Conditional Nested Sequents $\mathcal{NS}$

(2012-2014)

$\Gamma(P, \neg P) \quad (AX)$

$\Gamma(\tau) \quad (AX_{\tau})$

$\Gamma(\bot) \quad (AX_{\bot})$

$\Gamma(A) \quad (\neg)$

$\Gamma(\neg \neg A) \quad (\neg)$

$\Gamma([A : A], [\neg A]) \quad \Gamma([A : A], [\neg A])$

$\Gamma(\neg (A \Rightarrow B), A) \quad \Gamma(\neg (A \Rightarrow B), \neg B)$

$\Gamma(\neg (A \Rightarrow B)) \quad (MP)$

$\Gamma, (C \Rightarrow D), [A : A] \Rightarrow \Gamma, (C \Rightarrow D), [A : C] \Rightarrow \Gamma, (C \Rightarrow D), [C : A] \Rightarrow \Gamma, (C \Rightarrow D), [A : A] \Rightarrow (CSO)$

**Clarifications:** Conditional logics extend classical logic with formulas of the form $A \Rightarrow B$: intuitively, $A \Rightarrow B$ is true in a world $x$ if $B$ is true in the set of worlds where $A$ is true and that are most similar to $x$. The calculi $\mathcal{NS}$ manipulate nested sequents, a generalization of ordinary sequent calculi where sequents are allowed to occur within sequents. A nested sequent $\Gamma = A_1, \ldots, A_m, [B_1 : \Gamma_1], \ldots, [B_n : \Gamma_n]$ is inductively defined by the formula $\mathcal{F}(\Gamma) = A_1 \lor \ldots \lor A_m \lor (B_1 \Rightarrow \mathcal{F}(\Gamma_1)) \lor \ldots \lor (B_n \Rightarrow \mathcal{F}(\Gamma_n))$. $\Gamma(d)$ represents a sequent $\Gamma$ containing a context (a unique empty position) filled by the (nested) sequent $A$. Besides the rules shown above, the calculi $\mathcal{NS}$ also include standard rules for propositional connectives.

**History:** The calculi $\mathcal{NS}$ have been introduced in [1] and extended in [2]. The theorem prover NESCOND, implementing $\mathcal{NS}$ in Prolog, has been presented in [3].

**Remarks:** Completeness is a consequence of cut admissibility. $\mathcal{NS}$ calculi can be used to obtain a PSPACE decision procedure for the respective conditional logics (optimal for CK and extensions with ID and MP).

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