Deadlock and lock freedom in the linear π-calculus

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(with contributions from Tzu-Chun Chen, Luca Novara, Andrea Tosatto)

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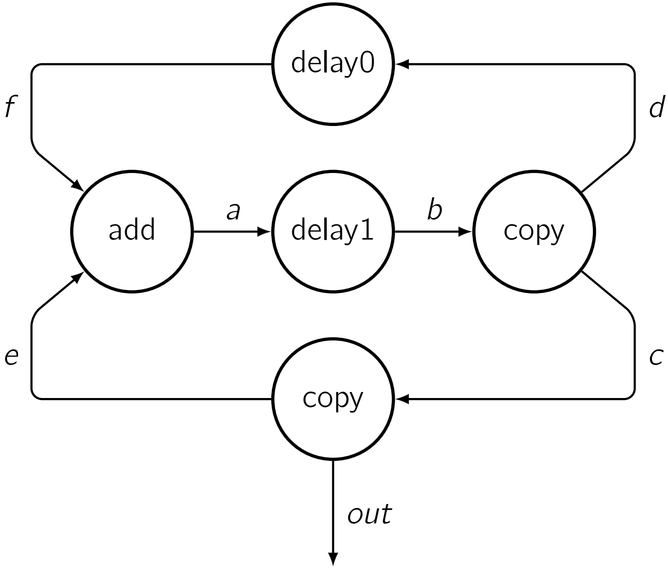
Demo

Demo



Stages

1. linearity analysis
   ⇒ partition channels into **linear** and **non-linear**

2. protocol analysis (optional)
   ⇒ infer **communication structure**

3. deadlock analysis
   ⇒ no pending communications in **stable** states

4. lock analysis
   ⇒ pending communications in **all** states can be completed

Why the focus on linear channels?

- Kahn process networks [Kahn '74]
- $\sim 50\%$ channels are linear [Kobayashi, Pierce, Turner '99]
- binary sessions [Kobayashi '07, Demangeon and Honda '11, Dardha et al. '12]
- multiparty sessions [Padovani '13, Pérez et al. '14]

Outline

1. Introduction
2. Linearity analysis
3. Protocol analysis
4. Deadlock analysis
5. Lock analysis
6. Final remarks

Linearity analysis: how it works

$$^{\kappa_1,\kappa_2}[t]$$

`new a in { a!3 | a?x }`

- $\alpha_0 = \alpha_1 + \alpha_2$
- $^{\rho,\rho}[\mathtt{int}] = {}^{0,1}[\mathtt{int}] + {}^{1,0}[\mathtt{int}]$
- $\rho = 0 + 1$
- $\rho = 1 + 0$
- $\alpha_0 = {}^{1,1}[\mathtt{int}]$

type combination $\neq$ unification

Linearity analysis: how it works

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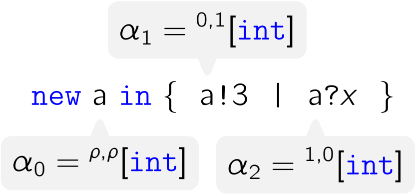

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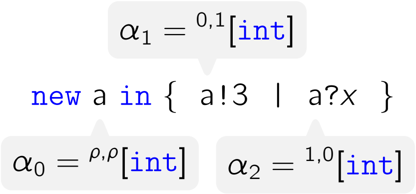

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type combination $\neq$ unification

Linearity analysis: how it works

$$ {}^{\kappa_1,\kappa_2}[t] $$

$\alpha_1 = {}^{0,1}[\texttt{int}]$

$\texttt{new a in \{ a!3 | a?}x\texttt{ \}}$

$\alpha_0 = {}^{\rho,\rho}[\texttt{int}]$ $\alpha_2 = {}^{1,0}[\texttt{int}]$

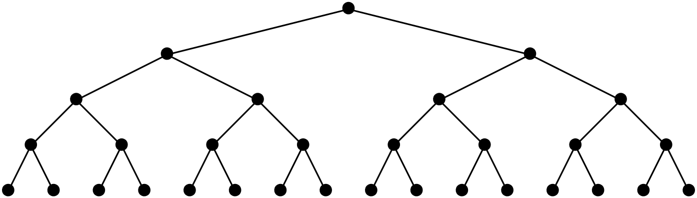
- $\alpha_0 = \alpha_1 + \alpha_2$
- ${}^{\rho,\rho}[\texttt{int}] = {}^{0,1}[\texttt{int}] + {}^{1,0}[\texttt{int}]$
- $\rho = 0 + 1$
- $\rho = 1 + 0$
- $\alpha_0 = {}^{1,1}[\texttt{int}]$

type combination $\neq$ unification

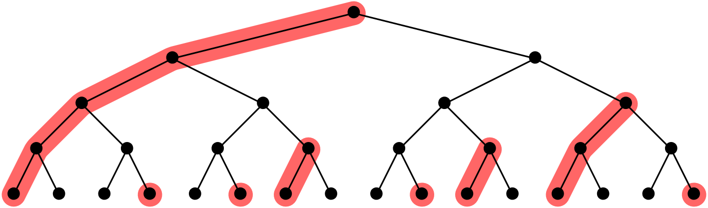
Demo: trees

```
  *case take? of
   { Leaf         ⇒ {}
   ; Node(c,l,r) ⇒ c!0 | take!l | skip!r }
|
  *case skip? of
   { Leaf         ⇒ {}
   ; Node(_,l,r) ⇒ skip!l | take!r }
|
  take!t | skip!t
```

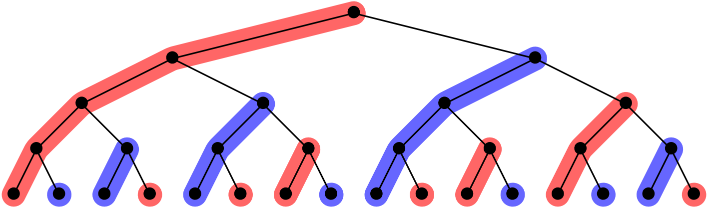
Channels used by `take` (red) and `skip` (blue)



Channels used by `take` (red) and `skip` (blue)



Channels used by `take` (red) and `skip` (blue)



Outline

1. Introduction
2. Linearity analysis
3. **Protocol analysis**
4. Deadlock analysis
5. Lock analysis
6. Final remarks

$s?(x).s!(x+1)$

binary
sessions

linear
π-calculus

session
types

linear
channel
types

`?int.!int...`

$s?(x).s!(x+1)$ $\qquad$ $s?(x,s').\texttt{new}\ s''\ \texttt{in}\ s'!(x+1,s'')$

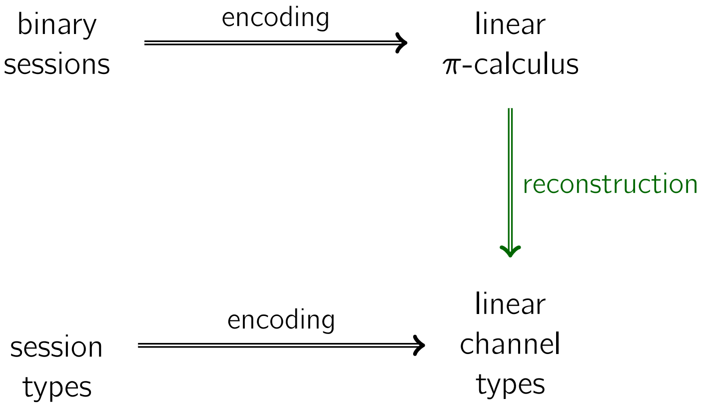
binary sessions $\xRightarrow{\text{encoding}}$ linear π-calculus

session types $\xRightarrow{\text{encoding}}$ linear channel types

$?\texttt{int}.!\texttt{int}\ldots$ $\qquad$ ${}^{1,0}[\texttt{int} \times {}^{0,1}[\texttt{int} \times \cdots]]$

$s?(x).s!(x+1)$

$s?(x,s').\texttt{new}\ s''\ \texttt{in}\ s'!(x+1,s'')$



binary sessions

encoding

linear π-calculus

reconstruction

session types

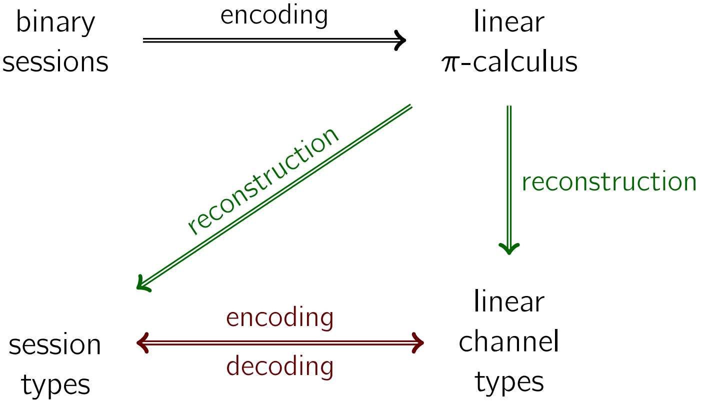
encoding

linear channel types

$?\texttt{int}.!\texttt{int}\ldots$

$^{1,0}[\texttt{int} \times {}^{0,1}[\texttt{int} \times \cdots]]$

$s?(x).s!(x+1)$ $\qquad$ $s?(x,s').\texttt{new}\ s''\ \texttt{in}\ s'!(x+1,s'')$



binary sessions

encoding

linear π-calculus

reconstruction

reconstruction

session types

encoding

decoding

linear channel types

$?\texttt{int}.!\texttt{int}\ldots$ $\qquad$ ${}^{1,0}[\texttt{int} \times {}^{0,1}[\texttt{int} \times \cdots]]$

Demo: math server

```
*server?s.
 case s? of
 { Quit    ⇒ {}
 ; Plus c1 ⇒ c1?(x,c2).
             c2?(y,c3).
             new c4 in { c3!(x + y, c4) | server!c4 }
 ; Eq c1   ⇒ c1?(x:Int,c2).
             c2?(y,c3).
             new c4 in { c3!(x = y, c4) | server!c4 }
 ; Neg c1  ⇒ c1?(x,c2).
             new c3 in { c2!(0 - x, c3) | server!c3 } }
```

Outline

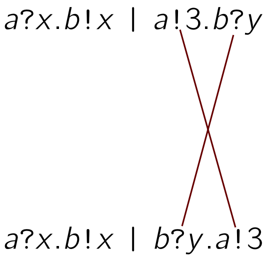
Different processes, same typing

$$a?x.b!x \mid a!3.b?y$$

Different processes, same typing

$$\mathtt{a} : {}^{1,1}[\mathtt{int}], \mathtt{b} : {}^{1,1}[\mathtt{int}] \vdash a\mathtt{?}x.b\mathtt{!}x \mid a\mathtt{!}3.b\mathtt{?}y$$

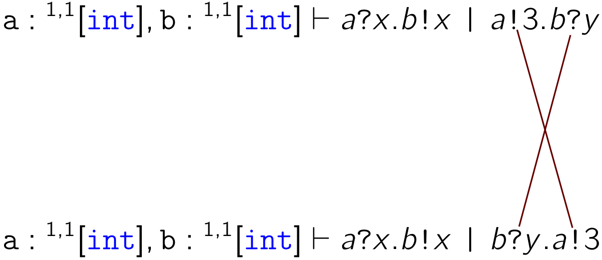
Different processes, same typing



$$\mathtt{a} : {}^{1,1}[\mathtt{int}], \mathtt{b} : {}^{1,1}[\mathtt{int}] \vdash a?x.b!x \mid a!3.b?y$$

$$a?x.b!x \mid b?y.a!3$$

Different processes, same typing



$$\mathtt{a} : {}^{1,1}[\mathtt{int}], \mathtt{b} : {}^{1,1}[\mathtt{int}] \vdash a\mathtt{?}x.b\mathtt{!}x \mid a\mathtt{!}3.b\mathtt{?}y$$

$$\mathtt{a} : {}^{1,1}[\mathtt{int}], \mathtt{b} : {}^{1,1}[\mathtt{int}] \vdash a\mathtt{?}x.b\mathtt{!}x \mid b\mathtt{?}y.a\mathtt{!}3$$

Basic strategy for deadlock analysis

❶ assign each linear channel a level $\in \mathbb{Z}$

$${}^{\kappa_1,\kappa_2}[t]^{h}$$

❷ make sure that channels are used in strict order

$$a\ \ ?x.b\ \ !x \mid b\ ?y.a\ \ !3$$

Basic strategy for deadlock analysis

1. assign each linear channel a level $\in \mathbb{Z}$

$$\kappa_1,\kappa_2[t]^h$$

2. make sure that channels are used in strict order

$$a^m?x.b^n!x \mid b^n?y.a^m!3$$

Basic strategy for deadlock analysis

❶ assign each linear channel a level $\in \mathbb{Z}$

$$^{\kappa_1,\kappa_2}[t]^{h}$$

❷ make sure that channels are used in strict order

$$\overbrace{a^{m}?x.b^{n}}^{<}!x \mid \overbrace{b^{n}?y.a^{m}}^{<}!3$$

A problem with recursive processes

```
*link?(x ,y ).
 x ?(z,a ).              -- x blocks y and a
 new b  in y !(z,b ).    -- y blocks a and b
 link!(a ,b )
```

A problem with recursive processes

```
*link?(x^0,y^1).
 x^0?(z,a ).                -- x blocks y and a
 new b  in y^1!(z,b ).      -- y blocks a and b
 link!(a ,b )
```

A problem with recursive processes

```
*link?(x⁰,y¹).
 x⁰?(z,a²).               -- x blocks y and a
 new b  in y¹!(z,b ).     -- y blocks a and b
 link!(a²,b )
```

A problem with recursive processes

```
*link?(x^0,y^1).
 x^0?(z,a^2).                -- x blocks y and a
 new b^3 in y^1!(z,b^3).     -- y blocks a and b
 link!(a^2,b^3)
```

A problem with recursive processes

```
*link?(x^0,y^1).
 x^0?(z,a^2).                   -- x blocks y and a
 new b^3 in y^1!(z,b^3).        -- y blocks a and b
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Problem

- the levels of `a` and `b` don't match those of `x` and `y`
- type error

A problem with recursive processes

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```

Problem

- ⊖ the levels of `a` and `b` don't match those of `x` and `y`
- ⊖ type error

Solution

- ⊕ the mismatch is OK as long as it is a translation
- ⊕ allow level polymorphism

Type reconstruction: how it works

```
*link?(x ,y ).
 x ?(z,a ).             --
 new b  in y !(z,b ).   --
 link!(a ,b )           --
```

1. perform linearity analysis
2. put integer variables in place of (unknown) levels
3. compute constraints
4. use ILP solver

Type reconstruction: how it works

```
*link?(x^n,y^m).
 x^n?(z,a^h).              --
 new b^k in y^m!(z,b^k).   --
 link!(a^h,b^k)            --
```

1. perform linearity analysis
2. put integer variables in place of (unknown) levels
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Type reconstruction: how it works

$\texttt{*link?(x}^{n}\texttt{,y}^{m}\texttt{).}$

$\quad\texttt{x}^{n}\texttt{?(z,a}^{h}\texttt{).}$ $\qquad\qquad$ -- $n < m \wedge n < h$

$\quad\texttt{new b}^{k}\texttt{ in y}^{m}\texttt{!(z,b}^{k}\texttt{).}$ $\qquad$ --

$\quad\texttt{link!(a}^{h}\texttt{,b}^{k}\texttt{)}$ $\qquad\qquad$ --

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Type reconstruction: how it works

$\texttt{*link?(x}^{n}\texttt{,y}^{m}\texttt{).}$

$\texttt{x}^{n}\texttt{?(z,a}^{h}\texttt{).}$ -- $n < m \wedge n < h$

$\texttt{new b}^{k}\texttt{ in y}^{m}\texttt{!(z,b}^{k}\texttt{).}$ -- $m < h \wedge m < k$

$\texttt{link!(a}^{h}\texttt{,b}^{k}\texttt{)}$ --

1. perform linearity analysis
2. put integer variables in place of (unknown) levels
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Type reconstruction: how it works

$$\begin{array}{ll}
\texttt{*link?(x}^{n}\texttt{,y}^{m}\texttt{).} & \\
\quad\texttt{x}^{n}\texttt{?(z,a}^{h}\texttt{).} & \texttt{--}\ n < m \wedge n < h \\
\quad\texttt{new b}^{k}\texttt{ in y}^{m}\texttt{!(z,b}^{k}\texttt{).} & \texttt{--}\ m < h \wedge m < k \\
\quad\texttt{link!(a}^{h}\texttt{,b}^{k}\texttt{)} & \texttt{--}\ h = n + t \wedge k = m + t
\end{array}$$

1. perform linearity analysis
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Type reconstruction: how it works

$\texttt{*link?(x}^{n}\texttt{,y}^{m}\texttt{).}$

$\texttt{ x}^{n}\texttt{?(z,a}^{h}\texttt{).}$ -- $n < m \land n < h$

$\texttt{ new b}^{k}\texttt{ in y}^{m}\texttt{!(z,b}^{k}\texttt{).}$ -- $m < h \land m < k$

$\texttt{ link!(a}^{h}\texttt{,b}^{k}\texttt{)}$ -- $h = n + t \land k = m + t$

1. perform linearity analysis
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Outline

Deadlocks vs locks

$$\texttt{new}\ \underset{{}^{1,1}[\texttt{int}]^{n}}{\mathsf{a}}\ \texttt{in}\ \{\ \overset{{}^{0,1}[\texttt{int}]^{n}}{\mathsf{a}}\,!\,3 \mid c\,!\,\underset{{}^{1,0}[\texttt{int}]^{n}}{\mathsf{a}} \mid *c?x.c\,!\,x\ \}$$

Strategy for lock analysis

❶ assign each linear channel a finite number $k \in \mathbb{N}$ of tickets

$$^{\kappa_1,\kappa_2}[t]^{h}_{k}$$

❷ each time a channel travels, one ticket is consumed

❸ channels with no tickets cannot travel

Lock analysis and type reconstruction

$\texttt{*link?(x}^{0}\texttt{,y}^{1}\texttt{).}$
$\quad\texttt{x}^{0}\texttt{?(z,a}^{2}\texttt{).}$
$\quad\texttt{new b}^{3}\texttt{ in y}^{1}\texttt{!(z,b}^{3}\texttt{).}$
$\quad\texttt{link!(a}^{2}\texttt{,b}^{3}\texttt{)}$

① perform linearity analysis

② put natural variables in place of (unknown) levels and tickets

③ compute constraints

④ use ILP solver

Lock analysis and type reconstruction

$$\begin{array}{l} *\texttt{link?}(\texttt{x}^{0}_{0},\texttt{y}^{1}_{0}). \\ \quad \texttt{x}^{0}_{0}\texttt{?}(\texttt{z},\texttt{a}^{2}). \\ \quad \texttt{new b}^{3}\ \texttt{in y}^{1}_{0}\texttt{!}(\texttt{z},\texttt{b}^{3}). \\ \quad \texttt{link!}(\texttt{a}^{2},\texttt{b}^{3}) \end{array}$$

1. perform linearity analysis
2. put natural variables in place of (unknown) levels and tickets
3. compute constraints
4. use ILP solver

Lock analysis and type reconstruction

$*\texttt{link?(x}^{0}_{0}\texttt{,y}^{1}_{0}\texttt{).}$

$\quad\texttt{x}^{0}_{0}\texttt{?(z,a}^{2}_{1}\texttt{).}$

$\quad\texttt{new b}^{3}\texttt{ in y}^{1}_{0}\texttt{!(z,b}^{3}\texttt{).}$

$\quad\texttt{link!(a}^{2}_{1}\texttt{,b}^{3}\texttt{)}$

1. perform linearity analysis
2. put natural variables in place of (unknown) levels and tickets
3. compute constraints
4. use ILP solver

Lock analysis and type reconstruction

$$\begin{array}{l}
*\mathtt{link?}(\mathtt{x}_0^0,\mathtt{y}_0^1). \\
\quad \mathtt{x}_0^0\mathtt{?}(\mathtt{z},\mathtt{a}_1^2). \\
\quad \mathtt{new}\ \mathtt{b}_2^3\ \mathtt{in}\ \mathtt{y}_0^1\mathtt{!}(\mathtt{z},\mathtt{b}_1^3). \\
\quad \mathtt{link!}(\mathtt{a}_1^2,\mathtt{b}_1^3)
\end{array}$$

1. perform linearity analysis
2. put natural variables in place of (unknown) levels and tickets
3. compute constraints
4. use ILP solver

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Related work

No levels

- sessions (without interleaving)
- session types as linear logic propositions

Discrete levels

- Kobayashi [2002, 2006, . . .]
- Bettini et al. [2008]
- Padovani, Vasconcelos, Vieira [2014]
- . . .

Dense levels

- Giachino, Kobayashi, Laneve [2014]

Summary of products

- 🗎 Padovani, **Deadlock and Lock Freedom in the Linear π-Calculus** (LICS'14)

- 🗎 Padovani, **Type Reconstruction for the Linear π-Calculus with Composite and Equi-Recursive Types** (FoSSaCS'14)

- 🗎 Padovani, Chen, Tosatto, **Type Reconstruction Algorithms for Deadlock-Free and Lock-Free Linear π-Calculi** (submitted)

- 🗎 Padovani and Novara, **Types and Effects for Deadlock-Free Higher-Order Programs** (submitted)

- 🗎 Padovani and Tosatto, **Hypha** (available at `http://di.unito.it/hypha`)