Chapter VIII
Introducing Social Issues into a Minority Game by Using an Agent Based Model

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ABSTRACT

In this chapter, the authors perturb a Minority Game (MG) with some sociological issues, first by implementing a social network among the involved agents, through which they can somehow communicate their decision to a group of "friends," a local subset of those participating the game. Thus, the emergent aggregate behaviour will be very far from that of the original MG; the stress here is on the possibility of an agent changing his or her own decision, after getting the information from other n agents. Two different communication protocols among the agents will be examined: a synchronous one and the more realistic asynchronous one. Additionally, in some experiments a memory is introduced, acting as a selection mechanism. Last, some special agents (Opinion Leaders) whose influence over the others is higher than normal, are implemented in order to study how this can change the aggregate results.

INTRODUCTION

Game Theory (GT) is a distinct and interdisciplinary approach to the study of strategic behaviour. The disciplines most involved in game theory are mathematics, economics, and the other social and behavioural sciences. GT was founded by the great mathematician John von Neumann.

The key link between neoclassical economics and game theory is rationality. Neoclassical economics is based on the assumption that human beings are absolutely rational in their economic choices. The kind of rationality which is usually assumed in economics–perfect, logical, deductive rationality–is extremely useful in generating solutions to theoretical problems, but it fails to account for situations in which rationality is bounded (because agents can not cope with the complexity of the situation) or when ignorance about other agents' ability and willingness to apply perfect rationally leads to subjective beliefs about the situation. Even in those situations, agents are not completely irrational: They adjust their behaviour based on what they think other agents are going to do, and these expectations are generated endogenously by information about what other agents have done in the past. On the basis of these expectations, the agent takes an action, which in turn becomes a precedent that influences the behaviour of future agents. This creates a feedback loop: Expectations arise from precedents and then create the actions which, in turn, constitute the precedents for the next step.

GT was intended to confront just this problem: to provide a theory of economic and strategic behaviour when people interact directly, rather than through the market. In game theory, "games" have always been a metaphor for more serious interactions in human society.

The Minority Game (MG) is a simple, generalized framework, belonging to the GT field, which represents the collective behaviour of agents in an idealized situation where they have to compete through adaptation for some finite resource.

While the MG was born as the mathematical formulation of the "El Farol Bar" problem considered by Arthur (1994), it goes beyond this one, since it generalizes the study of how many individuals may reach a collective solution to a problem under adaptation of each one's expectations about the future. In Arthur (1994), the "El Farol Bar" problem was posed as an example of inductive reasoning in scenarios of bounded rationality.

The original formulation of this problem is as follows: N people, at every step, take an individual decision among two possibilities. Number one is to stay at home, number two is to go to a bar. Since the space in the bar is limited (finite resource), the time there is enjoyable if and only if the number of the people there is less than a fixed threshold (aN, where $a<1$). Every agent has his or her own expectation of the number of people in the bar, and according to their forecast decides whether to go or not. The only information available to the agents is the number of people attending the bar in the recent past; this means that there is no deductively rational solution to this problem, but there can be plenty of models trying to infer the future number according to the past ones.

An interesting aspect of the problem is that if most agents think that the number of people going to the bar is $> aN$, then they won't go, thus invalidating their own prevision. Computer simulations of this model shows that the attendance fluctuates around aN in a $(aN,(1-a)N)$ structure of people attending/not attending. The bar problem has been applied to some proto-market models: At each time step agents can buy (go to the bar) or sell an asset and after each time step, the price of the asset is determined by a simple supply-demand rule.

The MG has been first described in Challet and Zhang (1997) as a mathematical formalization and generalization of the bar problem. It is assumed that an odd number of players take a decision at each step of the simulation; the agents that take the minority decision win, while the others loose. Stepping back to the bar problem, we can see it as a MG with two possible actions: $a1 = 1$ (to go to the bar) and $a2 = -1$ (not to go to the bar). After each round, the cumulative action value $A(t)$ is calculated as the sum of each value given to the single actions. The minority rule sets the comfort level at $A(t) = 0$, so that agent is given a payoff $-ai(t)g[A(t)]$ at each time step with g being an odd function of $A(t)$.

The MG has been chosen in this work since it's a model that could be used as a metaphor in many fields–it's intrinsically interdisciplinary (see Cappellini and Lamieri,2007 for an economic application on the dynamism of industrial sectors)and its structure is well known and quite straightforward enough to be described through an agent-based model. The perturbations to the original model described in this chapter, cover different sides. In order to extend the MG it would be possible to change:

- Topology (from regular mono/bi-dimensional to a social network).
- Cognitively the rules of reward and learning (evolutionary MG).
- How the information and feedbacks are spread and received (local information).
- The regime of interactions (global or local minorities).

By relaxing the original hypotheses, a model is obtained with local interactions and information, approaching somehow certain physical models more oriented to the Social Sciences, e.g., the Ising model and, in particular, the Sznajd model.

The purpose of the Ising model is that of imitating a phenomenon where individual elements modify their individual behaviour in order to conform to the behaviour of other individuals nearby. This happens for many phenomena in nature, e.g., cardiac thin filament activation with nearest-neighbour cooperative interactions (Rice, et al., 2003). The Sznajd Model (see Sznajd-Weron & Sznajd, 2000; Sznajd-Weron, 2005; Stauffer, et al., 2000; Stauffer, 2001 for numerical implementation) is a socio-physics model of opinion formation, which is based on the Ising model. As an example, each spin can simulate a voter. Their opinions vary according to a trade union maxim: "United we Stand, Divided we Fall."

The main differences among those models and the one presented here are that the Sznajd model uses majority rule (here minority rule is applied) and, above all, that the model presented here uses a random topology, while the Sznajd model uses a regular one.

INTRODUCING COMMUNICATION: SOCIAL NETWORKS AND GRAPHS

The bar problem, as well as the MG in its original formulation, state that there is no communication among the agents involved in the simulation; the first idea presented here is to introduce a sort of a social network into the model, in order to see how the links among certain agents can change the aggregate results. A social network is defined as "a set of nodes—e.g. persons, organizations—linked by a set of social relationship—e.g. friendship, transfer of funds, overlapping membership—of a specific type" (Laumann, et al., 1978).

Here, the minority rule will be very easy: A set of N agents chooses between (-1) and (1). Those in the minority (denoted with $n < N$) win and get a payoff equal to N/n: the fewer agents that stay in the minority, the higher the payoff. Also, the social network involved will be quite simple, just linking an agent to others with a relation limited to the possibility of asking a question: "Will you choose (–1) or (1)?" Not all the agents will be connected, though, so that some of them will have to make a prevision just considering the past few results, exactly like in the original MG.

In the example shown in Table 1, there are five agents involved in the simulation: Agent 1 can ask agents 2, 3, and 4, while agent 2 can ask agent 3, and number 3 can ask numbers 1 and 5; agent 4 can then ask numbers 1 and 5, while number 5 is a lonely agent (he or she can't ask anyone, even if two other agents can ask what he or she will do).

Any kind of network can be described in terms of a graph, composed of nodes and a set of lines, with edges joining the nodes. In a mathematician's terminology, a graph is a collection of points and lines connecting some (possibly empty) subset of

Table 1. Definition of relations among agents

	1	2	3	4	5
1		x	x	x	
2			x		
3	x			x	
4	x				x
5					

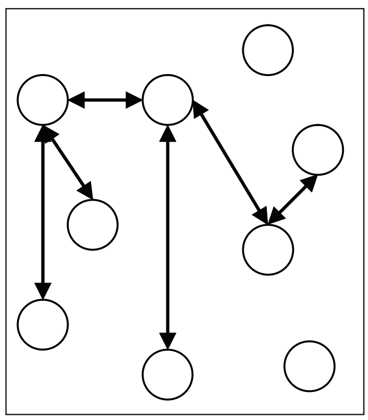
them. The points of a graph are most commonly known as graph vertices, but may also be called nodes or simply points. Similarly, the lines connecting the vertices of a graph are most commonly known as graph edges, but may also be called arcs or lines. The study of graphs is known as graph theory, and was first systematically investigated by König in the 1930s (Gardner, 1984). Graphs come in a wide variety, withthe most common type beingthose with at most one edge (i.e., either one edge or no edges) that may connect any two vertices. Such graphs are called simple graphs and are the ones used in the present analysis. The edges of graphs may also be imbued with directedness. A normal graph in which edges are undirected is said to be undirected. Otherwise, if arrows may be placed on one or both endpoints of the edges of a graph to indicate directedness, the graph is said to be directed.

In this work, the graph used to represent the social network linking the agents together is bi-directed, i.e., each edge points in both directions as once. This seems realistic, since this network can be thought of as a group of friends, or, in general, people who know each other. If A knows B, then it's quite obvious that B knows A in turn; those situations in which a subject disseminates his or her opinion to others and isn't touched by their decisions (e.g., advertisement, political campaigns, and so forth) is voluntarily not considered. That's because we suppose that this sort of dissemination comes "a priori," i.e., before this analysis starts; the interest here is in studying how a set of agents mutually connected into a network can influence one another and come to a final overall result. In Figure 1, an example of a network used in the model is shown. It's possible to notice that some nodes (agents) can be left totally unlinked, thus having to take their decision just basing on their own forecasts.

THE SIMULATION FRAMEWORK

A community of reactive agents that must take a decision is used; the decision could be simply binary (e.g., to sell or to buy in a stock market, to go or not to go to a pub, and so forth) or more complex (e.g., choosing whom to vote for at the next election, choosing the colour for a car, and so on). While the mechanism behind the constitution of an opinion in human beings is beyond the purpose of this work, we'll analyze how a social network interconnecting a community of agents can influence their choices and, in particular, how it could determine changes of their own opinions. That's why simple, reactive agents have been used: No plans are required to carry out the initial decision that could even be randomly generated, and the only action they have to perform is to evaluate

Figure 1. Agents communicating over a network



the opinions of their "friends," who are the other agents linked with them, and choose whether or not to be influenced by them.

Among the many toolkits and frameworks that can be used to build agent based simulations, JAS (http://jaslibrary.sourceforge.net) was selected for this work, since it includes graphical support for Social Network Analysis.

At the beginning of the simulation during the setup, a simple world is created, populated by N agents. These agents can be considered as the vertexes of a social network and the links among them (relations) as the edges. The network is directed and every arc is composed of two edges with opposite directions. Every agent has a list of F (friends) other agents, whom she/he can ask. This list is composed of the neighbours, i.e., the vertexes linked to the examined vertex (the agent). The nodes are randomly generated, and the links are created one by one.

The neighbourhood is intended as having sociological, as well as physical closeness. According to Laumann, et al. (1978), the relations between the involved agents are considered as friendship. This social relationship is characterized by a random creation but is also very stable in short/medium term. The links in this simulation are directed and bi-directional, as is the friendship.

Here follows a brief description of the simulation process:

- At the beginning of each simulation step, every agent has its own forecast. The forecast is absolutely random between two choices –1 and +1.
- The decision taken by each agent (before communicating with others) is denoted with a "certainty index" equal to 1 (100%).
- Now an agent is randomly chosen. She/He starts asking the first in the list; if this one has the same prevision, then the certainty index is increased by a value of *1/F*, while if the prevision is different, than the certainty index is lowered by *1/F*.
- After having asked a statement to all the friends in her/his list, the agent takes the final decision: If the certainty index is equal to or greater than one, then the decision will be the original one. If it's lower than 1, then the decision will be the other possible one
- Another agent is then randomly chosen, and so on (the same agent can't be chosen twice during the same turn). Note that an agent that's been asked can still change his or her mind, based on the agents he or she will ask in turn

Before starting the simulation, two core parameters can be changed: the number of agents involved and the number of links among the agents. Three runs of the simulation are examined: one with 1,000 agents and 500 total links (an average of one link every two agents), another with 100 agents and 500 links (an average of five links for every agent), and the last one with 100 agents and 5,000 links (fifty links for every agent). In every run the MG is iterated 1,000 times.

ASYNCHRONOUS COMMUNICATION

Two communication protocols are implemented in the model. In the asynchronous protocol, agents act sequentially. So the first agents which act take their decision, and from then on they reply to the other agents with the new decision taken. The synchronous protocol states that the agents always communicate their original opinion to the others: They broadcast their opinion to all the agents who are linked to them. Finally, after having collected all the opinions of their friends, they reconsider their choice. The difference among the two protocols is studied by using the same starting parameters in the simulation (*ceteris paribus*).

The asynchronous case is examined first, where the agents act sequentially. So the first agents to act take a decision, and from then on they reply to the other agents with the new decision taken. As an example, we can think of a group of people using the phone two by two to communicate; in this way the last agent to speak already has complete information about the definitive choices of all the others to whom she spoken before.

In the output graph time can be read on the x-axis (1,000 iterations of the game), and two lines are plotted: The red one (the lower one in the graphs) depicts the decisions that changed while the blue one (the upper one) is for unchanged decisions. On the y-axis the number of decisions is shown (changed or not); the scale (10^1, 10^2, and 10^3) depends on the agents number.

The standard example is a world of 100 agents and 500 relations (Figure 2), in which an average of 65 out of 100 preserve their original decisions.

In a second run a different situation is depicted, in which the agents have many more relations among themselves: An average of fifty for every inhabitant (Figure 3).

A simple, common sense rule states that the more relations, the higher the probability to change opinion. This example proves the rule to be right and the presented model to be consistent with

Figure 2. 100 agents and 500 relations

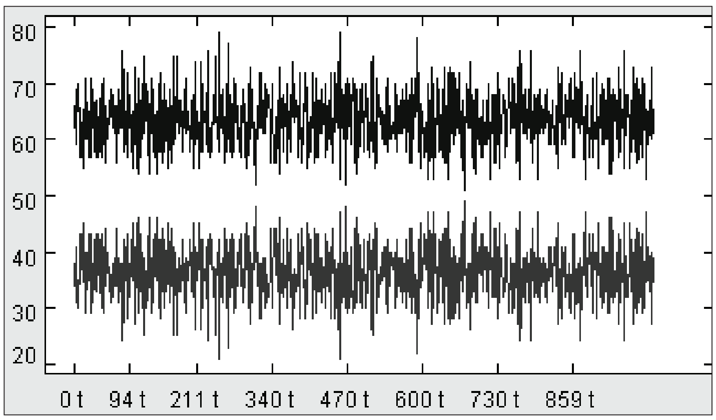
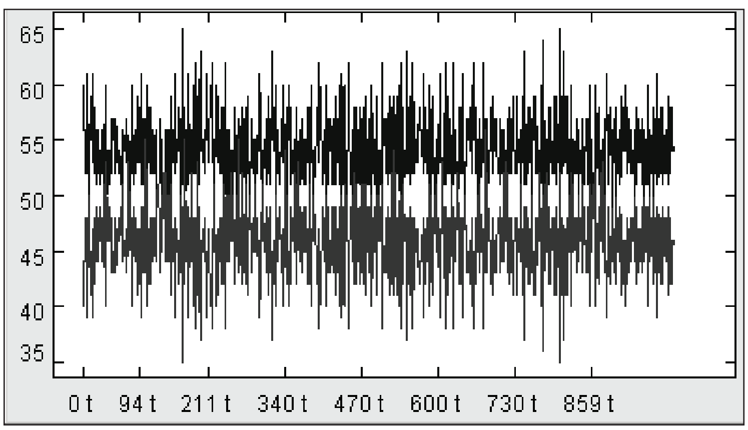


Figure 3. 100 agents and 5,000 relations



real world results; a counter example can now be given, i.e., a poor relations world, such as the one in Figure 4 with one thousand inhabitants and a total of just five hundred relations.

Here, less than 20 percent of the agents changed their opinion. In order to test the extreme situation, a world with no relations among the agents has been imagined (like in the original MG).

Obviously, in a world with one thousand unlinked agents no opinions change (Figure 5).

SYNCHRONOUS COMMUNICATION

The synchronous communication process is now explored, which can be compared to a situation in which a group of friends are physically in the main square of a small village, deciding what to do in the evening. They are speaking loud, all together, and so communication is "instantaneously" broadcasted and decisions are taken at the same time.

Figure 4. 1,000 agents and 500 relations

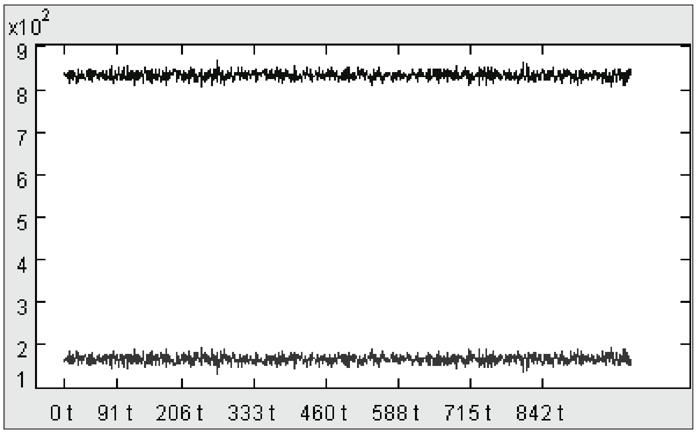
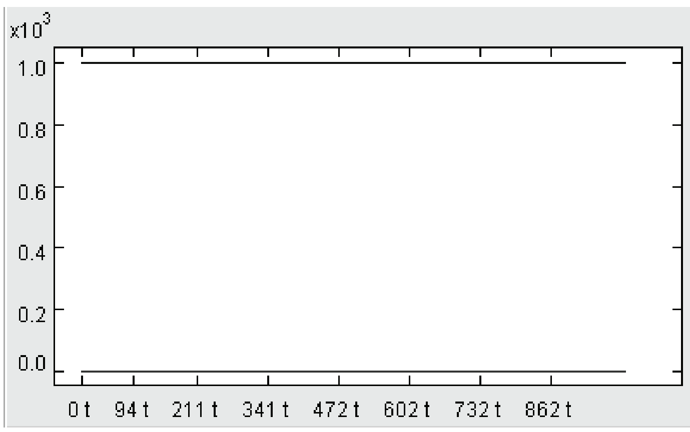


Figure 5.1,000 Agents and Zero Relations



Now the agents always communicate to the others their original opinion: they broadcast their opinion to all the agents which are linked to them. Finally, after they collect all the opinions of their friends, they evaluate the certainty index and reconsider their choice.

The simulation was executed with the new rule but with all the other parameters same as before (*ceteris paribus*).

In the first example (Figure 6) there are ten percent more opinion changes compared to the sequential model.

The best result is in the second run (Figure 7): the world rich of relations. The two lines overlap (even if there is a high variance in data).

A second simple rule coming from this analysis can be expressed: A synchronous communication among the agents increases their attitude to change opinion, which is at least ten percent higher.

The proof is the third run, in which again there is a higher result when compared to the asynchronous case.

MEMORY AND REWARDING

In this section the stress is on how the introduction of a simple kind of memory, based on past turns, can change the previous results This is one of the most simple strategies implemented in

Figure 6. 100 agents and 500 relations

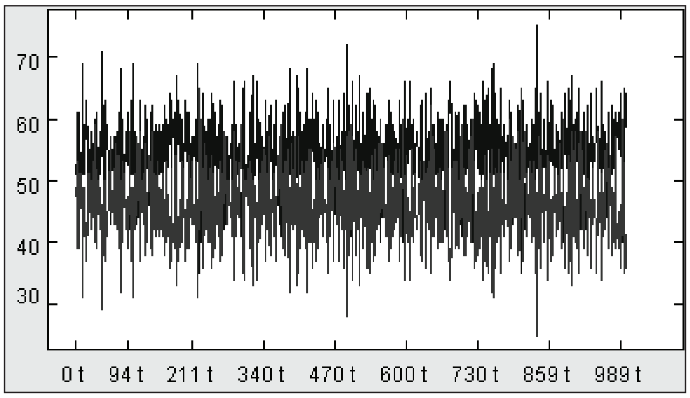
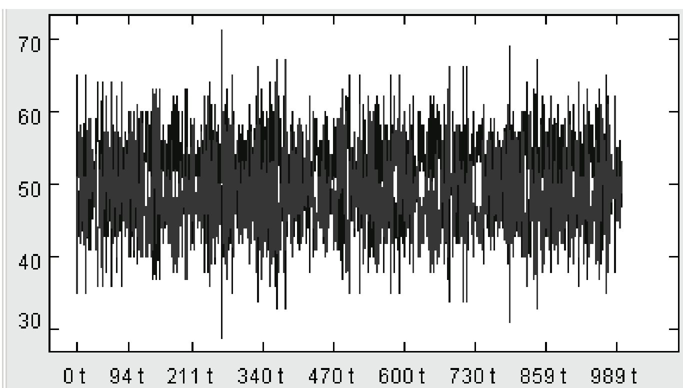


Figure 7. 100 agents and 5,000 relations



the original MG, though the aim is not the final result, be it win or lose, but the way the agents behave, i.e., change their original opinion, when their "mind" changes somehow.

Here a payoff system to reward the players in the minority is introduced. The memory is a list of length *N* (technically the same length can be used for all the agents or randomized, by using a range from 1 to 20). In each "box," the last cumulative choice of the group to which the agent belongs is added. The value is normalized and is +1 if the sum of choices is higher than zero, or -1 if the sum is less than zero. The agent uses her/his memory by reading the list, and summing the last group choices. The agent choice will be +1 if the sum is lower than 0, meaning that the mode of the group

Figure 8. 1,000 agents and 500 relations

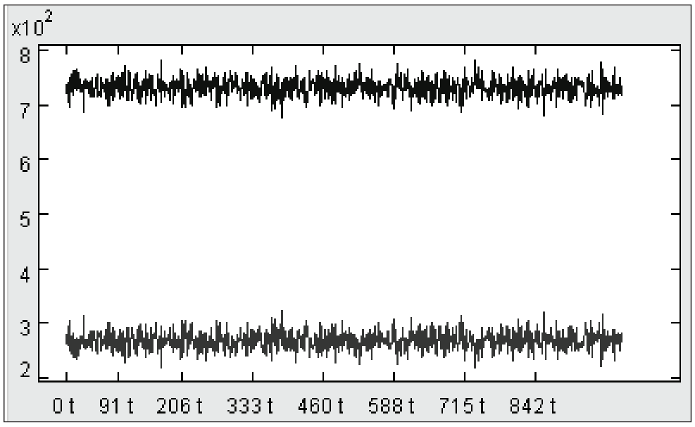
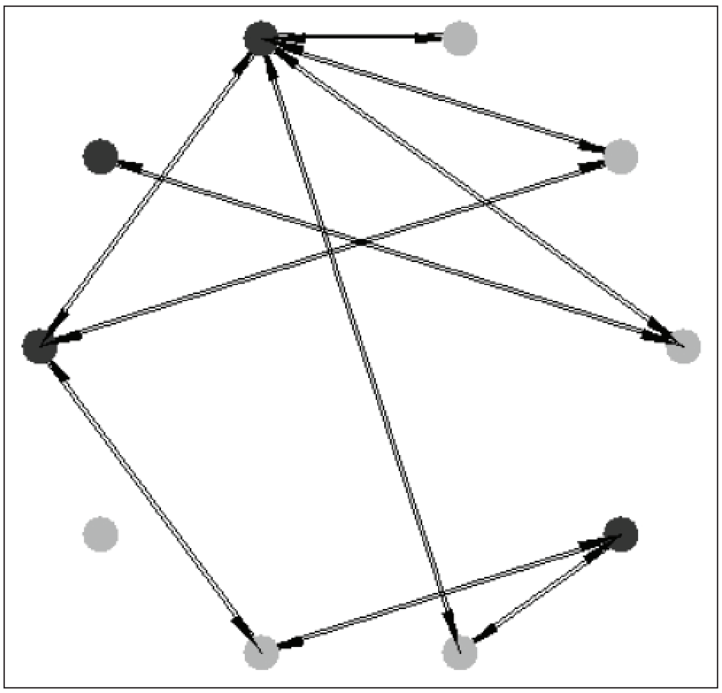


Figure 10. 10 agents and 10 relations topology



is –1 and –1 in the opposite situation, or it can be random if there is no prevailing result.

A network graph is also introduced, in which the topology can be observed as the agents change their colours, red for “+1” and green for “-1”.The relations (links) among the agents are bi-directional ones, and represented by the black arrows connecting the nodes. This means that if A can ask B, then B can, in turn, ask A. An example of this can be observed in Figure 9.

This figure depicts an interesting experiment composed of 10 agents and 10 relations, using memory and sequential communication.

Looking at the graph, we can observe that every group is in equilibrium. In fact, according to bounded rationality, each agent knows only the information about his own neighbours. Observing each agent’s point of view, there are triplets Green-Red-Green or Red-Green-Red in perfect equilibrium, in which every agent respects the minority rule. The agents reach an elevated global optimum (Figure 11) of eight out of ten.

The stability of the system is strengthened by the steady distribution observed in Figure 11. In fact, the node that changes opinion is usually the isolated one.

The rewarding system counts one point for every agent that chooses a (local) minority option.

Figure 11. 10 agents and 10 relations, rewards

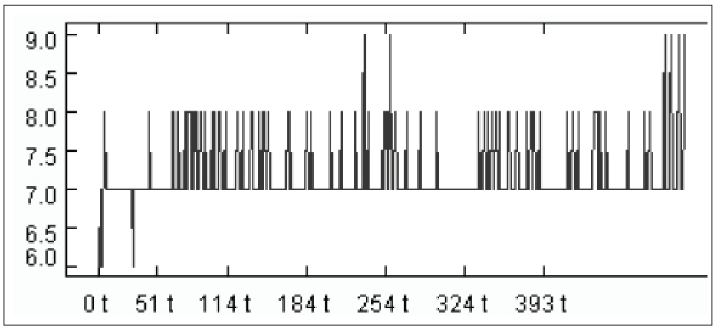
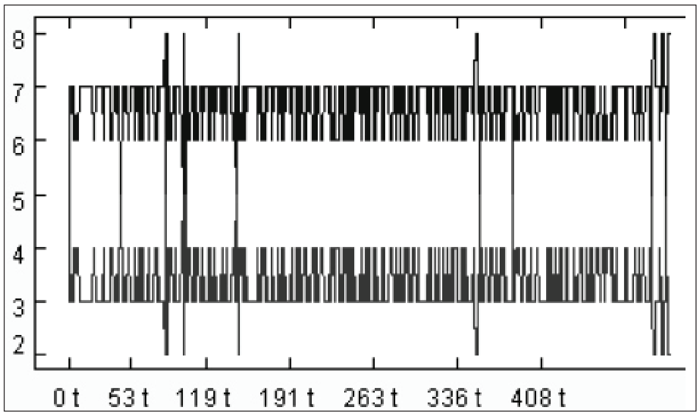


Figure 12. 10 agents and 10 relations, changed choices



OPINION LEADERS—DECISIONAL POWER

In the attempt to create a more realistic situation, some special agents are introduced in the simulation. Normal agents change their mind according to a simple percentage rule, by computing how many of their neighbours have the same opinion. They then can influence in turn the others with the same system. Opinion Leaders (OLs) have a stronger influence and are less likely to change their own mind.

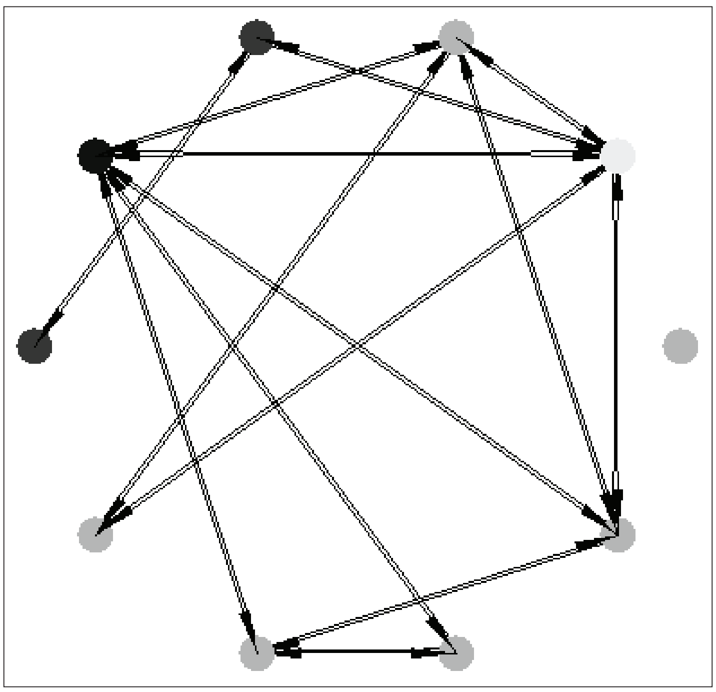
An OL is a person who is considered a credible source of information for others on a specific topic and who is sought out for that information. OLs are influential because they have certain characteristics which make them attractive to others. Whatever the reasons, OLs play a very important role in the community because their behaviours and values are emulated by others and they may be viewed as representing his/her community in many fields.

The key characteristic of an OL is that he or she is trusted to evaluate new information in the context of (local) group norms. The influence of each opinion leader might be limited to her/his own social network, or it may extend across many networks. The other main features of an OL are that she/he must be:

- Sensitive to local environment and group norms
- Approachable and have good listening skills
- Perceived as clinically competent and caring
- Perceived as excellent evaluators

OLs are not necessarily are in official positions, early adopters or even innovators in their choices. That's why usually it's necessary to use *sociogram* techniques and surveys to identify them. It's therefore evident that OLs play a very

Figure 4. The network layer: 10 agents and 14 relations



important role in the formation of trends and decisions in a society.

In the simulation, some agents with the role of OL are implemented in order to see how they can change the aggregate trend in a MG with communication.

In the model, the OL is a minority agent with different thresholds. The basic idea is that OLs influence more people, and besides they must be quite sure about their decision. We can imagine she/he as an advocate or a supporter of a certain social or political cause. During agent creation, the two with the maximum number of links are selected. Their opinions are forced to be opposite, so that they become advocates for the two opposite causes in the game (binary choice). The weight related to OL ideas is also different; it's twice or ten times the original value.

An OL can change her/his own opinion, but usually this should be less frequent than for "normal" agents. At every step, she/he can randomly change opinion with a probability of 5 percent or of 87.5 percent (certainty index > 1,75) or more if his/her "friends" have taken his/her same decision (this is a higher value when compared to the 50% + 1 of the normal agents, certainty index > 1).

RESULTS WITH OL

An example of results with OL is represented in a network layer of ten vertexes and fourteen edges, as shown in Figure 4. The minority agents are represented by red and green, as described before, in blue as the OL that chose –1, and the greens and in yellow the other one.

Figure 5. 10 agents and 14 relations: agents' choices

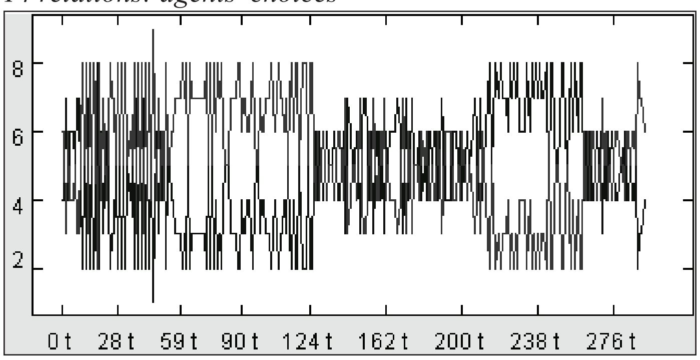
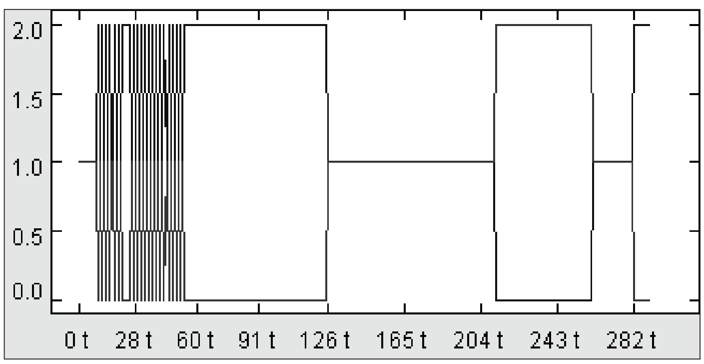


Figure 6. 10 agents and 14 relations: Ols' choices



The OL opinion has a weight double that of the others. It's extremely interesting to observe the graphs of total opinions (Figure 5) and of OLs' opinions (Figure 6).

Both the minority rule (except for the very high boundary) and a very rare random event are preserved. The initial paradigm is then conserved, while giving more realism to the model, e.g., in politics a common situation can be imagined with two main parties, of which OLs are supporters with different ideas. But in a dictatorship or in a period of "revolution," during which political positions aren't emerging, you can have that all the OLs support the same ideas, or that both continually change their opinions.

The correlation of the two series is 90.6%. So, in a small community, a person with many relations can change the aggregate "mood." This correlation could be an effective measure of OLs' influence on the population.

There are three main stylized structures/behaviours:From period 10 to 50 (the revolution/anarchy) you have a highly coordinated situation in which both OLs and almost 80 percent of the agents (eight out of ten) adopt the same idea. That implies an opinion switch the next turn.

On the other hand, from period 126 to 210 (the plain democracy), the OLs upheld different opinions, and in this case other agents are split between the two ideas. Finally, from period 210 to 260 and from 50 to 126 (the dictatorship), you can observe the polarization of the agents at opposite extremes of the OLs' common ideas. In a network with more vertexes, the Ols' influence decreases quickly.

LOCAL MINORITIES

Kauffman (1969), first described a disordered dynamical system that consists of N Boolean variables or spins in stable relation to each other (Kauffman Networks), used by gene regulatory systems (but also for spin glasses, evolution, social sciences, economics, and finance). Each gene changes its status (active or not) depending on some signals. Paczuski, et al. (2000) used that structure introducing a MG with personal limited information resources, but with a global reward mechanism.

Kalinowski, et al. (2000) describe a model in which agents who are placed in a circle are able to cooperate due to self-organization.

The term "Local" was introduced by Moelbert and Los Rios (2002). They depicted a one-dimensional, or square, lattice with communities of three or five individuals, each one interacting with two (four) nearest neighbours.

In Chau, et al. (2004), a new model was introduced, called the Networked MG (NMG). It is a modified MG model in which all players can make use of not only global information but also local information from their neighbours that are disseminated through a network. The local information of a player is based on the choices of this player and his/her nearest neighbours on the ring.

In the model presented here, a more complex topology is used (not a simple ring); besides, the reward mechanism is not the same for all the agents involved; metaphorically this could be thought as *n* different local MGs.

All the previous works are based on a bounded communication and they are generally closer to a type of Small Worlds scenario. They show that space correlation becomes important. This local communication is implemented among the agents, but also introduces another level of information: Every agent issues a statement before acting and the decision is subsequently based on that. The possibility to lie in the declaration is not considered in this case.

Johnson, et al. (1999), while describing an evolutionary version of the MG (EMG), found that the introduction of partial information instead of global and diffuse news forced agents to take a decision based on inductive—rather than deductive—thinking. The result is a self-segregation of individuals.

Kirley (2004) extended this research in order to introduce small world connections in it. This spatial approach, and a small degree of disorder, lead to an improvement of system efficiency: The agents can more effectively coordinate their behaviour.

Local Evolutionary minority games (LEMG, Burgos, et al., 2004) used an approach similar to Moelbert and Los Rios (2002) by introducing a Local perspective in global EMG model. It also found a dependence on network structure and a likeness with particular spin systems.

Finally, Namatame and Sato (2004) found coherent and systematic behaviours and a macroscopic pattern arising the strategic interaction of local rules.

In the literature there is a distinction between Local and Global models. A Local model contains the Global one as a particular case, where the neighbourhood is composed of all individuals (Burgos, et al., 2004).

The greater advantage in using agent-based models is to examine the dynamics of a system at a micro level, while the behaviour at the macro level is the aggregation of the micros.

The concept of Local Minority (LM) is introduced; the same concept was referred to as "relative minority" in a previous work (Remondino & Cappellini 2004). A LM is a group of individuals in the minority within a (partially) closed subset of the population. They also may not represent a global minority.

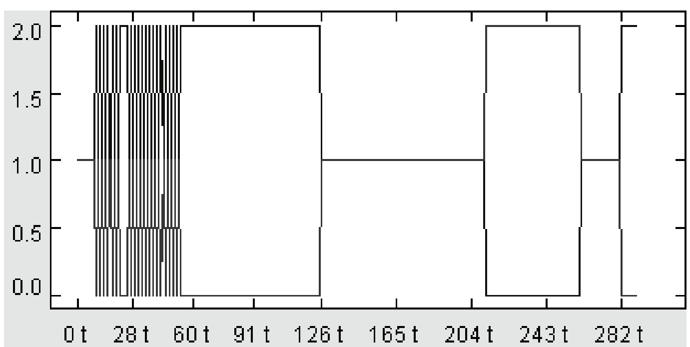
In Figure 7, a population spited into a chain of triplets (subsets of three individuals) can be observed. Their rewards depend on choices of neighbours only. In this configuration every agent could potentially be in one of the minorities. In fact five out seven agents are in different local minorities.

As a metaphor for local minorities, we can go back to a particular case of the MG—the bar problem quoted before. In that framework, local minorities can be represented by imagining that in the same pub there are many different rooms, with different features. For instance, one of them could have live music, the other one could be a smoking room, while the last one can be a no-smoking area. Of course, each room has a limited capacity so that the time spent there is enjoyable up to a certain threshold. So, it's advisable that the total amount of people is divided into local minorities (rooms), to make the time enjoyable for [an optimum number] many of them.

This perspective drives us towards some important considerations:

- The centrality of an exam at micro (meso) level of agent communities, instead of one of the total population, in order to understand the system dynamics.

Figure 7. Local minorities



- The representation of a bounded (partial) knowledge of the world. Is this an egoistic view? Is it important to be happier than my neighbours?
- Could this be an useful framework to study "word of mouth" or NIMBY (Not In My Backyard!) problems?
- The cumulative rewards for the individuals in the minority (minorities) could be greater than half of the number of agents: this means than more than one half of the population (the majority) is included in the local minorities.

CONCLUSION

While the original MG states that the agents involved must take a decision based on the historical data, their own experience and the forecasts about what the others will choose, in this chapter a form of communication of individual statements among them is introduced in order to see how the decision process would change. The stress here is not on the decision taken, be it the best or the worst, but on how the agents can change their decision when they are linked into a social network; in particular, this could be an empiric proof to a common sense rule: With a fixed number of agents, the more the links, the higher the probability to change opinion. An agent-based simulation was built, some real world parameters were tested, and the obtained results have been analyzed.

Two different communication protocols were employed among the agents: asynchronous and the more realistic synchronous, in order to see how this could affect the way the agents changed their opinions. Using the synchronous communication, the one in which an agent communicates with all others linked with him or her at the same time, the attitude to change opinion is at least 10 percent higher than in the asynchronous case, in which the agents act sequentially.

A sort of memory is then introduced, based on the past experiences, to act as a selection mechanism. In conjunction with communication, the thus composed simple cognitive system of agents creates local stable equilibria.

The framework described here gives some interesting results about how a network of connections among the agents who exchange their initial statement about a binary decision can change the way the aggregate behaves. In addition, other communication protocols can be analyzed using this framework.

Some specific agents were introduced lastly, called "Opinion Leaders" (OL), whose influence is higher than that of normal entities. In the real world an OL is a person who is considered a credible source of information for others on a specific topic and is sought out for that information. In this model, the agents defined as OLs are somehow special, in the sense that their opinion is "stronger" than the others', and is less subject to external influences. From the results, it emerges that in a small community, a person with many relations can change the aggregate "mood." This correlation could be an effective measure of an OL's influence on the population. By observing the results, a stylized political metaphor was introduced by identifying periods of "revolution/anarchy," "democracy," and "dictatorship" in the aggregate trend of decision making. This is, of course, just one of the many possible interpretations of the model presented model; for instance, the OLs can be thought of as the testimonials for some advertising campaign or advocates for a social cause, and so on.

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