Conditional Labelled Sequent Calculi SeqS

(2003-2007)

(AX) $\Gamma, x : P \vdash A, x : P$ (P atomic)

\[
\frac{\Gamma, x : A \Rightarrow B \vdash x \overset{\lambda}{\rightarrow} y, \Delta}{\Gamma, x : A \Rightarrow B \vdash A} \quad (\Rightarrow L)
\]

\[
\frac{\Gamma, x : A \Rightarrow B \vdash y, \Delta}{\Gamma, x : A \Rightarrow B \vdash y, \Delta} \quad (\Rightarrow R)
\]

\[
\frac{u : A \vdash u : B}{\Gamma, x \vdash y \vdash x \vdash y, \Delta} \quad (\text{EQ})
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\[
\frac{\Gamma, x \vdash y, \Delta}{\Gamma, x \vdash y, \Delta} \quad (\text{ID})
\]

\[
\frac{\Gamma, x \vdash A}{\Gamma, x \vdash x, \Delta} \quad (\text{MP})
\]

\[
\frac{\Gamma, x \vdash y, \Delta, x \vdash z}{\Gamma, x \vdash y \vdash A, \Delta} \quad (\text{CS}) (x \neq y, u \notin \Gamma, \Delta)
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\[
\frac{\Gamma, x \vdash A}{\Gamma, x \vdash y \vdash A, \Delta} \quad (\text{CEM}) (y \neq z, u \notin \Gamma, \Delta)
\]

Given a sequent $\Gamma$ and labels $x$ and $u$, $\Gamma[x/u]$ is the sequent obtained by replacing in $\Gamma$ all occurrences of $x$ with $u$.

Clarifications: Conditional logics extend classical logic with formulas of the form $A \Rightarrow B$. SeqS uses selection function semantics: $A \Rightarrow B$ is true in a world $w$ if $B$ is true in the set of worlds selected by the selection function $f$ for $A$ and $w$ (that are most similar to $w$). SeqS manipulates labelled formulas, where labels represent worlds, of the form $x : A$ ($A$ is true in $x$) and $x \overset{\lambda}{\rightarrow} y$ ($y$ belongs to $f(x, A)$). SeqS considers normal conditional logics, such that if $A$ and $B$ are true in the same worlds, then $f(w, A) = f(w, B)$. The rule (EQ) takes care of normality. Besides the rules shown, SeqS includes standard rules for propositional connectives.

History: The calculi SeqS have been introduced in [3]. The theorem prover CondLean, implementing SeqS calculi in Prolog, has been presented in [1, 2].

Remarks: Completeness is a consequence of the admissibility of cut. The calculi SeqS can be used to obtain a PSpace decision procedure for the respective conditional logics and to develop goal-directed proof procedures.


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