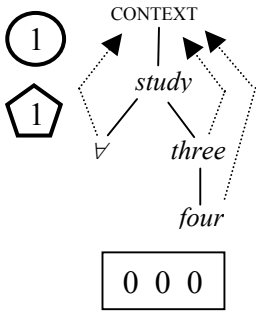


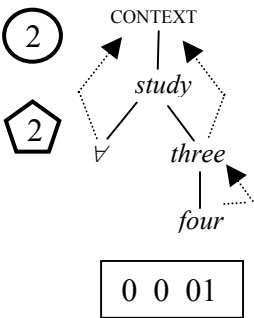
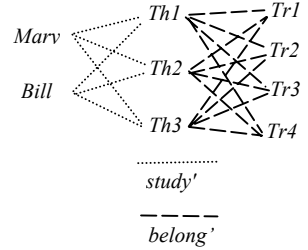
THE EIGHT INTERPRETATIONS OF THE SENTENCE

All students studied three theorems of four theories



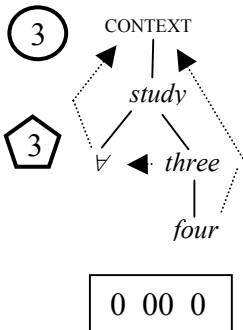
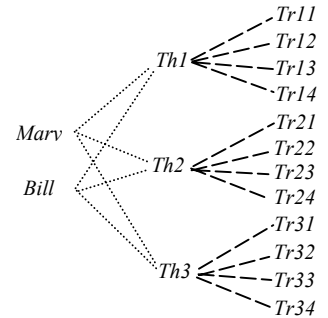
There are three theorems (Th1, Th2 and Th3), all of which belong to a well-defined group of four theories (Tr1, Tr2, Tr3, Tr4); all (relevant) students studied those three theorems

$$\begin{aligned} & \exists_{y_1} \exists_{y_2} \exists_{y_3} \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [y_1 \neq y_2 \neq y_3 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_y \forall_z [((y=y_1 \vee y=y_2 \vee y=y_3) \wedge (z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4)) \rightarrow \\ & \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \forall_y \forall_x [((y=y_1 \vee y=y_2 \vee y=y_3) \wedge \text{student}'(x)) \rightarrow \text{study}'(x, y)] \end{aligned}$$



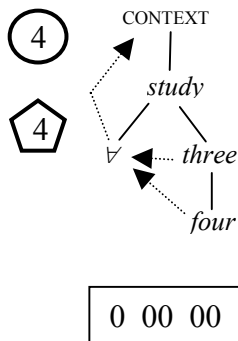
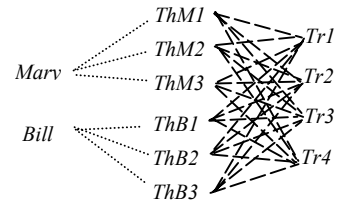
There are three theorems (Th1, Th2 and Th3), each of which belong to a group of four theories, but the theories may possibly be different for the different theorems (Tr11, Tr12, Tr13, Tr14, Tr21, Tr22, Tr23, Tr24, Tr31, Tr32, Tr33, Tr34); all (relevant) students studied those three theorems.

$$\begin{aligned} & \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ & \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \\ & \quad (\text{theorem}'(y) \wedge \\ & \quad \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{belong}'(y, z)]])] \wedge \\ & \forall_y \forall_x [((y=y_1 \vee y=y_2 \vee y=y_3) \wedge \text{student}'(x)) \rightarrow \\ & \quad \text{study}'(x, y)] \end{aligned}$$



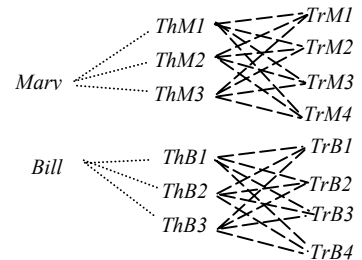
There is a well-defined group of four theories (Tr1, Tr2, Tr3, Tr4). All (relevant) student chose three theorems belonging to all four theories (but they could choose different theorems) and studied them.

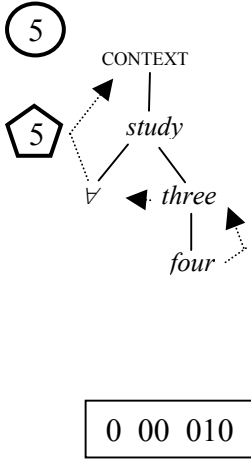
$$\begin{aligned} & \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \forall_x [\text{student}'(x) \rightarrow \\ & \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} [y_1 \neq y_2 \neq y_3 \wedge \\ & \quad \forall_y \forall_z [((y=y_1 \vee y=y_2 \vee y=y_3) \wedge \\ & \quad (z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4)) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$



All (relevant) students chose three theorems (ThM1, ThM2, ThM3, ThB1, ThB2, ThB3) and four theories (TrM1, TrM2, TrM3, TrM4, TrB1, TrB2, TrB3, TrB4), such that each theorem chosen by a student belongs to all the theories chosen by her/him. Each student studied her/his three theorems.

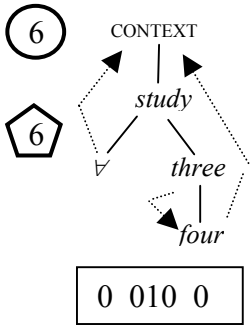
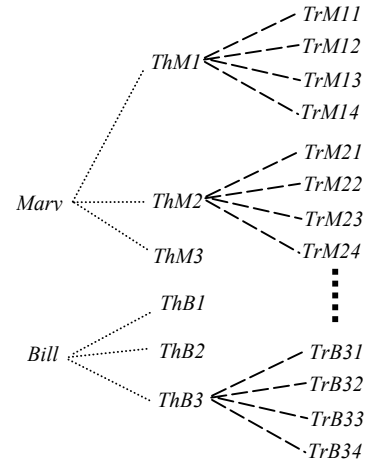
$$\begin{aligned} & \forall_x [\text{student}'(x) \rightarrow \\ & \quad \exists_{y_1} \exists_{y_2} \exists_{y_3} \exists_{z_1} \exists_{z_2} \exists_{z_3} \exists_{z_4} [y_1 \neq y_2 \neq y_3 \wedge z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \\ & \quad \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \\ & \quad \forall_y \forall_z [((y=y_1 \vee y=y_2 \vee y=y_3) \wedge \\ & \quad (z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4)) \rightarrow \\ & \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ & \quad \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \text{study}'(x, y)]]] \end{aligned}$$





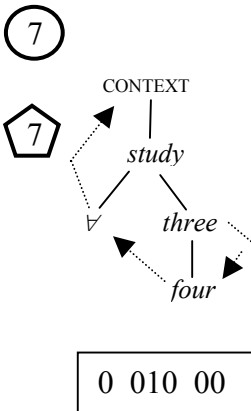
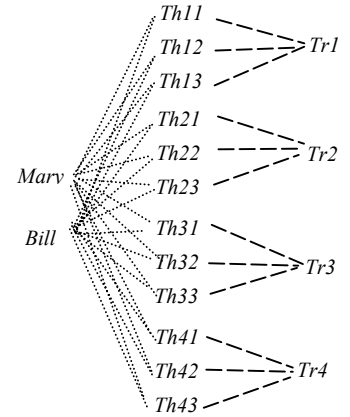
All (relevant) students chose three theorems (ThM1, ThM2, ThM3, ThB1, ThB2, ThB3) each of which must belong to any four theories (TrM11, TrM12, TrM13, TrM14, for theorem ThM1; TrM21, TrM22, TrM23, TrM24, for theorem ThM2 and so on). Each student studied her/his three theorems.

$$\forall_x [\text{student}'(x) \rightarrow \exists_{y_1 \exists_{y_2 \exists_{y_3}} [y_1 \neq y_2 \neq y_3 \wedge \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow (\text{theorem}'(y)) \wedge \exists_{z_1 \exists_{z_2 \exists_{z_3 \exists_{z_4}} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{belong}'(y, z)]]]]]] \wedge \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \text{study}'(x, y)]]]$$



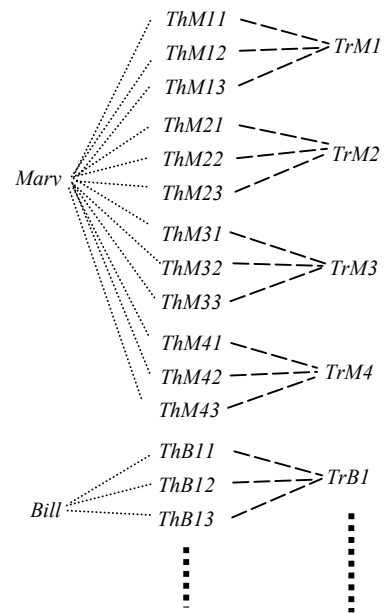
There are four theories (Tr1, Tr2, Tr3, Tr4), and three theorems for each theory. All (relevant) students studied all those theorems

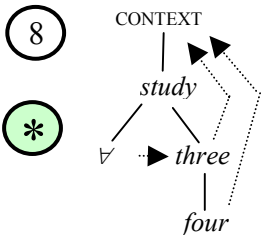
$$\exists_{z_1 \exists_{z_2 \exists_{z_3 \exists_{z_4}} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \exists_{y_1 \exists_{y_2 \exists_{y_3}} [y_1 \neq y_2 \neq y_3 \wedge \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \forall_x \forall_y [(\text{student}'(x) \wedge (y=y_1 \vee y=y_2 \vee y=y_3)) \rightarrow \text{study}'(x, y)]]]]]$$



For all the (relevant) students there are four theories. Each student studies three theorems for each of the theories associated with her/him.

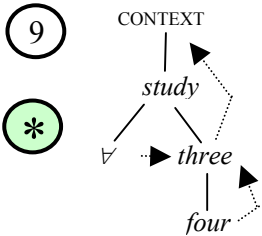
$$\forall_x [\text{student}'(x) \rightarrow \exists_{z_1 \exists_{z_2 \exists_{z_3 \exists_{z_4}} [z_1 \neq z_2 \neq z_3 \neq z_4 \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \text{theory}'(z)] \wedge \forall_z [(z=z_1 \vee z=z_2 \vee z=z_3 \vee z=z_4) \rightarrow \exists_{y_1 \exists_{y_2 \exists_{y_3}} [y_1 \neq y_2 \neq y_3 \wedge \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \forall_y [(y=y_1 \vee y=y_2 \vee y=y_3) \rightarrow \text{study}'(x, y)]]]]]$$





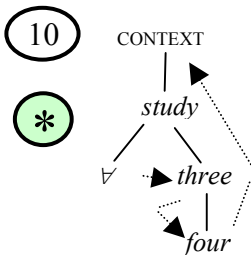
Conflates in 1.

01 0 0



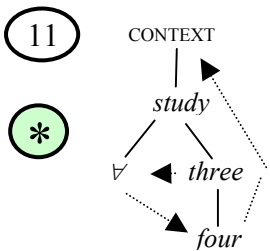
Conflates in 2.

01 0 01



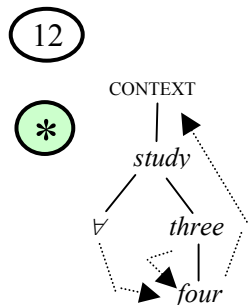
Conflates in 6.

01 010 0



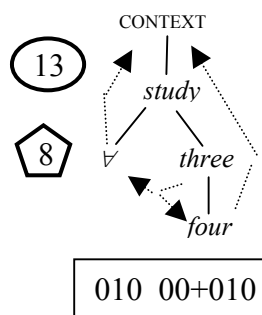
Conflates in 3.

010 00 0



Conflates in 6.

010 010 0



There are four theories (Tr1, Tr2, Tr3, Tr4). All (relevant) students chose three theorems from each theory and studied them.

$$\begin{aligned} &\exists_{z1}\exists_{z2}\exists_{z3}\exists_{z4} [z1 \neq z2 \neq z3 \neq z4 \wedge \\ &\quad \forall_z [(z=z1 \vee z=z2 \vee z=z3 \vee z=z4) \rightarrow \text{theory}'(z)] \wedge \\ &\quad \forall_x \forall_z [(\text{student}'(x) \wedge (z=z1 \vee z=z2 \vee z=z3 \vee z=z4)) \rightarrow \\ &\quad \quad \exists_{y1}\exists_{y2}\exists_{y3} [y1 \neq y2 \neq y3 \wedge \\ &\quad \quad \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \\ &\quad \quad \quad \quad (\text{theorem}'(y) \wedge \text{belong}'(y, z))] \wedge \\ &\quad \quad \quad \forall_y [(y=y1 \vee y=y2 \vee y=y3) \rightarrow \text{study}'(x,y)]]]] \end{aligned}$$

010 00+010 0

