

Inference of Global Progress Properties for Dynamically Interleaved Multiparty Sessions

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The problem

$$a(y).b(z).y?(x).z!\langle x \rangle$$
$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

- two distinct sessions
- each session is well typed
- the system gets stuck

The problem

$a(y).b(z).y?(x).z!\langle x \rangle$ $y : ?\text{int}$ $z : !\text{int}$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$ $y : !\text{int}$ $z : ?\text{int}$

- two distinct sessions
- each session is well typed
- the system gets stuck

The “interaction” type system

If $\vdash P$, then P never gets stuck

- ☺ Bettini, Coppo, D'Antoni, De Luca, Dezani-Ciancaglini, Yoshida, **Global Progress in Dynamically Interleaved Multiparty Sessions**, CONCUR 2008
- ☹ **not syntax-directed**

Outline

- ① Progress
- ② Key ideas of the type system
- ③ Two examples
- ④ Remarks

Progress 1/2

If $P \rightarrow^* \mathcal{E} [s?(x).P']$
Then $\rightarrow^* \mathcal{E}' [s?(x).P' \mid s : m \cdot h]$

If $P \rightarrow^* \mathcal{E} [s : m \cdot h]$
Then $\rightarrow^* \mathcal{E}' [s : m \cdot h \mid s?(x).P']$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

\downarrow_*

$$(\nu s)(\nu s')(s?(x).s'!\langle x \rangle \mid s'?(x).s!\langle c \rangle \mid s : \emptyset \mid s' : \emptyset)$$

A process without progress

$$a(y).b(z).y?(x).z!\langle x \rangle \mid \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

\downarrow^*

$$(\nu s)(\nu s')(s?(x).s'!\langle x \rangle \mid s'?(x).s!\langle c \rangle \mid s : \emptyset \mid s' : \emptyset)$$

Progress 2/2

A **good** process that looks like a **bad** one

$$P \rightarrow^* \mathcal{E}[s?(x).P' \mid \bar{b}(y).s!\langle 3 \rangle.Q']$$

A **bad** process that looks like a **good** one

$c(y).(a \text{ process that gets stuck})$

Progress 2/2

A **good** process that looks like a **bad** one

$$P \rightarrow^* \mathcal{E}[s?(x).P' \mid \bar{b}(y).s!\langle 3 \rangle.Q']$$

A **bad** process that looks like a **good** one

$$c(y).(a \text{ process that gets stuck})$$

Idea

- define progress modulo **catalyzers**
- catalyzer = missing participant that never gets stuck

Consequence

- session initiation can be considered **non-blocking**

Interaction type system: basic ideas

- ① associate processes with **dependencies** $a \prec b$

“an action of service a blocks an action of service b ”

- ② a process is well typed if it yields **no circular dependencies**

Computing service dependencies

$$a(y).b(z).y?(x).z!\langle x \rangle \quad a \prec b$$

$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \quad b \prec a$$

Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a \prec b$

$\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle$

$c(t).t!\langle a \rangle$

Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a \prec b$

$t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle$

$t!\langle a \rangle$



Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a \prec b$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$ $b \prec a$

Service names as messages

$a(y).b(z).y?(x).z!\langle x \rangle$ $a \prec b$

$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$

Idea

- identify a class of safe services even if mutually dependent
- restrict messages to services in this class

Nested services

Definition

a is a **nested service** if $\lambda \prec a$ implies that λ is a nested service

| | | Nested? |
|--|----------------------------|---------|
| $\bar{a}(y).\bar{a}(z).z?(x).y?(x')$ | $a \prec a$ | ✓ |
| $\bar{a}(y).\bar{b}(z).z?(x).y?(x')$ $\bar{b}(z).\bar{a}(y).y?(x).z?(x')$ | $b \prec a$ $a \prec b$ | ✓ |
| $\bar{a}(y).\bar{b}(z).y?(x).z?(x')$ | $y \prec b$ | ✗ |

Private services

$$a(y).(\nu b)(b(z).z?(x).y!\langle x \rangle)$$

- no catalyzer can help starting the session on b

Private services

$$a(y).(\nu b)(b(z).z?(x).y!\langle x \rangle)$$

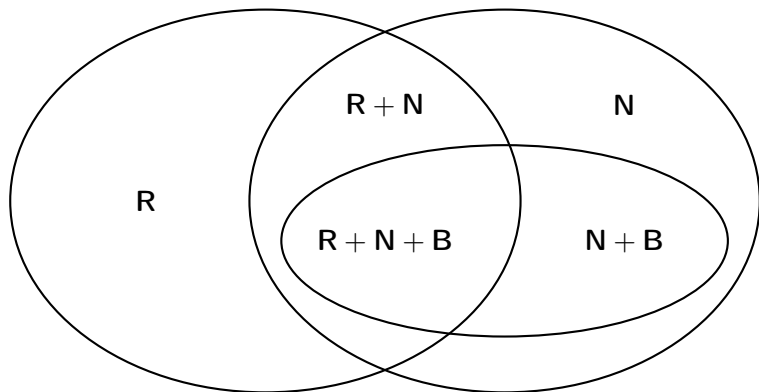
- no catalyzer can help starting the session on b

Definition

a is **boundable** if it is never followed by free channels

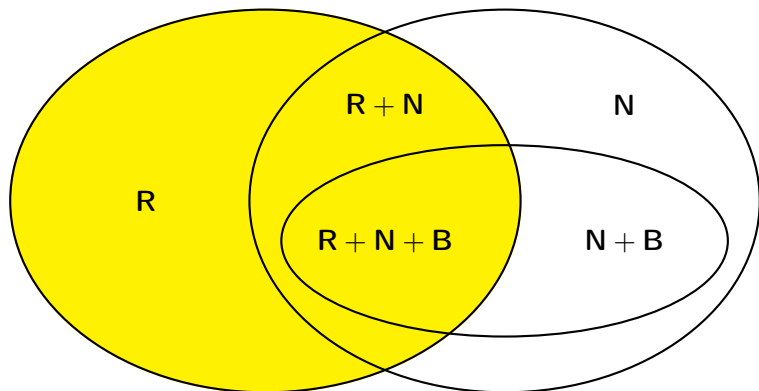
- b is nested but not boundable

Service classification



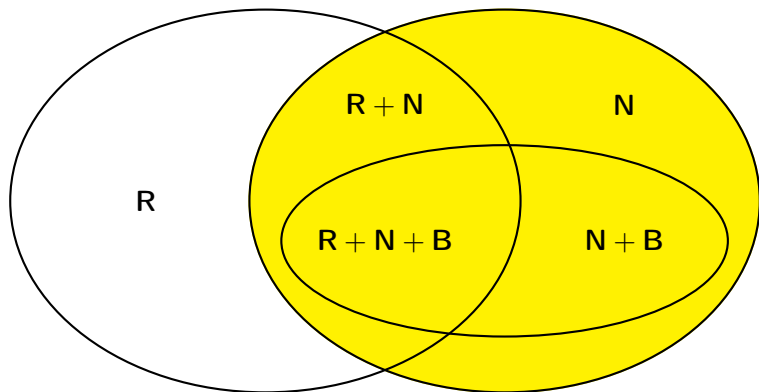
- each service can have up to three features ...
- ... which the interaction type system **guesses**

Service classification



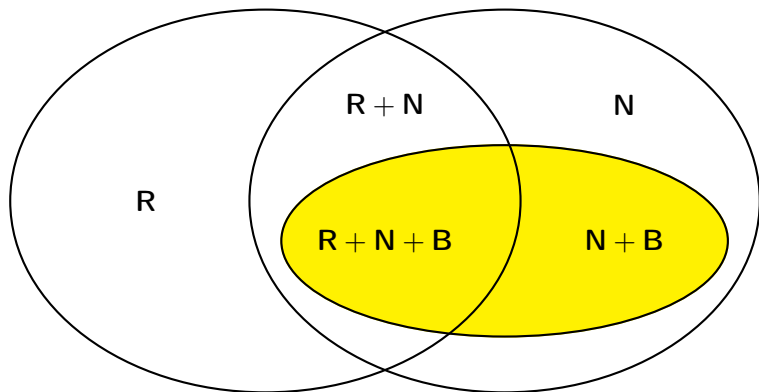
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Service classification



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Service classification



- each service can have up to three features ...
- ... which the interaction type system **guesses**

Algorithm judgments

$$P \Rightarrow D; R; N; B$$

If...

$$\begin{array}{l} D^\infty \subseteq N \setminus R \\ D \downarrow N \subseteq N \\ fs(P) \subseteq R \cup N \end{array}$$

Example 1

$$a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow$$

Example 1

$$\frac{\frac{\frac{0 \Rightarrow}{z!\langle x \rangle \Rightarrow}}{y?(x).z!\langle x \rangle \Rightarrow}}{b(z).y?(x).z!\langle x \rangle \Rightarrow}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

Example 1

all services have all features

$$\frac{\frac{\frac{0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{\quad}}{z!\langle x \rangle \Rightarrow}}{y?(x).z!\langle x \rangle \Rightarrow}}{b(z).y?(x).z!\langle x \rangle \Rightarrow}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

Example 1

$$\frac{\frac{\frac{0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{z!\langle x \rangle \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{y?(x).z!\langle x \rangle \Rightarrow}}{b(z).y?(x).z!\langle x \rangle \Rightarrow}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

Example 1

$$\frac{\frac{\frac{0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{z!\langle x \rangle \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{y?(x).z!\langle x \rangle \Rightarrow \{y \prec z\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{b(z).y?(x).z!\langle x \rangle \Rightarrow} \frac{}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow}$$

Example 1

$$\frac{
 \frac{
 \frac{
 0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}
 }{
 z! \langle x \rangle \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}
 }{
 y?(x).z! \langle x \rangle \Rightarrow \{y < z\}; \mathcal{S}; \mathcal{S}; \mathcal{S}
 }{
 b(z).y?(x).z! \langle x \rangle \Rightarrow \{y < b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}
 }{
 a(y).b(z).y?(x).z! \langle x \rangle \Rightarrow
 }$$

$B \subseteq N$

$D \downarrow N \subseteq N$

Example 1

$$\frac{\frac{\frac{0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{z!\langle x \rangle \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{y?(x).z!\langle x \rangle \Rightarrow \{y \prec z\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{b(z).y?(x).z!\langle x \rangle \Rightarrow \{y \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}}}{a(y).b(z).y?(x).z!\langle x \rangle \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}}$$

Example 1 (cont.)

$$\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Rightarrow$$

Example 1 (cont.)

$$\frac{\frac{\frac{0 \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{z?(x).y!\langle x \rangle \Rightarrow \{z \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Rightarrow$$

Example 1 (cont.)

$$\frac{\frac{\frac{0 \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{y!\langle x \rangle \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{z?(x).y!\langle x \rangle \Vdash \{z \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}}{\bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$a(y) \cdots \Vdash \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}$$

$$a(y) \cdots \mid \bar{a}(y) \cdots \Vdash$$

Example 1 (cont.)

$$\begin{array}{c}
 0 \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 y!\langle x \rangle \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 z?(x).y!\langle x \rangle \Vdash \{z \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 \bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 \bar{a}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}
 \end{array}$$

$$\frac{a(y) \cdots \Vdash \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}{a(y) \cdots \mid \bar{a}(y) \cdots \Vdash}$$

Example 1 (cont.)

$$\frac{0 \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}{y! \langle x \rangle \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$\frac{z?(x).y! \langle x \rangle \Vdash \{z \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}{\bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$\frac{\bar{a}(y).\bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$\frac{a(y) \cdots \Vdash \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{a \prec b, b \prec a\}; \mathcal{S} \setminus \{a, b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}}{a(y) \cdots \mid \bar{a}(y) \cdots \Vdash \{a \prec b, b \prec a\}; \mathcal{S} \setminus \{a, b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}}$$

$D^\infty \subseteq \mathbb{N} \setminus \mathbb{R}$

Example 1 (cont.)

$$\begin{array}{c}
 0 \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 y! \langle x \rangle \Vdash \emptyset; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 z?(x).y! \langle x \rangle \Vdash \{z \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 \bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 \bar{a}(y).\bar{b}(z).z?(x).y! \langle x \rangle \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S}
 \end{array}$$

~~$$\begin{array}{c}
 a(y) \cdots \Vdash \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{a}(y) \cdots \Vdash \{b \prec a\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 a(y) \cdots \mid \bar{a}(y) \cdots \Vdash \{a \prec b, b \prec a\}; \mathcal{S} \setminus \{a, b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}
 \end{array}$$~~

Example 2

$$\overline{c(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow$$

$$a(y) \cdots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}$$

$$a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a \prec b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}$$

Example 2

$$\begin{array}{c} \vdots \\ \hline \overline{\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\ \hline \bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \\ \hline t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \\ \hline \bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \\ \\ a(y) \cdots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\ \hline a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a \prec b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \end{array}$$

Example 2

$$\begin{array}{c}
 \vdots \text{ x must be nested, so } b \text{ too} \\
 \hline
 \bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S} \\
 \hline
 \bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\
 \hline
 t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \dots \\
 \hline
 \text{dependencies discharged} \\
 \hline
 \bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \dots
 \end{array}$$

$$\begin{array}{c}
 a(y) \dots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \dots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\
 \hline
 a(y) \dots \mid \bar{c}(t) \dots \Rightarrow \{a \prec b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}
 \end{array}$$

Example 2

⋮

$$\frac{}{\overline{b(z).z?(x).y!\langle x \rangle} \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{\overline{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$a(y) \cdots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}$$

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Example 2

$$\begin{array}{c}
 \vdots \\
 \hline
 \overline{\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S} \\
 \hline
 \overline{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\
 \hline
 \overline{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\
 \hline
 \overline{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle} \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S} \\
 \\
 \overline{a(y) \cdots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}} \\
 \hline
 a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a \prec b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}
 \end{array}$$

Example 2

⋮

$$\frac{}{\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \{b \prec y\}; \mathcal{S}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$\frac{}{\bar{c}(t).t?(x).\bar{x}(y).\bar{b}(z).z?(x).y!\langle x \rangle \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}}$$

$$a(y) \cdots \Rightarrow \{a \prec b\}; \mathcal{S}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\} \quad \bar{c}(t) \cdots \Rightarrow \emptyset; \mathcal{S} \setminus \{b\}; \mathcal{S}; \mathcal{S}$$

$$a(y) \cdots \mid \bar{c}(t) \cdots \Rightarrow \{a \prec b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}; \mathcal{S} \setminus \{b\}$$

Result

Theorem

If $P \Rightarrow D; R; N; B$, then P has progress

Proof.

The algorithm is sound and complete wrt the inference type system (cf. CONCUR 2008) □

Result

Theorem

If $P \Rightarrow D; R; N; B$, then P has progress

Proof.

The algorithm is sound and complete wrt the inference type system (cf. CONCUR 2008) (for finite processes only) □

Soon to come

Inference for recursive processes

Wrap up

- static analysis for (multiparty) session interleaving
- progress \neq absence of deadlock
 - diverging systems do not necessarily have progress
 - catalyzers may help reduction
- efficient inference algorithm

Future work

- many simple program patterns are **ill typed**
 - ☞ more flexible type discipline is required
- π -calculus \neq **programming language**
 - ☞ richer/more compositional types are needed
- type systems for liveness properties are **complex**
 - ☞ traditional concepts/techniques (fairness, subtyping, coinductive reasoning, ...) must be revisited

A simple ill-typed process with progress

$$\begin{array}{ll} \text{def } X(y, z) = y!\langle 3 \rangle.z?(x).X(y, z) \text{ in} & b \prec a \\ \text{def } Y(y, z) = y?(x).z!\langle x \rangle.Y(y, z) \text{ in} & a \prec b \\ a(y).b(z).X(y, z) \mid \bar{a}(y).\bar{b}(z).Y(y, z) & \end{array}$$