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Preferential vs Rational Description Logics: which one for Reasoning About Typicality?

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Abstract. Extensions of Description Logics (DLs) to reason about typicality and defeasible inheritance have been largely investigated. In this paper, we consider two such extensions, namely (i) the extension of DLs with a typicality operator $\mathbf{T}$, having the properties of Preferential nonmonotonic entailment **P**, and (ii) its variant with a typicality operator having the properties of the stronger Rational entailment **R**. The first one has been proposed in [6]. Here, we investigate the second one and we show, by a representation theorem, that it is equivalent to the approach to preferential subsumption proposed in [3]. We compare the two extensions, preferential and rational, and argue that the first one is more suitable than the second one to reason about typicality, as the latter leads to very unintuitive inferences.

1 Introduction

Description logics (DLs) represents one of the most important formalisms of knowledge representation. Their success can be explained by two key advantages. On the one hand, DLs have a well-defined semantics based on first-order logic; on the other hand, they offer a good trade-off between expressivity and complexity.

In a DL framework, a knowledge base (KB) contains two components: an intensional part, called the TBox, containing the definition of concepts (and possibly roles) as well as a specification of inclusion relations among them, and an extensional part, called the ABox, containing instances of concepts and roles. Since the very objective of the TBox is to build a taxonomy of concepts, the need of representing prototypical properties and of reasoning about defeasible inheritance of such properties naturally arises.

The traditional approach is to handle defeasible inheritance by integrating in DLs some kind of nonmonotonic reasoning mechanism. This has led to study nonmonotonic extensions of DLs [1, 2, 4]. However, as the authors themselves have pointed out, all these proposals present either semantical or computational difficulties (or both). Finding a suitable nonmonotonic extension for inheritance with exceptions is therefore far from obvious.

Here, we consider an alternative approach, based on nonmonotonic entailment as defined by Kraus, Lehmann and Magidor (KLM) in [7, 8]. This approach is adopted by [6] and [3]. The main advantage of this approach over previous ones is that the semantics of the resulting description logics is very simple and close to standard semantics for DLs. Furthermore, at least for what concerns [6] there is a calculus for the proposed logic, and the logic can be extended in order to deal with inheritance with exceptions.

We start by considering the logic $\mathcal{ALC} + \mathbf{T}$ proposed in [6], that extends the description logic $\mathcal{ALC}$ by a typicality operator $\mathbf{T}$. The intended meaning of $\mathbf{T}$, for any concept C, is that $\mathbf{T}(C)$ singles out the instances of C that are considered as "typical". Thus, an assertion like "typical writers are brillant" is represented by $\mathbf{T}(\mathit{Writer}) \sqsubseteq \mathit{Brillant}$. An $\mathcal{ALC} + \mathbf{T}$ TBox can consistently contain the above inclusion together with $\mathbf{T}(\mathit{Writer} \sqcap \mathit{Depressed}) \sqsubseteq \neg \mathit{Brillant}$ ("typical depressed writers are not brillant"). It is worth noticing that, if the same properties were expressed by ordinary inclusions, such as $\mathit{Writer} \sqsubseteq \mathit{Brillant}$, we would simply get that there are not depressed writers, thus the KB would collapse. This collapse is avoided in $\mathcal{ALC} + \mathbf{T}$, as it is not assumed that $\mathbf{T}$ is monotonic, that is to say $C \sqsubseteq D$ does not imply $\mathbf{T}(C) \sqsubseteq \mathbf{T}(D)$.

The semantics of the $\mathbf{T}$ operator is defined by a set of postulates that are essentially a restatement of axioms and rules of nonmonotonic entailment in preferential logic **P**, as defined by KLM. The semantics of $\mathbf{T}$ is given by means of a preference relation $<$ on individuals, so that typical instances of a concept C can be defined as the instances of C that are minimal with respect to $<$.

In this paper, we consider whether the properties of $\mathbf{T}$ in $\mathcal{ALC}+\mathbf{T}$ are the correct ones, by comparing them with the properties that would result if we adopt the stronger KLM rational logic **R** described in [8]. We provide some examples to show that **P** is better suited than **R** since **R** would force some inferences that we consider counterintuitive. Using **R**, for instance, we would be forced to conclude that typical writers are not brillant from the simple fact that there is a certain Mr. John who is a typical brillant person (he has, for instance, a lot of social success), who is a writer but who is not a typical writer (since he has never succeeded in publishing anything). We consider this as an unwanted inference, and therefore argue that the properties of **R** are too strong for $\mathbf{T}$, and that **P** must be preferred.

In Section 4 we provide a representation theorem to show that the extension of DLs by a typicality operator $\mathbf{T}$ having the properties of **R** is equivalent to the approach in [3]. The approach in [3] therefore inherits the above criticisms for extensions of DLs that use **R**.

2 The Logic $\mathcal{ALC} + \mathbf{T}$

In this section we briefly recall the description logic $\mathcal{ALC} + \mathbf{T}$ introduced in [6]. A knowledge base is a pair (TBox,ABox). TBox contains subsumptions of the form $C \sqsubseteq D$ and $\mathbf{T}(C) \sqsubseteq D$. ABox contains expressions of the form $C(a)$, $\mathbf{T}(C)(a)$, and aRb.

The semantics for $\mathbf{T}$ makes use of a preference relation among individuals that, roughly speaking, measures the relative typicality of individuals: if $a < b$, than a is more typical than b. Intuitively, typical members of a concept C are minimal elements of C wrt $<$.

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Definition 1 (Semantics of $\mathcal{ALC}+\mathbf{T}$) *A model of $\mathcal{ALC}+\mathbf{T}$ is any structure $\langle \Delta, I, < \rangle$, where: Δ is the domain; I is the extension function that maps each concept C to $C^I \subseteq \Delta$, and each role R to a $R^I \subseteq \Delta^I \times \Delta^I$. $<$ is a strict partial order satisfying the so called Smoothness Condition, i.e. for all $S \subseteq \Delta$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$. I is defined in the usual way (as for $\mathcal{ALC}$) and, in addition, $(\mathbf{T}(C))^I = Min_<(C^I) = \{x : x \in C^I$ and $\nexists y \in C^I$ s.t. $y < x\}$.*

A model $\mathcal{M} = \langle \Delta, I, < \rangle$ satisfies a KB if for all inclusions $C \sqsubseteq D$ in TBox, and all elements $x \in \Delta$, if $x \in C^I$ then $x \in D^I$. $\mathcal{M}$ satisfies ABox if: (i) for all $C(a)$ in ABox, $a^I \in C^I$, (ii) for all aRb in ABox, $(a^I, b^I) \in R^I$. Notice that this semantics is very similar to the one of Preferential logic **P** as defined by KLM [7].

The semantics for $\mathcal{ALC}+\mathbf{T}$ can be equivalently expressed by using a model $\langle \Delta, I, f_{\mathbf{T}} \rangle$ in which $f_{\mathbf{T}}(S)$ selects the *typical* instances of S, and in case $S = C^I$ for a concept C, it selects the typical instances of C. In this semantics, $(\mathbf{T}(C))^I = f_{\mathbf{T}}(C^I)$, and $f_{\mathbf{T}}$ has the following intuitive properties for all subsets S of Δ:

$(f_{\mathbf{T}}-1)$ $f_{\mathbf{T}}(S) \subseteq S$

$(f_{\mathbf{T}}-2)$ if $S \neq \emptyset$, then also $f_{\mathbf{T}}(S) \neq \emptyset$

$(f_{\mathbf{T}}-3)$ if $f_{\mathbf{T}}(S) \subseteq R$, then $f_{\mathbf{T}}(S) = f_{\mathbf{T}}(S \cap R)$

$(f_{\mathbf{T}}-4)$ $f_{\mathbf{T}}(\bigcup S_i) \subseteq \bigcup f_{\mathbf{T}}(S_i)$

$(f_{\mathbf{T}}-5)$ $\bigcap f_{\mathbf{T}}(S_i) \subseteq f_{\mathbf{T}}(\bigcup S_i)$

$(f_{\mathbf{T}}-1)$ enforces that typical elements of S belong to S. $(f_{\mathbf{T}}-2)$ enforces that if there are elements in S, then there are also *typical* such elements. $(f_{\mathbf{T}}-3)$ expresses a weak form of monotonicity, namely *cautious monotonicity*. The next properties constraint the behavior of $f_{\mathbf{T}}$ wrt $\cap$ and $\cup$ in such a way that they do not entail monotonicity.

3 Extension of $\mathcal{ALC}+\mathbf{T}$ with a modular preference relation: the logic $\mathcal{ALC}+\mathbf{T_R}$

If we add to the conditions above for $f_{\mathbf{T}}$ the following condition:

$(f_{\mathbf{T}}-\mathbf{R})$ if $f_{\mathbf{T}}(S) \cap R \neq \emptyset$, then $f_{\mathbf{T}}(S \cap R) \subseteq f_{\mathbf{T}}(S)$

of Rational Monotonicity, we obtain a stronger DL based on Rational Entailment [8]. $(f_{\mathbf{T}}-\mathbf{R})$ forces again a form of monotonicity: if there is a typical S having the property R, then all typical S and Rs inherit the properties of typical Ss. We call $\mathcal{ALC}+\mathbf{T_R}$ the logic obtained by adding $(f_{\mathbf{T}}-\mathbf{R})$ to the properties $(f_{\mathbf{T}}-1)-(f_{\mathbf{T}}-5)$. As for $\mathcal{ALC}+\mathbf{T}$, the semantics of $\mathcal{ALC}+\mathbf{T_R}$ can be formulated in terms of possible world structures $\langle \Delta, I, < \rangle$ in which $<$ is *modular*, i.e. for each x, y, z, if $x < y$, then either $z < y$ or $x < z$.

Definition 2 (Semantics of $\mathcal{ALC}+\mathbf{T_R}$) *A model $\mathcal{M}$ is any structure $\langle \Delta, I, < \rangle$, where Δ, I and $<$ are defined as in Definition 1, furthermore $<$ is modular.*

The equivalence between this semantics and the one formulated with $f_{\mathbf{T}}$ is proven by the following representation theorem:

Theorem 1 *A KB is satisfiable in a model described in Definition 2 iff it is satisfiable in a model $\langle \Delta, I, f_{\mathbf{T}} \rangle$ where $f_{\mathbf{T}}$ satisfies $(f_{\mathbf{T}}-1)-(f_{\mathbf{T}}-5)$ and $(f_{\mathbf{T}}-\mathbf{R})$, and $(\mathbf{T}(C))^I = f_{\mathbf{T}}(C^I)$.*

The following facts hold in $\mathcal{ALC}+\mathbf{T_R}$:

(**R**) $\neg(\mathbf{T}(A) \sqcap B \sqsubseteq \bot)$ implies $\mathbf{T}(A \sqcap B) \sqsubseteq \mathbf{T}(A)$

(*) $\neg(\mathbf{T}(A) \sqcap B \sqsubseteq \bot)$ implies $\mathbf{T}(B) \sqcap A \sqsubseteq \mathbf{T}(A)$

Both properties allow us to draw conclusions from the simple fact that there is *one* individual that (i) is a typical instance of the concept A and that (ii) has the property B. From (**R**), we derive that *all* typical A and Bs are typical As. From $(*)$ we derive something about typical Bs, even if A and B are unrelated properties. In particular, we derive that typical Bs that also have the property A are typical As. From (*) we derive the counterintuitive example of the Introduction, where from $(a)\mathbf{T}(Brillant)(john)$, $(b)\,Writer(john)$, $(c)\neg\mathbf{T}(Writer)(john)$ and an empty TBox, we can conclude that $(d)\mathbf{T}(Writers) \sqsubseteq \neg Brillant$. As a further example, given an ABox containing $(1)\mathbf{T}(Graduated)(andras)$, $(2)SoccerPlayer(andras)$, $(3)\mathbf{T}(SoccerPlayer)(lilian)$, $(4)\,Graduated(lilian)$, and an empty TBox, we can get that $(5)\mathbf{T}(SoccerPlayer)(andras)$, which does not make sense given that $lilian$ is a different person not related to $andras$, hence we do not want to use $lilian$'s properties to make inferences about $andras$.

In our opinion, the inferences that hold in $\mathcal{ALC}+\mathbf{T_R}$ are rather arbitrary and counterintuitive. In conclusion, we believe that the logic **R** is too strong and unsuitable to reason about typicality.

4 $\mathcal{ALC}+\mathbf{T_R}$ in the literature

$\mathcal{ALC}+\mathbf{T_R}$ is equivalent to the logic for defeasible subsumptions in DLs proposed by [3], when considered with $\mathcal{ALC}$ as the underlying DL. The idea underlying the approach by [3] is very similar to that underlying $\mathcal{ALC}+\mathbf{T}$ and $\mathcal{ALC}+\mathbf{T_R}$: some objects in the domain are more typical than others. In the approach by [3], x is more typical than y if $x \geq y$. The properties of $\geq$ in [3] correspond to those of $<$ in $\mathcal{ALC}+\mathbf{T_R}$. At a syntactic level the two logics differ, so that in [3] one finds the defeasible inclusions $C \precsim D$ instead of $\mathbf{T}(C) \sqsubseteq D$ of $\mathcal{ALC}+\mathbf{T_R}$. But the idea is the same: in the two cases the inclusion holds if the most preferred (typical) Cs are also Ds. Indeed, it can be shown that the logic of preferential subsumption can be translated into $\mathcal{ALC}+\mathbf{T_R}$ by replacing $C \precsim D$ with $\mathbf{T}(C) \sqsubseteq D$:

Theorem 2 *When the underlying DL is $\mathcal{ALC}$, a knowledge base is satisfiable by a preferential model of [3] iff its translation is satisfiable in an $\mathcal{ALC}+\mathbf{T_R}$ model.*

5 Conclusions

We have investigated the role of rational monotonicity in the context of nonmonotonic extensions of DLs. We have first compared two approaches based on the KLM semantics, namely: 1. $\mathcal{ALC}+\mathbf{T}$, and 2. $\mathcal{ALC}+\mathbf{T_R}$ (which is equivalent to the approach by [3]). We have provided some examples to show that the former is more appropriate than the latter when reasoning about typicality. Of course, both $\mathcal{ALC}+\mathbf{T}$ and $\mathcal{ALC}+\mathbf{T_R}$ are monotonic, so they must be completed by some kind of nonmonotonic mechanism. For $\mathcal{ALC}+\mathbf{T}$, some work has been done in [5].

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