From Lock Freedom to Progress Using Session Types

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The problem

$$(\nu ab)(\nu cd)\left(\begin{array}{l} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$$\begin{array}{ll} a : ?\mathit{int} & d : !\mathit{int} \\ b : !\mathit{int} & c : ?\mathit{int} \end{array}$$

- two distinct sessions
- each session is well typed
- the system makes no progress

From lock freedom to progress

- Bettini *et al.*, **Global Progress in Dynamically Interleaved Multiparty Sessions**, CONCUR 2008
- Coppo *et al.*, **Inference of Global Progress Properties for. . .** , BEAT and COORDINATION 2013
  - for multiparty sessions
  - asynchronous communication
  - session types for linear channels

- Kobayashi, **A Type System for Lock-Free Processes**, Inf. and Comp., 2002
  - for the (almost) pure π-calculus
  - synchronous communication
  - usages for non-linear channels

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Is it a good idea?

$$\Gamma, u : S \vdash u : S \ \text{(NAME)} \qquad \Gamma \vdash \mathsf{true}, \mathsf{false} : \mathsf{bool} \qquad \frac{\Gamma \vdash e_i : \mathsf{bool} \qquad (i = 1, 2)}{\Gamma \vdash e_1 \ \mathsf{and} \ e_2 : \mathsf{bool}} \ \text{(BOOL),(AND)}$$

$$\frac{\Gamma \vdash u : \mathsf{G} \quad \Gamma \vdash P \triangleright \Delta, y : \mathsf{G} \upharpoonright \mathsf{p} \quad \mathsf{p} = \mathsf{mp}(\mathsf{G})}{\Gamma \vdash \overline{u}[\mathsf{p}](y).P \triangleright \Delta} \ \text{(MCAST)} \qquad \frac{\Gamma \vdash u : \mathsf{G} \quad \Gamma \vdash P \triangleright \Delta, y : \mathsf{G} \upharpoonright \mathsf{p} \quad \mathsf{p} < \mathsf{mp}(\mathsf{G})}{\Gamma \vdash u[\mathsf{p}](y).P \triangleright \Delta} \ \text{(MACC)}$$

$$\frac{\Gamma \vdash e : S \quad \Gamma \vdash P \triangleright \Delta, c : T}{\Gamma \vdash c!\langle \Pi, e \rangle.P \triangleright \Delta, c : !\langle \Pi, S \rangle.T} \ \text{(SEND)} \qquad \frac{\Gamma, x : S \vdash P \triangleright \Delta, c : T}{\Gamma \vdash c?(\mathsf{q}, x).P \triangleright \Delta, c : ?(\mathsf{q}, S).T} \ \text{(RCV)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta, c : T}{\Gamma \vdash c!\langle\langle \mathsf{p}, c' \rangle\rangle.P \triangleright \Delta, c : !\langle \{\mathsf{p}\}, \mathsf{T} \rangle.T, c' : \mathsf{T}} \ \text{(DELEG)} \qquad \frac{\Gamma \vdash P \triangleright \Delta, c : T, y : \mathsf{T}}{\Gamma \vdash c?((\mathsf{q}, y)).P \triangleright \Delta, c : ?(\mathsf{q}, \mathsf{T}).T} \ \text{(SRCV)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta, c : T_j \quad j \in I}{\Gamma \vdash c \oplus \langle \Pi, l_j \rangle.P \triangleright \Delta, c : \oplus \langle \Pi, \{l_i : T_i\}_{i \in I} \rangle} \ \text{(SEL)} \qquad \frac{\Gamma \vdash P_i \triangleright \Delta, c : T_i \quad \forall i \in I}{\Gamma \vdash c \& (\mathsf{p}, \{l_i : P_i\}_{i \in I}) \triangleright \Delta, c : \& (\mathsf{p}, \{l_i : T_i\}_{i \in I})} \ \text{(BRANCH)}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \quad \Gamma \vdash Q \triangleright \Delta'}{\Gamma \vdash P \mid Q \triangleright \Delta, \Delta'} \ \text{(PAR)} \qquad \frac{\Gamma \vdash e : \mathsf{bool} \quad \Gamma \vdash P \triangleright \Delta \quad \Gamma \vdash Q \triangleright \Delta}{\Gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ P\ \mathsf{else}\ Q \triangleright \Delta} \ \text{(IF)}$$

$$\frac{\Delta \ \text{end only}}{\Gamma \vdash \mathbf{0} \triangleright \Delta} \ \text{(INACT)} \qquad \frac{\Gamma, a : G \vdash P \triangleright \Delta}{\Gamma \vdash (\nu a : G)P \triangleright \Delta} \ \text{(NRES)}$$

$$\frac{\Gamma \vdash e : S \quad \Delta \ \text{end only}}{\Gamma, X : S\, T \vdash X\langle e, c \rangle \triangleright \Delta, c : T} \ \text{(VAR)} \qquad \frac{\Gamma, X : S\, \mathbf{t}, x : S \vdash P \triangleright y : T \qquad \Gamma, X : S\, \mu\mathbf{t}.T \vdash Q \triangleright \Delta}{\Gamma \vdash \mathsf{def}\ X(x, y) = P\ \mathsf{in}\ Q \triangleright \Delta} \ \text{(DEF)}$$

Is it a good idea?

$$\Gamma, u : S \vdash u : S \ (\textsc{Name}) \qquad \Gamma \vdash \ \mathsf{true}, \mathsf{false} : \mathsf{bool} \qquad \frac{\Gamma \vdash e_i : \mathsf{bool} \qquad (i = 1, 2)}{\Gamma \vdash e_1 \ \mathsf{and} \ e_2 : \mathsf{bool}} \ (\textsc{Bool}),(\textsc{And})$$

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$$\frac{\Gamma \vdash e : S \quad \Gamma \vdash P \rhd \Delta, c : T}{\Gamma \vdash c!\langle \Pi, e\rangle.P \rhd \Delta, c : !\langle \Pi, S\rangle.T} \ (\textsc{Send}) \qquad \frac{\Gamma, x : S \vdash P \rhd \Delta, c : T}{\Gamma \vdash c?(\mathsf{q}, x).P \rhd \Delta, c : ?(\mathsf{q}, S).T} \ (\textsc{Rcv})$$

$$\frac{\Gamma \vdash P \rhd \Delta, c : T}{\Gamma \vdash c!\langle\langle \mathsf{p}, c'\rangle\rangle.P \rhd \Delta, c : !\langle \{\mathsf{p}\}, \mathsf{T}\rangle.T, c' : \mathsf{T}} \ (\textsc{Deleg}) \qquad \frac{\Gamma \vdash P \rhd \Delta, c : T, y : \mathsf{T}}{\Gamma \vdash c?((\mathsf{q}, y)).P \rhd \Delta, c : ?(\mathsf{q}, \mathsf{T}).T} \ (\textsc{SRcv})$$

$$\frac{\Gamma \vdash P \rhd \Delta, c : T_j \quad j \in I}{\Gamma \vdash c \oplus \langle \Pi, l_j\rangle.P \rhd \Delta, c : \oplus\langle \Pi, \{l_i : T_i\}_{i \in I}\rangle} \ (\textsc{Sel}) \qquad \frac{\Gamma \vdash P_i \rhd \Delta, c : T_i \quad \forall i \in I}{\Gamma \vdash c\&(\mathsf{p}, \{l_i : P_i\}_{i \in I}) \rhd \Delta, c : \&(\mathsf{p}, \{l_i : T_i\}_{i \in I})} \ (\textsc{Branch})$$

$$\frac{\Gamma \vdash P \rhd \Delta \quad \Gamma \vdash Q \rhd \Delta'}{\Gamma \vdash P \mid Q \rhd \Delta, \Delta'} \ (\textsc{Par}) \qquad \frac{\Gamma \vdash e : \mathsf{bool} \quad \Gamma \vdash P \rhd \Delta \quad \Gamma \vdash Q \rhd \Delta}{\Gamma \vdash \mathsf{if} \ e \ \mathsf{then} \ P \ \mathsf{else} \ Q \rhd \Delta} \ (\textsc{If})$$

$$\frac{\Delta \ \mathsf{end \ only}}{\Gamma \vdash \mathbf{0} \rhd \Delta} \ (\textsc{Inact}) \qquad \frac{\Gamma, a : G \vdash P \rhd \Delta}{\Gamma \vdash (\nu a : G)P \rhd \Delta} \ (\textsc{NRes})$$

$$\frac{\Gamma \vdash e : S \quad \Delta \ \mathsf{end \ only}}{\Gamma, X : S\,T \vdash X\langle e, c\rangle \rhd \Delta, c : T} \ (\textsc{Var}) \qquad \frac{\Gamma, X : S\,\mathbf{t}, x : S \vdash P \rhd y : T \qquad \Gamma, X : S\,\mu\mathbf{t}.T \vdash Q \rhd \Delta}{\Gamma \vdash \mathsf{def} \ X(x, y) = P \ \mathsf{in} \ Q \rhd \Delta} \ (\textsc{Def})$$

$+$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad a \in \mathscr{R}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \tilde{a}[\mathsf{p}](y).P \blacktriangleright \mathscr{D}\{a/y\}^+} \ \{\textsc{InitR}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad a \in \mathscr{N}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \tilde{a}[\mathsf{p}](y).P \blacktriangleright \mathscr{D} \setminus\!\!\setminus y} \ \{\textsc{InitN}\}$$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad a \in \mathscr{B} \quad \mathsf{fc}(P) \subseteq \{y\}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \tilde{a}[\mathsf{p}](y).P \blacktriangleright \mathscr{D} \setminus\!\!\setminus y} \ \{\textsc{InitB}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad \mathsf{fc}(P) \subseteq \{y\}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \tilde{x}[\mathsf{p}](y).P \blacktriangleright \mathscr{D} \setminus\!\!\setminus y} \ \{\textsc{InitV}\}$$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad e \in \mathscr{S} \Rightarrow e \in \mathscr{N} \cup \mathscr{B}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c!\langle \Pi, e\rangle.P \blacktriangleright \mathscr{D}} \ \{\textsc{Send}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c?(\mathsf{q}, x).P \blacktriangleright (\mathsf{pre}(c, \mathsf{fc}(P)) \cup \mathscr{D})^+} \ \{\textsc{Rcv}\}$$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c!\langle\langle \mathsf{p}, c'\rangle\rangle.P \blacktriangleright (\{\curlywedge(c) \prec \curlywedge(c')\} \cup \mathscr{D})^+} \ \{\textsc{Deleg}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad \mathscr{D} \setminus \mathscr{S} \subseteq \{\curlywedge(c) \prec y\}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c?((\mathsf{q}, y)).P \blacktriangleright \mathscr{D} \setminus \{y\}} \ \{\textsc{SRcv}\}$$

$$\frac{}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \mathbf{0} \blacktriangleright \emptyset} \ \{\textsc{Inact}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \quad a \in \mathscr{B}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \setminus \{a\} \vdash (\nu a : \mathsf{G})P \blacktriangleright \mathscr{D} \setminus \{a\}} \ \{\textsc{NRes}\}$$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_1 \blacktriangleright \mathscr{D}_1 \quad \Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_2 \blacktriangleright \mathscr{D}_2}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_1 \mid P_2 \blacktriangleright (\mathscr{D}_1 \cup \mathscr{D}_2)^+} \ \{\textsc{Par}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_1 \blacktriangleright \mathscr{D}_1 \quad \Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_2 \blacktriangleright \mathscr{D}_2}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \mathsf{if} \ e \ \mathsf{then} \ P_1 \ \mathsf{else} \ P_2 \blacktriangleright (\mathscr{D}_1 \cup \mathscr{D}_2)^+} \ \{\textsc{If}\}$$

$$\frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D}}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c \oplus \langle \Pi, l\rangle.P \blacktriangleright \mathscr{D}} \ \{\textsc{Sel}\} \qquad \frac{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash P_i \blacktriangleright \mathscr{D}_i \quad \forall i \in I}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash c\&(\mathsf{p}, \{l_i : P_i\}_{i \in I}) \blacktriangleright (\mathsf{pre}(c, \bigcup_{i \in I} \mathsf{fc}(P_i)) \cup \bigcup_{i \in I} \mathscr{D}_i)^+} \ \{\textsc{Branch}\}$$

$$\frac{e \in \mathscr{S} \Rightarrow e \in \mathscr{N} \cup \mathscr{B}}{\Theta, X[y] \blacktriangleright \mathscr{D};\mathscr{R};\mathscr{N};\mathscr{B} \vdash X\langle e, c\rangle \blacktriangleright \mathscr{D}\{\curlywedge(c)/y\}} \ \{\textsc{Var}\}$$

$$\frac{\Theta, X[y] \blacktriangleright \mathscr{D};\mathscr{R};\mathscr{N};\mathscr{B} \vdash P \blacktriangleright \mathscr{D} \qquad \Theta, X[y] \blacktriangleright \mathscr{D};\mathscr{R};\mathscr{N};\mathscr{B} \vdash Q \blacktriangleright \mathscr{D}'}{\Theta;\mathscr{R};\mathscr{N};\mathscr{B} \vdash \mathsf{def} \ X(x, y) = P \ \mathsf{in} \ Q \blacktriangleright \mathscr{D}'} \ \{\textsc{Def}\}$$

Is it a good idea?

$$\Gamma, u:S \vdash u:S \ (\textsc{Name}) \qquad \Gamma \vdash \mathsf{true},\mathsf{false}:\mathsf{bool} \qquad \frac{\Gamma \vdash e_i:\mathsf{bool} \quad (i=1,2)}{\Gamma \vdash e_1 \ \mathsf{and}\ e_2:\mathsf{bool}}\ (\textsc{Bool}),(\textsc{And})$$

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$$\frac{\Gamma \vdash P \triangleright \Delta, c:T}{\Gamma \vdash c!\langle\langle \mathsf{p}, c'\rangle\rangle.P \triangleright \Delta, c:!\langle \{\mathsf{p}\}, \mathsf{T}\rangle.T, c':\mathsf{T}}\ (\textsc{Deleg}) \qquad \frac{\Gamma \vdash P \triangleright \Delta, c:T, y:\mathsf{T}}{\Gamma \vdash c?((\mathsf{q},y)).P \triangleright \Delta, c:?(\mathsf{q},\mathsf{T}).T}\ (\textsc{SRcv}) \ *$$

$$\frac{\Gamma \vdash P \triangleright \Delta, c:T_j \quad j \in I}{\Gamma \vdash c \oplus \langle \Pi, l_j\rangle.P \triangleright \Delta, c:\oplus\langle \Pi, \{l_i:T_i\}_{i\in I}\rangle}\ (\textsc{Sel}) \qquad \frac{\Gamma \vdash P_i \triangleright \Delta, c:T_i \quad \forall i \in I}{\Gamma \vdash c\&(\mathsf{p},\{l_i:P_i\}_{i\in I}) \triangleright \Delta, c:\&(\mathsf{p},\{l_i:T_i\}_{i\in I})}\ (\textsc{Branch}) \ *$$

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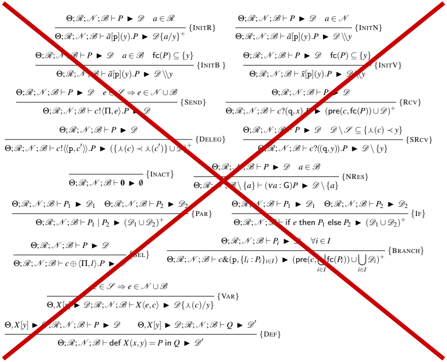


- one constraint on type rules for inputs $*$
- three constraints on session types

Outline

1. Processes & Types
2. Constraints
3. Examples
4. Remarks

The language

P	$::=$		**Process**
		$\mathbf{0}$	(idle process)
	$\mid$	$u?(x).P$	(input)
	$\mid$	$u!e.P$	(output)
	$\mid$	$P \mid P$	(composition)
	$\mid$	$(\nu ab)P$	(session)
	$\mid$	$\text{def } X(\vec{u}) = P \text{ in } P$	(definition)
	$\mid$	$X\langle\vec{u}\rangle$	(invocation)

+ unbounded FIFO queues (asynchronous communication)

Strategy

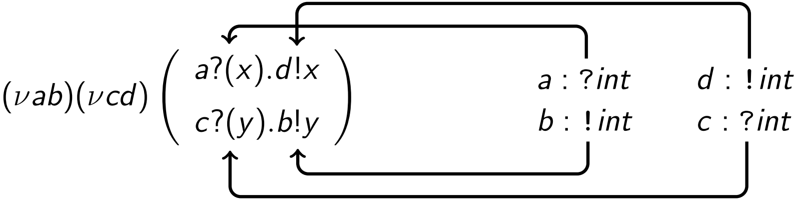
$$(\nu ab)(\nu cd)\left(\begin{array}{l} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$a : ?int \qquad d : !int$

$b : !int \qquad c : ?int$

1. Associate actions (in types) with timestamps t_a, t_b, ...
2. Determine constraints between timestamps $t_a < t_d$, ...
3. See whether the constraints admit a solution ($\Rightarrow$ well founded order for actions in the process)

Strategy



1. Associate actions (in types) with timestamps t_a, t_b, ...
2. Determine constraints between timestamps $t_a < t_d$, ...
3. See whether the constraints admit a solution
   ($\Rightarrow$ well founded order for actions in the process)

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Session types with timestamps

$$
\begin{array}{rcl}
T & ::= & \mathsf{end} \\
& | & \texttt{?}T.S \\
& | & \texttt{!}T.S \\
& | & \mathsf{rec}\ \alpha.T \\
& | & \alpha
\end{array}
$$

Session types with timestamps

$$
\begin{array}{rcl}
T & ::= & \mathsf{end} \\
& | & \langle \delta_1, \delta_2 \rangle ?\, T.S \\
& | & \langle \delta_1, \delta_2 \rangle !\, T.S \\
& | & \mathsf{rec}\ \alpha.T \\
& | & \alpha
\end{array}
$$

- δ_1 = obligation = "time limit for the action to begin"
- δ_2 = capability = "time limit for the action to end"

Constraint $\boxed{\mathbf{C1}}$: input prefixes

$$\frac{\Gamma, u : T, x : S \vdash P}{\Gamma, u : \langle \delta_1, \delta_2 \rangle ?S.T \vdash u?(x).P}$$

Constraint $\boxed{\mathbf{C1}}$: input prefixes

$$\frac{\Gamma, u : T, x : S \vdash P \qquad \delta_2 < \mathsf{ob}(\Gamma(v))^{\;v \in \mathsf{dom}(\Gamma)}}{\Gamma, u : \langle \delta_1, \delta_2 \rangle ?S.T \vdash u?(x).P}$$

blocking action ends before **blocked** actions begin

Constraint C2: duality

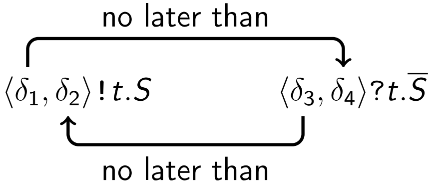
$$\frac{\Gamma, a : T, b : \overline{T} \vdash P}{\Gamma \vdash (\nu ab)P}$$

$\langle \delta_1, \delta_2 \rangle ! t.S$

$\langle \delta_3, \delta_4 \rangle ? t.\overline{S}$

Constraint C2: duality

$$\frac{\Gamma, a : T, b : \overline{T} \vdash P}{\Gamma \vdash (\nu ab)P}$$



Constraint C2: duality

$$\frac{\Gamma, a : T, b : \overline{T} \vdash P}{\Gamma \vdash (\nu ab)P}$$

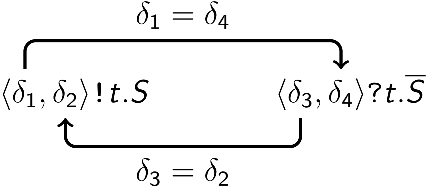
$\delta_1 \leq \delta_4$

$\langle \delta_1, \delta_2 \rangle \mathbf{!} t.S \qquad \langle \delta_3, \delta_4 \rangle \mathbf{?} t.\overline{S}$

$\delta_3 \leq \delta_2$

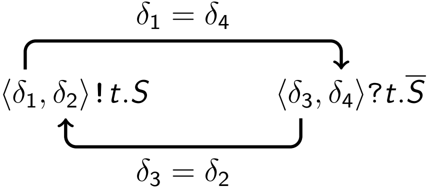
Constraint C2: duality

$$\frac{\Gamma, a : T, b : \overline{T} \vdash P}{\Gamma \vdash (\nu ab)P}$$



Constraint C2: duality

$$\frac{\Gamma, a : T, b : \overline{T} \vdash P}{\Gamma \vdash (\nu ab)P}$$



$$\overline{\langle \delta_1, \delta_2 \rangle ! t.S} = \langle \delta_2, \delta_1 \rangle ? t.\overline{S}$$

Example #1

$$(\nu ab)(\nu cd)\left(\begin{array}{c} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$$\begin{array}{ll} a : \langle\delta_1, \delta_2\rangle ?\,int & d : \langle\delta_3, \delta_4\rangle !\,int \\ b : \langle\delta_2, \delta_1\rangle !\,int & c : \langle\delta_4, \delta_3\rangle ?\,int \end{array}$$

Example #1

$\delta_2 < \delta_3$

$$(\nu ab)(\nu cd)\left(\begin{array}{l} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$$\begin{array}{ll} a : \langle \delta_1, \delta_2 \rangle ?\mathit{int} & d : \langle \delta_3, \delta_4 \rangle !\mathit{int} \\ b : \langle \delta_2, \delta_1 \rangle !\mathit{int} & c : \langle \delta_4, \delta_3 \rangle ?\mathit{int} \end{array}$$

Example #1

$\delta_2 < \delta_3$

$$(\nu ab)(\nu cd)\begin{pmatrix} a?(x).d!x \\ c?(y).b!y \end{pmatrix}$$

$$\begin{array}{ll} a : \langle \delta_1, \delta_2 \rangle ?\mathit{int} & d : \langle \delta_3, \delta_4 \rangle !\mathit{int} \\ b : \langle \delta_2, \delta_1 \rangle !\mathit{int} & c : \langle \delta_4, \delta_3 \rangle ?\mathit{int} \end{array}$$

Example #1

$\delta_2 < \delta_3$

$$(\nu ab)(\nu cd)\left(\begin{array}{c} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$$\begin{array}{ll} a : \langle \delta_1, \delta_2 \rangle ?int & d : \langle \delta_3, \delta_4 \rangle !int \\ b : \langle \delta_2, \delta_1 \rangle !int & c : \langle \delta_4, \delta_3 \rangle ?int \end{array}$$

Example #1

$\delta_2 < \delta_3$

$$(\nu ab)(\nu cd)\left(\begin{array}{c} a?(x).d!x \\ c?(y).b!y \end{array}\right)$$

$\delta_3 < \delta_2$

$$\begin{array}{ll} a : \langle \delta_1, \delta_2 \rangle ?\mathit{int} & d : \langle \delta_3, \delta_4 \rangle !\,\mathit{int} \\ b : \langle \delta_2, \delta_1 \rangle !\,\mathit{int} & c : \langle \delta_4, \delta_3 \rangle ?\mathit{int} \end{array}$$

Constraint C3: messages

$(\nu ab)a!b$

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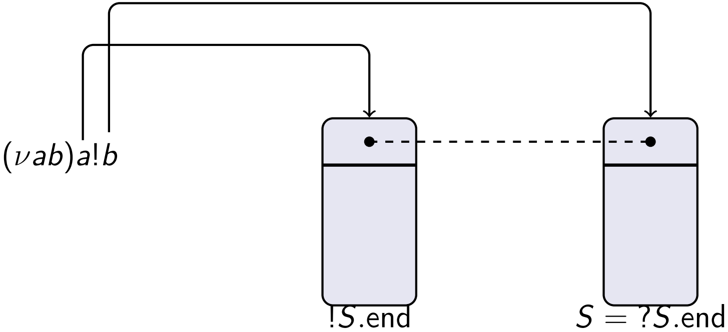

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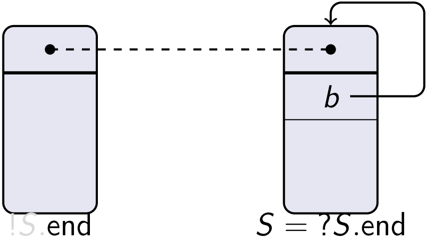

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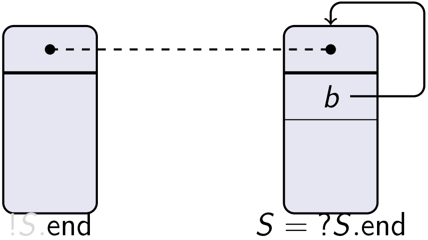


$\langle \delta_1, \delta_2 \rangle \, ! \, S$

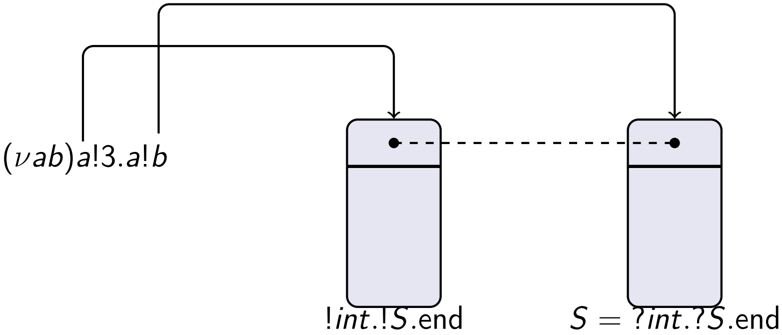
$\delta_2 < \mathrm{ob}(S)$

can't use a message
before it is delivered

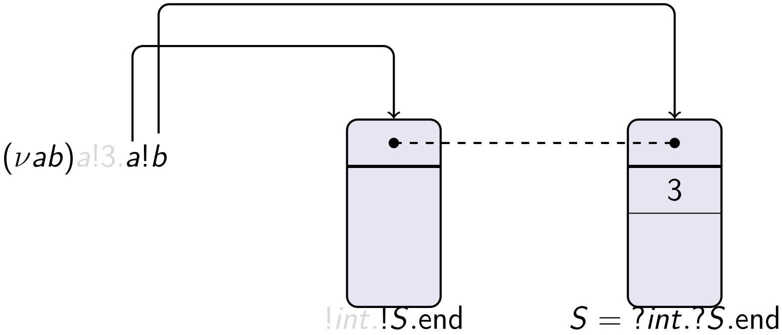
Constraint C4: asynchrony

$(\nu ab)a!3.a!b$

Constraint C4: asynchrony



Constraint C4: asynchrony



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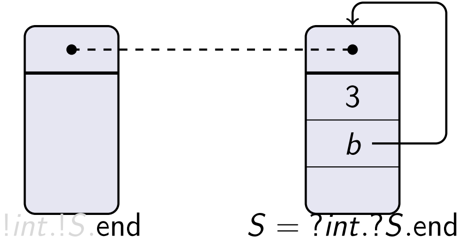

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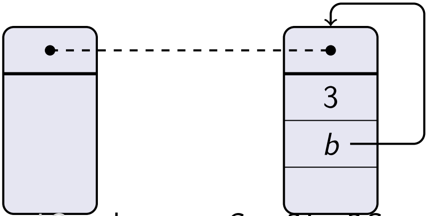

$!int.!S.\mathsf{end}$ $\quad S = ?int.?S.\mathsf{end}$

$\delta_3 < \delta_1$ by C3

$\langle \delta_2, \delta_1 \rangle \, ! \, int.\langle \delta_4, \delta_3 \rangle \, ! \, S$

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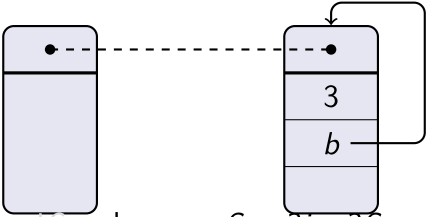

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$\delta_3 < \delta_1$ by C3

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$\delta_1 \leq \delta_3$

capabilities of consecutive outputs must be ordered

Example #2

$$\langle\overbrace{\delta_1, \delta_2}\rangle ?t.T \qquad \text{vs} \qquad \langle\delta\rangle ?t.T$$

$$\text{def } F(x) = x!3 \text{ in}$$
$$(\nu ab)(F\langle b\rangle \mid (\nu cd)(F\langle d\rangle \mid c?(x).a?(y)))$$

$$\left.\begin{array}{l} a, c : \langle\delta\rangle ?int \\ b, d : \langle\delta\rangle !int \end{array}\right\} \delta < \delta \qquad \left.\begin{array}{l} a, c : \langle\delta_1, \delta_2\rangle ?int \\ b, d : \langle\delta_2, \delta_1\rangle !int \end{array}\right\} \delta_2 < \delta_1$$

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$$\langle \overbrace{\delta_1, \delta_2} \rangle ?t.T \quad \text{vs} \quad \langle \delta \rangle ?t.T$$

same type

$$\text{def } F(x) = x!3 \text{ in}$$
$$(\nu ab)(F\langle b \rangle \mid (\nu cd)(F\langle d \rangle \mid c?(x).a?(y)))$$

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$$\left.\begin{array}{l} a, c : \langle \delta \rangle ?int \\ b, d : \langle \delta \rangle !\, int \end{array}\right\} \delta < \delta \qquad \left.\begin{array}{l} a, c : \langle \delta_1, \delta_2 \rangle ?int \\ b, d : \langle \delta_2, \delta_1 \rangle !\, int \end{array}\right\} \delta_2 < \delta_1$$

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$$\langle\overbrace{\delta_1, \delta_2}\rangle?t.T \qquad \text{vs} \qquad \langle\delta\rangle?t.T$$

def $F(x) = x!3$ in
$(\nu ab)(F\langle b\rangle \mid (\nu cd)(F\langle d\rangle \mid c?(x).a?(y)))$

$$\left.\begin{array}{l} a, c : \langle\delta\rangle?int \\ b, d : \langle\delta\rangle!int \end{array}\right\} \delta < \delta$$

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Wrap up

- type system for ensuring progress
- simpler than [2]: C1 on inputs + C2–4 on types
- finer than [2]: timestamping actions vs sessions
- simpler than [1]: duality vs reliability (and subtyping)

☛ C4 is new: asynchrony matters

[1] Kobayashi, **A Type System for Lock-Free Processes**, Inf. and Comp., 2002

[2] Bettini *et al.*, **Global Progress in Dynamically Interleaved Multiparty Sessions**, CONCUR 2008

Problem #1: simple processes are **ill typed**

$$(\nu ab)(\nu cd)(X\langle a, d\rangle \mid Y\langle b, c\rangle)$$

$$X(a, d) = a!3.d?(x).X\langle a, d\rangle \qquad a : \overline{T}, d : S$$

$$Y(b, c) = b?(x).c!x.Y\langle b, c\rangle \qquad b : T, c : \overline{S}$$

$$T = \langle \delta_1, \delta_2\rangle ?int.T$$

$$S = \langle \delta_3, \delta_4\rangle ?int.S$$

☛ more flexible type discipline is (likely) needed

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Problem #2: π processes $\neq$ **real programs**

$$\frac{\Gamma, u : T, x : S \vdash P \qquad \delta_2 < \mathsf{ob}(\Gamma(v))\ ^{v \in \mathsf{dom}(\Gamma)}}{\Gamma, u : \langle \delta_1, \delta_2 \rangle ?S.T \vdash u?(x).P}$$

☛ richer/more compositional types are needed

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What if this occurs inside a function?

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What if this occurs inside a function?

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What's next?

- attack problems #1 and #2 (BETTY WG1&3)
- multiparty sessions and shared channels (exercise)
- inference tool (Haskell implementation, coming soon)