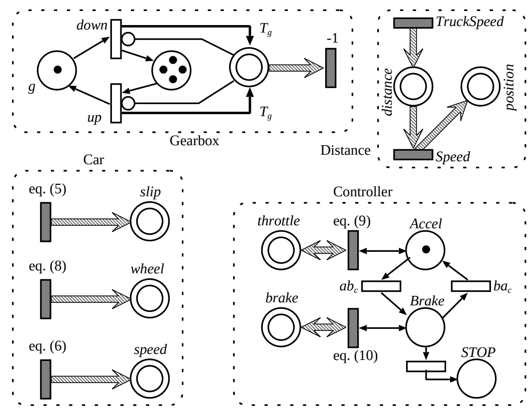

Figure 13. The detailed controller model

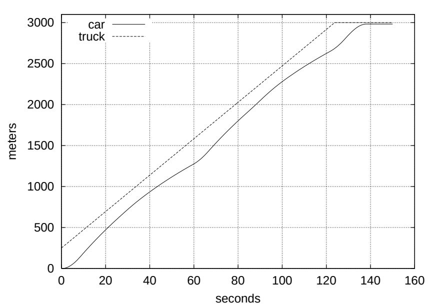

Figure 14. Car and Truck positions as function of time

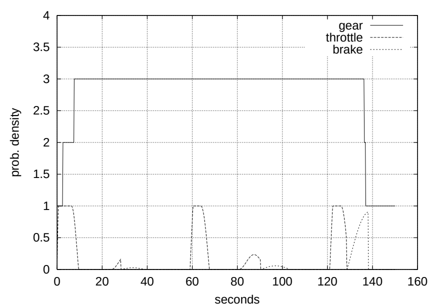

Figure 15. Gear, throttle and brake position as function of time

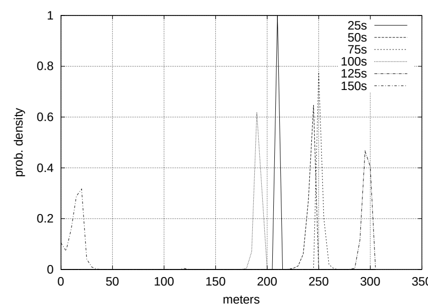

Figure 16. Distance (between car and truck) distribution at various time instants

6 Conclusions

In this paper we have presented a FSPN model of a safety car controller and have shown that FSPN is a suitable formalism to model such systems. Several experiments were carried out under different assumptions on the behavior of the drivers and on the circumstances.

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