Deadlock and lock freedom in the linear π-calculus

Luca Padovani – Torino

Objective

Assuring properties of communicating processes

- no deadlocks
- no locks

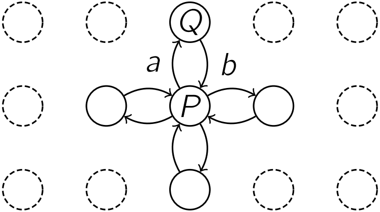
Problem undecidable in general

- Brand, Zafiropulo, **On communicating finite-state machines**, Journal of ACM, 1983
- Cécé, Finkel, **Verification of programs with half-duplex communication**, Inf. & Comp., 2005

Method

- types

Example: full-duplex communication




```
while true do
   send(a, s_P)
   s_Q := receive(b)
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Difficult to handle with current type systems

- recursion
- cyclic network
- several channels

Outline

1. The linear π-calculus
2. Types for deadlock prevention
3. Results
4. Concluding remarks

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The linear π-calculus

Unlimited channel

$$p^{\omega}[t]$$

≥ 0 communications

Linear channel

$$p[t]$$

1 communication

Kobayashi, Pierce, Turner, **Linearity and the pi-calculus**, TOPLAS 1999

Why the linear π-calculus?

"communications on linear channels [...] account for up to 50% of communications in a typical program"

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Linear vs "linearized" channels

$a!\langle 3\rangle.a!\langle 4\rangle \dots$ $\qquad$ $(\nu b)a!\langle 3, b\rangle.(\nu c)b!\langle 4, c\rangle \dots$

- Kobayashi, **Type systems for concurrent programs**, 2002
- Dardha, Giachino, Sangiorgi, **Session types revisited**, 2012

Formulating the problem

Theorem (Kobayashi, Pierce, Turner, 1999)

In a well-typed process, each linear channel is used for communication at most once

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- ⊕ the process is well typed (a and b are linear)
- ⊖ no communication is possible

Formulating the problem

Theorem (Kobayashi, Pierce, Turner, 1999)

In a well-typed process, each linear channel is used for communication ~~at most once~~ **exactly once**

$$a?(x).b!\langle x\rangle \mid b?(y).a!\langle y\rangle$$

- ⊕ the process is well typed (a and b are linear)
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Outline

Strategy

1. associate each linear channel a level $\in \mathbb{Z}$

$$p[t]^{h}$$

2. make sure that channels are used in strict order

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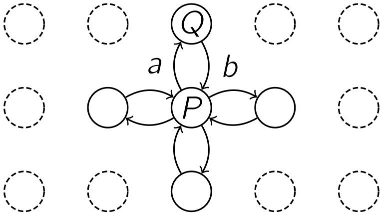
Typing rules

Input
$$\frac{\Gamma, x : t \vdash P \qquad h < |\Gamma|}{\Gamma, u : ?[t]^{h} \vdash u?(x).P}$$

Output
$$\frac{\Gamma \vdash e : t \qquad h < |t|}{\Gamma, u : \,![t]^{h} \vdash u!\langle e \rangle}$$

Problem: most recursive processes are **ill typed**



while **true** **do**

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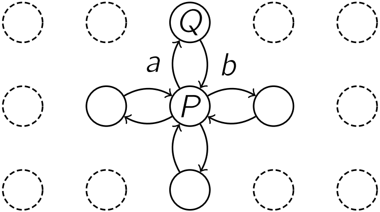


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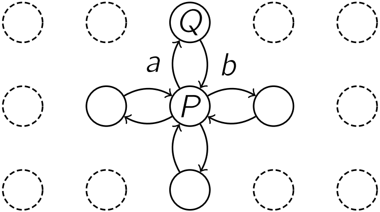


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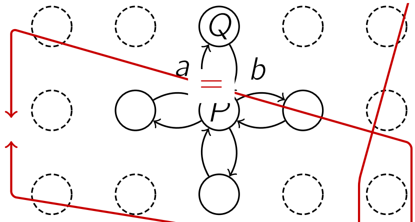


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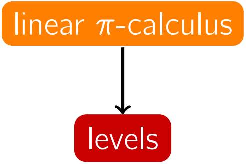
Giving up linearity



Giving up linearity

linear π-calculus

levels

more structured types $\Rightarrow$
fewer constraints on levels

Usages

- Kobayashi, *Inf. & Comp.*, 2002
- ...

Session types

- Dezani *et al.* CONCUR 2008
- ...many more

Example: full-duplex communication

- ⊕ recursive processes are well typed. . .
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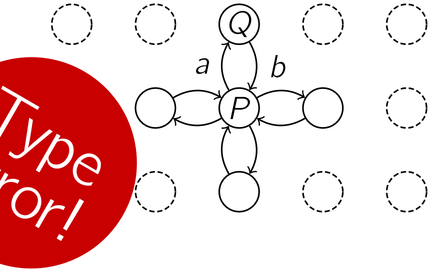

$$
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\end{array}
$$

recursion + cycles = mutual channel dependencies

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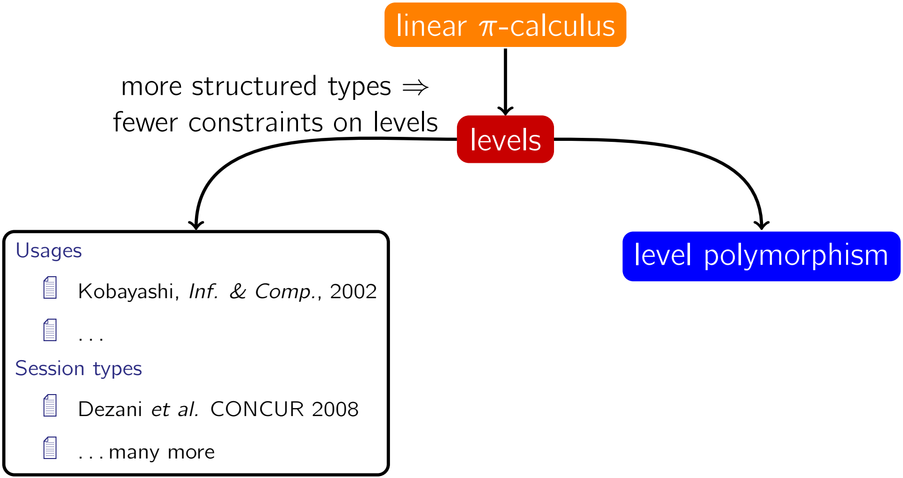

Level polymorphism

$$a?(b^{7}).b?(x).c^{8}!\langle x\rangle \qquad a?(b^{7}, c^{8}).b?(x).c!\langle x\rangle$$

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It depends on the continuation after an **input** on the channel

$a?(x).P$

some free linear channel in $P \Rightarrow a$ monomorphic

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Typing of linear π-calculus (Kobayashi, Pierce, Sangiorgi, 1999)

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no free linear

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Key observation

All channels used for replicated inputs are level-polymorphic
This is great news! recursion $\Rightarrow$ replication

Typing rules (revised)

Linear output

$$\frac{\Gamma \vdash e : t \qquad h < |t|}{\Gamma, u : ![t]^{h} \vdash u!\langle e \rangle}$$

Unlimited output

$$\frac{\Gamma \vdash e : \$^{k}\, t}{\Gamma, u : !^{\omega}[t] \vdash u!\langle e \rangle}$$

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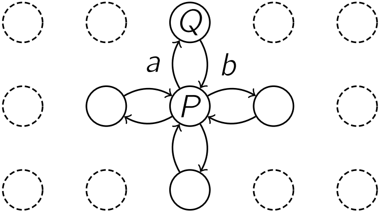

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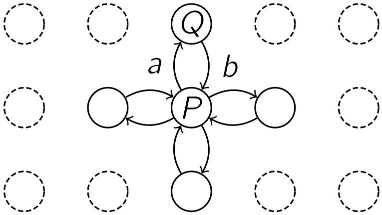


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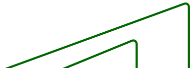

Outline

Result 1: deadlock freedom

Definition

P is **deadlock free** if $P \rightarrow^* (\nu \tilde{a})Q \not\rightarrow$ implies $\neg\mathrm{wait}(a, Q)$ for all a

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P closed and well typed implies P deadlock free

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Theorem

P closed and well typed implies P deadlock free

- but this is well typed...

$$c!\langle a\rangle \mid *c?(x).c!\langle x\rangle \mid a!\langle 1984\rangle$$

Result 2: lock freedom

Definition

P is **lock free** if $P \rightarrow^* (\nu \tilde{a})Q$ and $\mathrm{wait}(a, Q)$ implies

$$Q \rightarrow^* R \text{ such that } \mathrm{sync}(a, R)$$

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on on a is possible

$$\#[t]^{h}_{k}$$

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$$\#[t]^{h}$$

tickets = max no. of travels

Theorem

P closed and well typed with levels $\in \mathbb{N}$ implies P lock free

Result 3: expressiveness

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Theorem (not in proceedings, see long version)

If you can draw a **message sequence chart** for your protocol, then you can realize the protocol with a well-typed process

Result 3: expressiveness

- ⊖ characterizing well-typed processes is an open problem, but. . .

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If you can draw a **message sequence chart** for your protocol, then you can realize the protocol with a well-typed process

- ⊕ variable number of channels
- ⊕ variable number of processes
- ⊕ variable network topology

Outline

Some shortcomings

- Global ordering of events

$$a!\langle 3\rangle \mid b!\langle 4\rangle \mid \texttt{if}\ \cdots\ \begin{array}{l}\texttt{then}\ a?(x).b?(y)\cdots\\ \texttt{else}\ b?(y).a?(x)\cdots\end{array}$$

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$$a!\langle 3\rangle \mid b!\langle 4\rangle \mid \texttt{if}\ \cdots\ \texttt{then}\ a?(x).b?(y)\cdots$$
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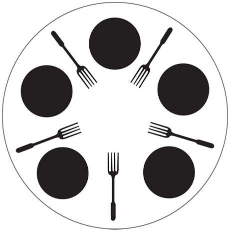
- Linear/unlimited dichotomy



Giachino, Kobayashi, Laneve, **Deadlock analysis of unbounded process networks**, CONCUR 2014 (to appear)

Further work

- Type reconstruction (with Tzu-Chun Chen and Andrea Tosatto)
  - Igarashi, Kobayashi, **Type Reconstruction for Linear π-Calculus with I/O Subtyping**, Inf. & Comp. 2000
  - Padovani, **Type Reconstruction for the Linear π-Calculus with Composite and Equi-Recursive Types**, FoSSaCS 2014

$$\left.\begin{array}{r}\text{level/ticket variables}\\ \text{linear constraints}\end{array}\right\}\text{linear (integer) programming problem}$$

- Extension to higher-order languages (with Luca Novara)
  - Padovani, Novara, **Types for Deadlock-Free Higher-Order Concurrent Programs**, 2014 (tech report on my homepage)