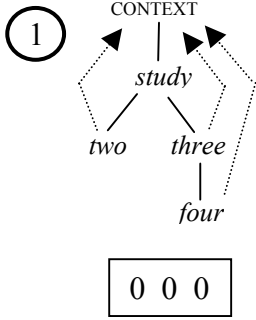


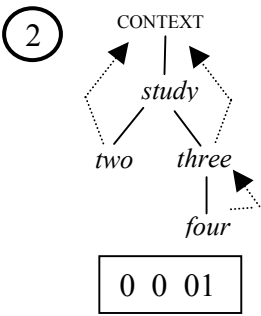
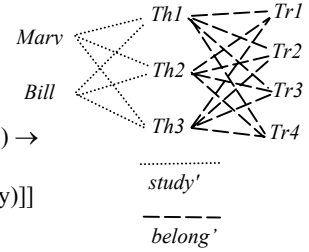
THE THIRTEEN INTERPRETATIONS OF THE SENTENCE

Two students studied three theorems of four theories



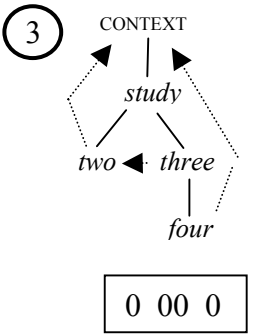
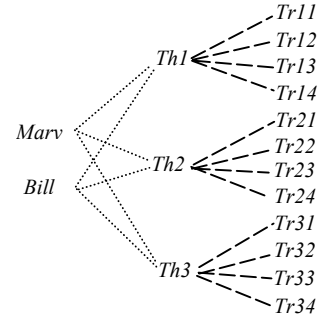
There are two students (Mary and Bill) each of which studied the same three theorems (Th1, Th2 and Th3), all of which belong to a well-defined group of four theories (Tr1, Tr2, Tr3, Tr4).

$$\begin{aligned} &\exists_{x_1}\exists_{x_2}\exists_{y_1}\exists_{y_2}\exists_{y_3}\exists_{z_1}\exists_{z_2}\exists_{z_3}\exists_{z_4} \\ &[x_1 \neq x_2 \wedge y_1 \neq y_2 \neq y_3 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ &\forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\forall_y \forall_z [((y = y_1 \vee y = y_2 \vee y = y_3) \wedge (z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4)) \rightarrow \\ &\quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ &\forall_x \forall_y [((x = x_1 \vee x = x_2) \wedge (y = y_1 \vee y = y_2 \vee y = y_3)) \rightarrow \text{study}'(x, y)] \end{aligned}$$



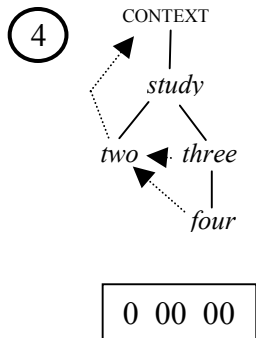
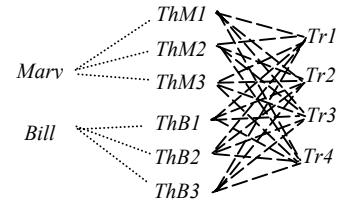
There are two students (Mary and Bill) each of which studied the same three theorems (Th1, Th2 and Th3), each of which belong to a group of four theories, but the theories may possibly be different for the different theorems (Tr11, Tr12, Tr13, Tr14, Tr21, Tr22, Tr23, Tr24, Tr31, Tr32, Tr33, Tr34).

$$\begin{aligned} &\exists_{x_1}\exists_{x_2}\exists_{y_1}\exists_{y_2}\exists_{y_3}[x_1 \neq x_2 \wedge y_1 \neq y_2 \neq y_3 \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ &\forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ &\quad (\text{theorem}'(y) \wedge \\ &\quad \exists_{z_1}\exists_{z_2}\exists_{z_3}\exists_{z_4}[z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ &\quad \quad \text{belong}'(y, z)]])] \wedge \\ &\forall_x \forall_y [((x = x_1 \vee x = x_2) \wedge (y = y_1 \vee y = y_2 \vee y = y_3)) \rightarrow \\ &\quad \text{study}'(x, y)] \end{aligned}$$



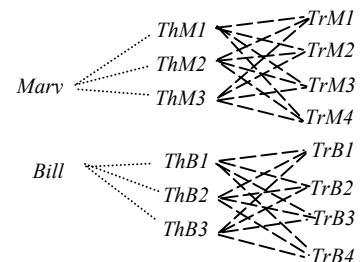
There are two students (Mary and Bill) and a well-defined group of four theories (Tr1, Tr2, Tr3, Tr4). Each student chooses three theorems belonging to all four theories, but they may choose different theorems

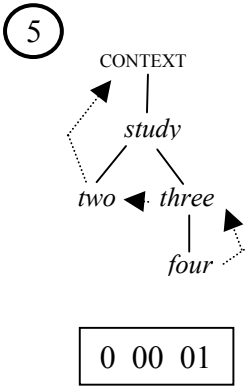
$$\begin{aligned} &\exists_{x_1}\exists_{x_2}\exists_{z_1}\exists_{z_2}\exists_{z_3}\exists_{z_4}[z_1 \neq z_2 \neq z_3 \neq z_4 \wedge x_1 \neq x_2 \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ &\forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \\ &\quad \exists_{y_1}\exists_{y_2}\exists_{y_3}[y_1 \neq y_2 \neq y_3 \wedge \\ &\quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ &\quad \quad (\text{theorem}'(y) \wedge \\ &\quad \quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ &\quad \quad \quad \text{belong}'(y, z)]])] \wedge \\ &\forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \text{study}'(x, y)] \end{aligned}$$



There are two students (Mary and Bill). Each student chooses three theorems (ThM1, ThM2, ThM3, ThB1, ThB2, ThB3) and four theories (TrM1, TrM2, TrM3, TrM4, TrB1, TrB2, TrB3, TrB4). Each theorem chosen by a student must belong to all the theories chosen by her/him.

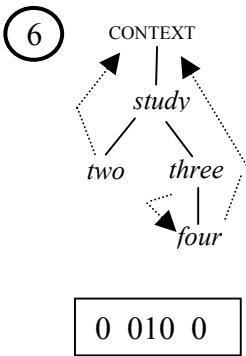
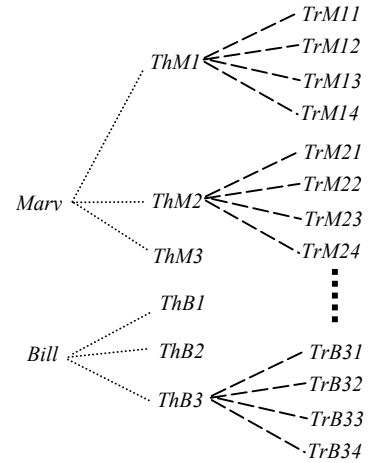
$$\begin{aligned} &\exists_{x_1}\exists_{x_2}[x_1 \neq x_2 \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ &\forall_x [(x = x_1 \vee x = x_2) \rightarrow \\ &\quad \exists_{y_1}\exists_{y_2}\exists_{y_3}\exists_{z_1}\exists_{z_2}\exists_{z_3}\exists_{z_4}[y_1 \neq y_2 \neq y_3 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \forall_y \forall_z [((y = y_1 \vee y = y_2 \vee y = y_3) \wedge \\ &\quad \quad (z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4)) \rightarrow \\ &\quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ &\quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \text{study}'(x, y)] \end{aligned}$$





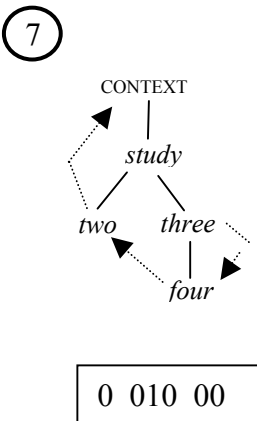
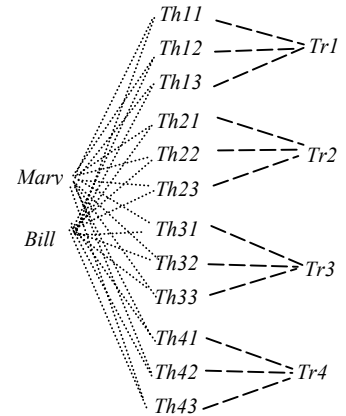
There are two students (Mary and Bill). Each student chooses three theorems (ThM1, ThM2, ThM3, ThB1, ThB2, ThB3) each of which must belong to any four theories (TrM11, TrM12, TrM13, TrM14, for theorem ThM1; TrM21, TrM22, TrM23, TrM24, for theorem ThM2 and so on).

$$\begin{aligned} & \exists_{x_1} \exists_{x_2} [x_1 \neq x_2 \wedge \\ & \forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_x [(x = x_1 \vee x = x_2) \rightarrow \\ & \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ & \quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \\ & \quad \quad \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \quad \quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ & \quad \quad \quad \text{theory}'(z)] \wedge \\ & \quad \quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ & \quad \quad \quad \text{belong}'(y, z)]]] \wedge \\ & \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



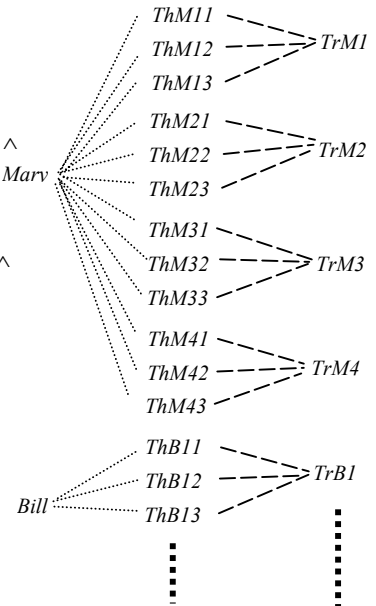
There are two students (Mary and Bill), four theories (Tr1, Tr2, Tr3, Tr4), and three theorems for each theory. Each student studies those theorems

$$\begin{aligned} & \exists_{x_1} \exists_{x_2} \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [x_1 \neq x_2 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ & \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ & \quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \forall_x \forall_y [((x = x_1 \vee x = x_2) \wedge \\ & \quad \quad (y = y_1 \vee y = y_2 \vee y = y_3)) \rightarrow \\ & \quad \quad \text{study}'(x, y)]]] \end{aligned}$$

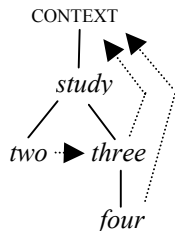


There are two students (Mary and Bill) and, for each of them, four theories (TrM1, TrM2, TrM3, TrM4). Each student studies three theorems for each of the theories associated with her/him.

$$\begin{aligned} & \exists_{x_1} \exists_{x_2} [x_1 \neq x_2 \wedge \\ & \forall_x [(x = x_1 \vee x = x_2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_x [(x = x_1 \vee x = x_2) \rightarrow \\ & \quad \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \quad \forall_z [(z = z_1 \vee z = z_2 \vee z = z_3 \vee z = z_4) \rightarrow \\ & \quad \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ & \quad \quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ & \quad \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \quad \forall_y [(y = y_1 \vee y = y_2 \vee y = y_3) \rightarrow \\ & \quad \quad \quad \text{study}'(x, y)]]]]] \end{aligned}$$



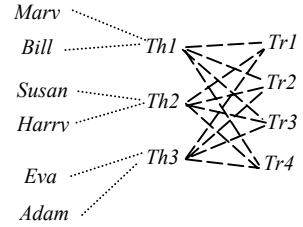
8



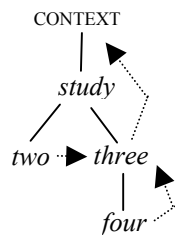
01 0 0

There are three theorems (Th1, Th2 and Th3) and four theories (Tr1, Tr2, Tr3, Tr4). The theorems belong to all of the theories, and for each theorem there are two students, possibly different, who study it.

$$\begin{aligned} &\exists_{y_1} \exists_{y_2} \exists_{y_3} \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [y_1 \neq y_2 \neq y_3 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \forall_y \forall_z [(y=y_1 \vee y=y_2 \vee y=y_3) \wedge \\ &\quad \quad (z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \\ &\quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ &\quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ &\quad \quad \exists_{x_1} \exists_{x_2} [x_1 \neq x_2 \wedge \\ &\quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{student}'(x)] \wedge \\ &\quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



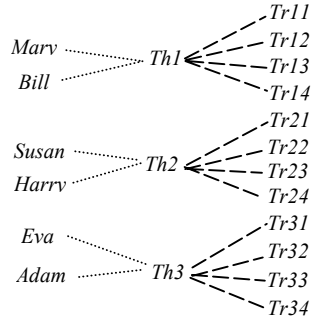
9



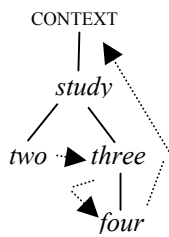
01 0 01

There are three theorems (Th1, Th2 and Th3). Each of them belongs to four theories, possibly different (Tr11, Tr12, Tr13, Tr14 and Tr21, Tr22, Tr23, Tr24). Moreover, for each theorem there are two students, possibly different, who study it.

$$\begin{aligned} &\exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ &\quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ &\quad \quad (\text{theorem}'(y) \wedge \\ &\quad \quad \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\quad \quad \quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \quad \quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \\ &\quad \quad \quad \quad \text{belong}'(y, z)]]] \wedge \\ &\quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ &\quad \quad \exists_{x_1} \exists_{x_2} [x_1 \neq x_2 \wedge \\ &\quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{student}'(x)] \wedge \\ &\quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



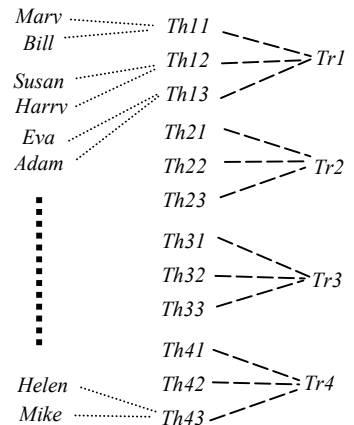
10



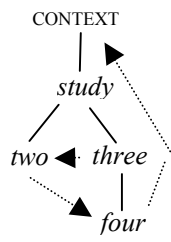
01 010 0

There are four theories (Tr1, Tr2, Tr3, Tr4). For each of them there are three, possibly different theorems (Th11, Th12, Th13, Th21, Th22, Th23 and Th31, Th32, Th33). For each theorem, there are two, possibly different, students who study them.

$$\begin{aligned} &\exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ &\quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \\ &\quad \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ &\quad \quad \quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ &\quad \quad \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ &\quad \quad \quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ &\quad \quad \quad \quad \exists_{x_1} \exists_{x_2} [x_1 \neq x_2 \wedge \\ &\quad \quad \quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{student}'(x)] \wedge \\ &\quad \quad \quad \quad \quad \forall_x [(x=x_1 \vee x=x_2) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



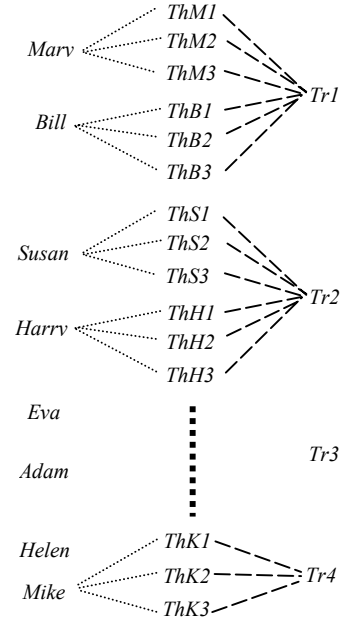
11



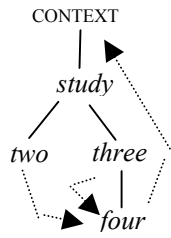
010 00 0

There are four theories (Tr1, Tr2, Tr3, Tr4). For each of them there are two, possibly different students (who chose them). For each student, there are three, possibly different, theorems who are studied by that student.

$$\begin{aligned} & \exists_{z1} \exists_{z2} \exists_{z3} \exists_{z4} [z1 \neq z2 \neq z3 \neq z4 \wedge \\ & \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \\ & \quad \exists_{x1} \exists_{x2} [x1 \neq x2 \wedge \\ & \quad \forall_x [(x=x1 \vee x=x2) \rightarrow \text{student}'(x)] \wedge \\ & \quad \forall_x [(x=x1 \vee x=x2) \rightarrow \\ & \quad \quad \exists_{y1} \exists_{y2} \exists_{y3} [y1 \neq y2 \neq y3 \wedge \\ & \quad \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ & \quad \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ & \quad \quad \quad \text{study}'(x, y)]]]]]] \end{aligned}$$



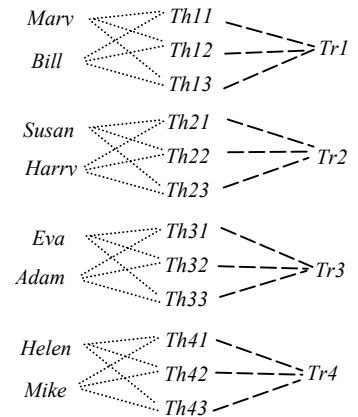
12



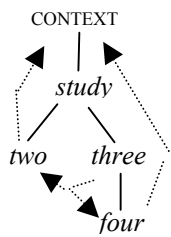
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There are four theories (Tr1, Tr2, Tr3, Tr4). For each of them there are two, possibly different, students (who chose them) and three, possibly different, theorems (Th11, Th12, Th13, Th21, Th22, Th23 and Th31, Th32, Th33). Each student who chose a given theory studies all the theorems of that theory.

$$\begin{aligned} & \exists_{z1} \exists_{z2} \exists_{z3} \exists_{z4} [z1 \neq z2 \neq z3 \neq z4 \wedge \\ & \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \\ & \quad \exists_{x1} \exists_{x2} \exists_{y1} \exists_{y2} \exists_{y3} [x1 \neq x2 \wedge y1 \neq y2 \neq y3 \wedge \\ & \quad \forall_x [(x=x1 \vee x=x2) \rightarrow \text{student}'(x)] \wedge \\ & \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \forall_x \forall_y [((x=x1 \vee x=x2) \wedge \\ & \quad \quad (y=y1 \vee y=y2 \vee y=y3)) \rightarrow \\ & \quad \quad \text{study}'(x, y)]]]] \end{aligned}$$



13



010 00+010 0

There are two students and four theories (Tr1, Tr2, Tr3, Tr4). Each student chooses three theorems form each theory and studies them.

$$\begin{aligned} & \exists_{x1} \exists_{x2} \exists_{z1} \exists_{z2} \exists_{z3} \exists_{z4} [z1 \neq z2 \neq z3 \neq z4 \wedge x1 \neq x2 \wedge \\ & \forall_x [(x=x1 \vee x=x2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_x \forall_z [((x=x1 \vee x=x2) \wedge (z=z1 \vee z=z2 \vee z=z3 \vee z=z4)) \rightarrow \\ & \quad \exists_{y1} \exists_{y2} \exists_{y3} [y1 \neq y2 \neq y3 \wedge \\ & \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ & \quad \quad \text{study}'(x, y)]]]] \end{aligned}$$

