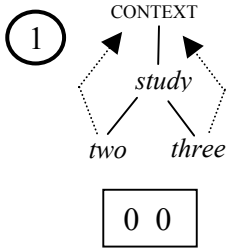


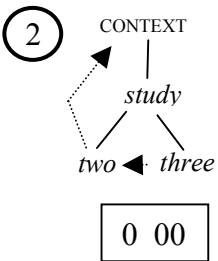
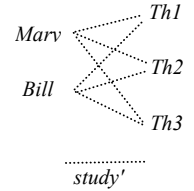
THE THREE INTERPRETATIONS OF THE SENTENCE

Two students studied three theorems



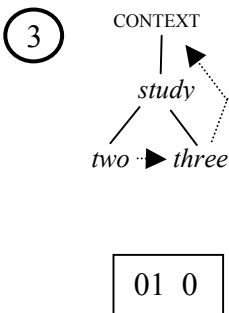
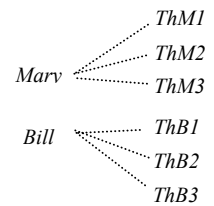
There are two students (Mary and Bill) each of which studied the same three theorems (Th1, Th2 and Th3).

$$\begin{aligned} & \exists_{x1} \exists_{x2} \exists_{y1} \exists_{y2} \exists_{y3} \\ & [x1 \neq x2 \wedge y1 \neq y2 \neq y3 \wedge \\ & \forall_x [(x = x1 \vee x = x2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_y [(y = y1 \vee y = y2 \vee y = y3) \rightarrow \text{theorem}'(y)] \wedge \\ & \forall_x \forall_y [((x = x1 \vee x = x2) \wedge (y = y1 \vee y = y2 \vee y = y3)) \rightarrow \text{study}'(x, y)] \end{aligned}$$



There are two students (Mary and Bill). Each student chooses three theorems and studies them.

$$\begin{aligned} & \exists_{x1} \exists_{x2} [x1 \neq x2 \wedge \\ & \forall_x [(x = x1 \vee x = x2) \rightarrow \text{student}'(x)] \wedge \\ & \forall_x [(x = x1 \vee x = x2) \rightarrow \\ & \quad \exists_{y1} \exists_{y2} \exists_{y3} [y1 \neq y2 \neq y3 \wedge \\ & \quad \quad \forall_y [(y = y1 \vee y = y2 \vee y = y3) \rightarrow \text{theorem}'(y)] \wedge \\ & \quad \quad \forall_y [(y = y1 \vee y = y2 \vee y = y3) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



There are three theorems (Th1, Th2 and Th3) and for each theorem there are two students, possibly different, who study it.

$$\begin{aligned} & \exists_{y1} \exists_{y2} \exists_{y3} [y1 \neq y2 \neq y3 \wedge \\ & \forall_y [(y = y1 \vee y = y2 \vee y = y3) \rightarrow \text{theorem}'(y)] \wedge \\ & \forall_y [(y = y1 \vee y = y2 \vee y = y3) \rightarrow \\ & \quad \exists_{x1} \exists_{x2} [x1 \neq x2 \wedge \\ & \quad \quad \forall_x [(x = x1 \vee x = x2) \rightarrow \text{student}'(x)] \wedge \\ & \quad \quad \forall_x [(x = x1 \vee x = x2) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$

