MINORITY GAME WITH COMMUNICATION OF STATEMENTS AND MEMORY ANALYSIS: A MULTI AGENT BASED MODEL

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Abstract: The Minority Game (MG) is a simple, generalized framework, belonging to the Game Theory field, which represents the collective behaviour of agents in an idealized situation where they have to compete through adaptation for some finite resource. It generalizes the study of how many individuals may reach a collective solution to a problem under adaptation of each one's expectations about the future. It is assumed that an odd number of players take a decision at each step of the simulation; the agents that take the minority decision win, while the others loose. The Minority Game in its original formulation states that there is no communication among the agents involved in the simulation; the idea in this paper is to introduce in the model a sort of a social network, through which the agents can exchange their initial statements about their next decision, in order to see how the links among certain agents can change the results of the simulation. A software model is built, in which the user can define the number of the agents involved and the number of links among them; some examples are studied and analyzed in order to find some general rule. Besides, two communication protocols are implemented in the model: the asynchronous one, in which the agents act sequentially. So the first agents which act take their decision, and from then on they reply to the other agents with the new decision taken. The synchronous protocol states that the agents always communicate to the others their original opinion: they broadcast their opinion to all the agents which are linked to them. Finally, after having collected all the opinions of their friends, they reconsider their choice. We examine the difference among the two protocols using the same starting parameters in the simulation. After examining some random choosing agents, we embed a sort of memory into them, so that they can reason on which has been the best choice by looking at the past n results. Some local minorities emerge from the model.

Keywords: Minority Game, social network, multi agent system, simulation, communication of statement, local minority

INTRODUCTION

Game Theory is a distinct and interdisciplinary approach to the study of strategic behaviour. The disciplines most involved in game theory are mathematics, economics and the other social and behavioural sciences. Game theory (like computational theory and so many other contributions) was founded by the great mathematician John von Neumann. The first important book was The Theory of Games and Economic Behaviour, which von Neumann wrote in collaboration with the great mathematical economist, Oskar Morgenstern. Certainly Morgenstern brought ideas from neoclassical economics into the partnership, but von Neumann, too, was well aware of them and had made other contributions to neoclassical economics.

The key link between neoclassical economics and game theory was and is rationality. Neoclassical economics is based on the assumption that human beings are absolutely rational in their economic choices. Specifically, the assumption is that each person maximizes her or his rewards - profits, incomes, or subjective benefits - in the circumstances that she or he faces. This hypothesis serves a double purpose in the study of the allocation of resources. First, it narrows the range of possibilities somewhat. Absolutely rational behaviour is more predictable than irrational behaviour. Second, it provides a criterion for evaluation of the efficiency of an economic system. If the system leads to a reduction in the rewards coming to some people, without producing more than compensating rewards to others (costs greater than benefits, broadly) then something is wrong. Pollution, the overexploitation of fisheries, and inadequate resources committed to research can all be examples of this.

In neoclassical economics, the rational individual faces a specific system of institutions, including property rights, money, and highly competitive markets. These are among the "circumstances" that the person takes into account in maximizing rewards. The implications of property rights, a money economy and ideally competitive markets is that the individual needs not consider her or his interactions with other individuals. She or he needs consider only his or her own situation and the "conditions of the market." But this leads to two problems. First, it limits the range of the theory. Where-ever competition is restricted (but there is no monopoly), or property rights are not fully defined, consensus neoclassical economic theory is inapplicable, and neoclassical economics has never produced a generally accepted extension of the theory to cover these cases. Decisions taken outside the money economy were also problematic for neoclassical economics.

Game theory was intended to confront just this problem: to provide a theory of economic and strategic behaviour when people interact directly, rather than through the market. In game theory, "games" have always been a metaphor for more serious interactions in human society. Game theory may be about poker and baseball, but it is not about chess, and it is about such serious interactions as market competition, arms races and environmental pollution. But game theory addresses the serious interactions using the metaphor of a game: in these serious interactions, as in games, the individual's choice is essentially a choice of a strategy, and the outcome of the interaction depends on the strategies chosen by each of the participants. On this interpretation, a study of games may indeed tell us something about serious interactions.

In neoclassical economic theory, to choose rationally is to maximize one's rewards. From one point of view, this is a problem in mathematics: choose the activity that maximizes rewards in given circumstances. Thus we may think of rational economic choices as the "solution" to a problem of mathematics. In game theory, the case is more complex, since the outcome depends not only on my own strategies and the "market conditions," but also directly on the strategies chosen by others, but we may still think of the rational choice of strategies as a mathematical problem - maximize the rewards of a group of interacting decision makers - and so we again speak of the rational outcome as the "solution" to the game.

THE MINORITY GAME

The Minority Game (MG) is a simple, generalized framework, belonging to the Game Theory field, which represents the collective behaviour of agents in an idealized situation where they have to compete through adaptation for some finite resource.

While the MG is born as the mathematical formulation of “El Farol Bar” problem considered by (Arthur, 1994), it goes way beyond this one, since it generalizes the study of how many individuals may reach a collective solution to a problem under adaptation of each one’s expectations about the future. In (Arthur, 1994) the “El Farol Bar” problem was posed as an example of inductive reasoning in scenarios of bounded rationality. The kind of rationality which is usually assumed in economics – perfect, logical, deductive rationality – is extremely useful in generating solutions to theoretical problems, but it fails to account for situations in which our rationality is bounded (because agents can not cope with the complexity of the situation) or when ignorance about other agents ability and willingness to apply perfect rationally lead to subjective beliefs about the situation. Even in those situations, agents are not completely irrational: they adjust their behaviour based on what they think other agents are going to do, and these expectations are generated endogenously by information about what other agents have done in the past. On the basis of these expectations, the agent takes an action, which in turn becomes a precedent that influences the behaviour of future agents. This creates a feedback loop: expectations arise from precedents and then create the actions which, in turn, constitute the precedents for the next step.

The original formulation of “El Farol Bar” problem is as follows: N people, at every step, take an individual decision among two possibilities. Number one is to stay at home; number two is to go to a bar. Since the space in the bar is limited (finite resource), the time there is enjoyable if and only if the number of the people there is less than a fixed threshold (aN, where a<1). Every agent has his own expectation on the number of people in the bar, and according to his forecast decides whether to go or not. The only information available to the agents is the number of people attending the bar in the recent past; this means that there is no deductively rational solution to this problem, but there can be plenty of models

trying to infer the future number according to the past ones.
The other very interesting aspect of the problem is that if most agents think that the number of people going to the bar is > aN then they won't go, thus invalidating their own prevision. Computer simulations of this model shows that the attendance fluctuates around aN in a (aN,(1 - a)N) structure of people attending/not attending. The "El Farol Bar" problem has been applied to some proto-market models: at each time step agents can buy (go to the bar) or sell an asset and after each time step, the price of the asset is determined by a simple supply-demand rule.

The MG has been first described in (Challet and Zhang, 1997) as a mathematical formalization and generalization of "El Farol Bar" problem. It is assumed that an odd number of players take a decision at each step of the simulation; the agents that take the minority decision win, while the others loose. Stepping back to "El Farol Bar" problem, we can see it as a minority game with two possible actions: a1 = 1 (to go to the bar) and a2 = -1 (not to go to the bar). After each round, the cumulative action value A(t) is calculated as the sum of each value given to the single actions. The minority rule sets the comfort level at A(t) = 0, so that agent is given a payoff -ai(t)g[A(t)] at each time step with g an odd function of A(t).

INTRODUCING COMMUNICATION: SOCIAL NETWORKS AND GRAPHS

The "El Farol Bar" problem, as well as the Minority game in its original formulation state that there is no communication among the agents involved in the simulation; the idea in this paper is to introduce in the model a sort of a social network, in order to see how the links among certain agents can change the results of the simulation. A social network is defined as "a set of nodes - e.g. persons, organizations - linked by a set of social relationship - e.g. friendship, transfer of funds, overlapping membership - of a specific type" (Laumann, et al., 1978).
In our case the minority rule will be very easy: a set of N agents will have to choose between (-1) and (1). Who is in the minority (denoted with n < N) wins and gets a payoff equal to N/n: the fewer agents stay in the minority, the higher the payoff. Also the social network involved will be quite simple, just linking an agent to others with a relation limited to the possibility of asking a question: "will you choose (-1) or (1)?". Not all the agents will be connected, though, so that some of them will have to make a prevision just considering the past few results, exactly like in the original MG.

In the example shown in Table 1 we have five agents involved in the simulation: agent 1 can ask agents 2, 3 and 4, while agent 2 can ask agent 3 and number 3 can ask number 1 and number 5; agent 4 can then ask number 1 and number 5, while number 5 is a lonely agent (he can't ask anyone, even if two other agents can ask him what he will do).

	1	2	3	4	5
1		x	x	x	
2			x		
3	x				x
4	x				x
5					

Table 1: definition of relations among agents

Any kind of network can be described in terms of a graph, composed of nodes and a set of lines, edges, joining the nodes. In a mathematician's terminology, a graph is a collection of points and lines connecting some (possibly empty) subset of them. The points of a graph are most commonly known as graph vertices, but may also be called nodes or simply points. Similarly, the lines connecting the vertices of a graph are most commonly known as graph edges, but may also be called arcs or lines. The study of graphs is known as graph theory, and was first systematically investigated by D. König in the 1930s (Gardner, 1984). Graphs come in a wide variety of different sorts. The most common type is graphs in which at most one edge (i.e., either one edge or no edges) may connect any two vertices. Such graphs are called simple graphs and are the ones we'll use in our analysis. The edges of graphs may also be imbued with directedness. A normal graph in which edges are undirected is said to be undirected. Otherwise, if arrows may be placed on one or both endpoints of the edges of a graph to indicate directedness, the graph is said to be directed.
In our work the graph used to represent the social network linking the agents together is bi-directed, i.e. each edge points on both directions as once. This seems realistic, since we can imagine our network as a group of friends, or in general people who know each other. If A knows B, then it's quite obvious that B knows A in turn; we don't voluntarily consider those situations in which a subject disseminates his opinion to others and isn't touched by their decisions (e.g.: advertisement, political campaigns, and so forth). That's because we suppose that this sort of dissemination comes "a priori", i.e. before our analysis starts; we are now interested in studying how a set of agents mutually

connected into a network can influence one another and come to a final overall result. In Figure 1 we depict an example of a network we use in our model. It's possible to notice that some nodes (agents) can be left totally unlinked, thus having to take their decision just basing on their own forecasts.

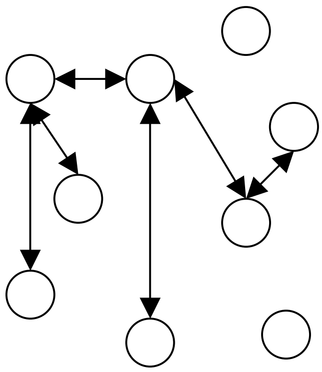

Figure 1: agents communicating over a network

AGENT BASED SIMULATION

The tool we used to get analytical results for our experiments is computer simulation; modelling is a way of solving problems that occur in the real world. It is applied when prototyping or experimenting with the real system is expensive or impossible. We can distinguish between analytical and simulation models. In analytical, or static, model the result functionally depends on the input (a number of parameters); it is possible to implement such model in a spreadsheet. However, analytical solution does not always exist, or may be very hard to find. Then simulation, or dynamic, modelling may be applied. A simulation model may be considered as a set of rules (e.g. equations, flowcharts, state machines, cellular automata) that define how the system being modelled will change in the future, given its present state. Simulation is the process of model "execution" that takes the model through (discrete or continuous) state changes over time. In general, for complex problems where time dynamics is important, simulation modelling is a better answer.

According to (Troitzsch, 1996), computer simulation in the social sciences has at least two types of origins: on one hand, it continues mathematical modelling and is no more than the numerical treatment of difference equations or the various kinds of differential equations (including partial and stochastic differential equations). Here, a machine is used to manipulate the symbols of the symbol system of mathematics, and this manipulation is more or less restricted to numerical treatment (although some computer help in symbolic computation is sometimes desirable, too). On the other hand, computer simulation is used in its own right, not as a substitution for more elegant mathematical solution algorithms, but as a means of manipulating the symbols of the symbol system of programming languages.

In (Ostrom 1988), agent based simulation is described as a third way to represent social models, being a powerful alternative to other two symbol systems: the verbal argumentation and the mathematical one. The former, which uses natural language, is a non computable way of modelling though a highly descriptive one; in the latter, while everything can be done with equations, the complexity of differential systems rises exponentially as the complexity of behaviour grows, so that describing complex individual behaviour with equations often becomes an intractable task. Simulation has some advantages over the other two: it can easily be run on a computer, through a program or a particular tool; besides it has a highly descriptive power, since it is usually built using a high level computer language, and, with few efforts, can even represent non-linear relationships, which are tough problems for the mathematical approach. According to (Gilbert, Terna 2000):

"The logic of developing models using computer simulation is not very different from the logic used for the more familiar statistical models. In either case, there is some phenomenon that the researchers want to understand better, that is the target, and so a model is built, through a theoretically motivated process of abstraction. The model can be a set of mathematical equations, a statistical equation, such as a regression equation, or a computer program. The behaviour of the model is then observed, and compared with observations of the real world; this is used as evidence in favour of the validity of the model or its rejection"

In Remondino (2003) we read that computer programs can be used to model either quantitative theories or qualitative ones; simulation has been successfully applied to many fields, and in particular to social sciences, where it allows to verify theories and create virtual societies. In order to simulate the described problem, multi-agent technique is used. Agent Based Modelling is the most interesting and advanced approach for simulating a complex system: in a social context, the single parts and the whole are often very hard to describe in detail. Besides, there are agent based formalisms which allow to study the emergency of social behaviour with the creation and study of models, known as artificial societies. Thanks to the ever increasing computational power, it's been possible to use such models to create software, based on intelligent agents, which aggregate behaviour is complex and difficult to predict, and can be used in open and distributed systems. The concept of Multi Agent System for social simulations is thus introduced: the single agents have a very simple structure. Only few details and actions are described for the entities: the behaviour of the whole system is a consequence of those of the single agents, but it's not necessarily the sum of them. This can bring to unpredictable results, when the simulated system is studied.

Agents have traditionally been categorized into one of the following types (Woolridge and Jennings, 1995): reactive, deliberative and hybrid.
When designing any agent-based system, it’s important to determine how sophisticated the agents' reasoning will be. Reactive agents simply retrieve pre-set behaviours similar to reflexes without maintaining any internal state. On the other hand, deliberative agents behave more like they are thinking, by searching through a space of behaviours, maintaining internal state, and predicting the effects of actions. Although the line between reactive and deliberative agents can be somewhat blurry, an agent with no internal state is certainly reactive, and one which bases its actions on the predicted actions of other agents is deliberative.
In Mataric (1995) we read that reactive agents maintain no internal model of how to predict future states of the world. They choose actions by using the current world state as an index into a table of actions, where the indexing function's purpose is to map known situations to appropriate actions. These types of agents are sufficient for limited environments where every possible situation can be mapped to an action or set of actions.
The purely reactive agent's major drawback is its lack of adaptability. This type of agent cannot generate an appropriate plan if the current world state was not considered a priori. In domains that cannot be completely mapped, using reactive agents can be too restrictive.

In the present work we have a community of reactive agents that must take a decision; this could be simply binary (e.g.: to sell or to buy in a stock market; to go or not to go to a pub, and so forth) or a more complex one (e.g.: choosing whom to vote for at the next elections; choosing the colour for a car, and so on). While the mechanism behind the constitution of an opinion in the human beings is beyond the purpose of this work, here we want to analyze how a social network interconnecting a community of agents can influence their choices and, in particular, how it could determine changes of their own opinions. That’s why we chose to use simple, reactive agents: no plans are required to carry on the initial decision, that could even be randomly generated, and the only action they have to perform is to evaluate the opinions of their “friends”, that are the other agents linked with them, and choose whether to be or not to be influenced by them.
There are many toolkits and frameworks that can be used to build agent based simulations; for this work JAS was selected (http://jaslibrary.sourceforge.net) since it includes graph support for Social Network Analysis. In the basic model we present in this paper we only examine how many agents change their own opinion, when increasing the number of direct relations among them; further work will address some other issues, such as the correctness of the agents’ choice, and so on.

THE SIMULATION FRAMEWORK

At the beginning of the simulation, during the setup, we create a simple world populated by N agents. These agents can be considered as the vertexes of a social network and the links among them (relations) as the edges. The network is directed and every arc is composed by two edges with opposite directions. Every agent has a list of F (friends) other agents (called friendsList) to whom he can ask. This list is composed by the neighbours, i.e. the vertexes linked to the examined vertex (the agent).

The neighbourhood is intended as a sociological, and not only physical, closeness. According to (Laumann et al. 1978) we describe the relations between our agents as friendship. This social relationship is characterized by a random creation but is also very stable in short/medium term. Our links are directed and bi-directional, as the friendship is.

Here follows a brief description of the simulation process:

- At the beginning of each simulation step, every agent has its own forecast. The forecast is absolutely random between two choices –1 and +1.
- The decision taken by each agent (before communicating with others) is denoted with a “certainty index” equal to 1 (100%).
- Now an agent is randomly chosen. He starts asking to the first in the list; if this one has the same prevision, then the certainty index is increased by a value of 1/F, while if the prevision is different, than the certainty index is lowered by 1/F
- After having asked a statement to all the friends in his list, the agent takes the final decision: if the certainty index is equal or greater than 1, then the decision will be the original one. If it’s lower than 1, then the decision will be the other possible one.

• Another agent is then randomly chosen, and so on (the same agent can't be chosen twice during the same turn). Note that an agent that's been asked can still change his mind, basing on the agents he will in turn ask

Before starting the simulation, we can change two core parameters: the number of the agents involved and the number of the links among the agents. Here we examine three runs of the simulation, one with 1000 agents and 500 total links (an average of one link every two agents); the other one with 100 agents and 500 links (an average of five links for every agent) and the last one with 100 agents and 5000 links (fifty links for every agent). In every run we iterate the minority game for 1000 times.

The model could be considered as some groups of friends that must choose between two alternatives: pub and disco. They communicate the selected choice to their friends, elaborate them and then take a final decision.

In the output graph we can read the time on x-axis (1000 iterations of the game), and we plot two lines: the red one (the lower one in the graphs) depicts the decisions changed while the blue one (the upper one) is for unchanged decisions.

In y-axis we read the number of decisions (changed or not) the scale (10^1, 10^2, 10^3) depends from agents number.

We choose as standard example a world of 100 agents and 500 relations (figure 2), in which an average of 65 out 100 preserve their original decisions.

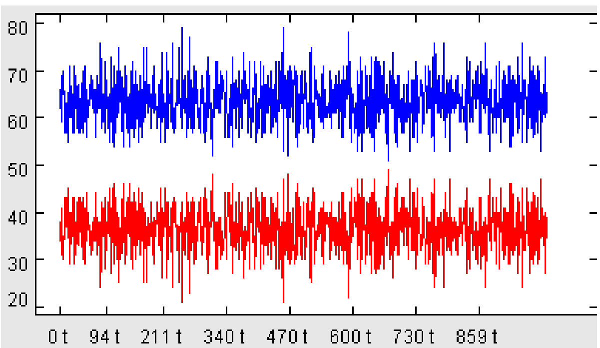

Figure 2: 100 agents and 500 relations

In a second run we imagine a different situation, in which the agents have many more relations among them: an average of fifty for every inhabitant (figure 3).

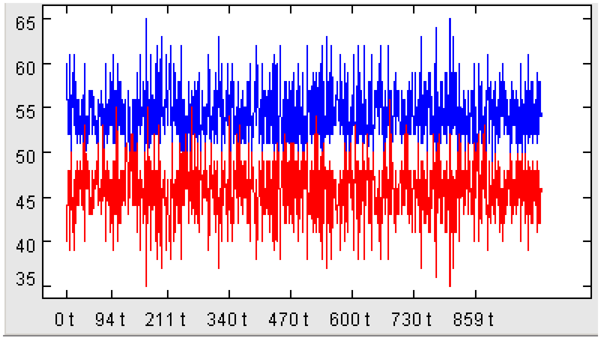

Figure 3: 100 agents and 5000 relations

A simple common sense rule states that the more relations, the higher is the probability to change opinion. This example proves the rule to be right and our model to be consistent with real world results; we can now try a counter example, i.e. a poor relations world, as the one in figure 4; one thousand inhabitants with a total of just five hundred relations.

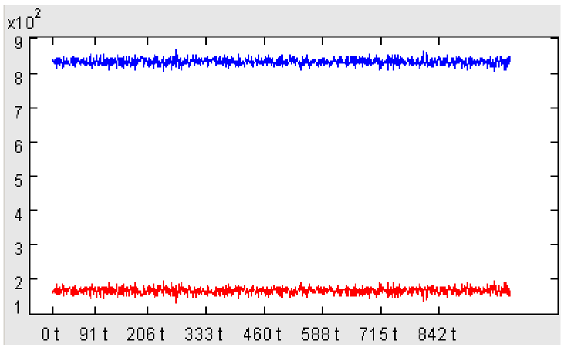

Figure 4: 1000 agents and 500 relations

Here we can observe that less than 20% of the agents changed their opinion. In order to test the extreme situation, we also imagined a world with no relations among the agents (like in the original MG).

Obviously in a world with one thousand unlinked agents we have no changing of opinion (figure 5).

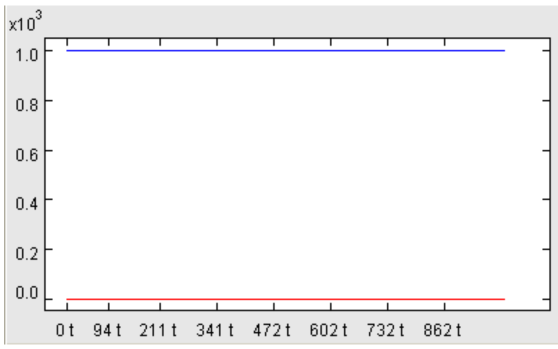

Figure 5:1000 Agents and Zero Relations

SYNCHRONOUS COMMUNICATION

A step further is the implementation of a different communication protocol among agents.

The first we used is an asynchronous one: the agents act sequentially. So the first agents to act take a decision, and from then on they reply to the other agents with the new decision taken. We wonder if this method can be realistic, so we decided to explore also a synchronous communication process, which seems more similar to the one we would have in a real world.

Now the agents always communicate to the others their original opinion: they broadcast their opinion to all the agents which are linked to them. Finally, after they collect all the opinions of their friends, they evaluate the certainty index and reconsider their choice.

We executed the simulation with the new rule and the same parameters as before.

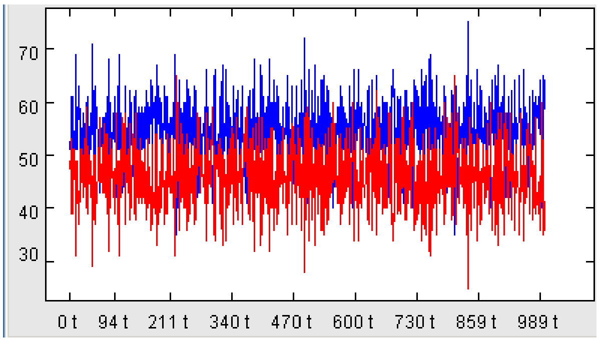

Figure 6: 100 agents and 500 relations

In the first example (figure 6) we have a ten percent more changed opinions, than we had in the sequential model.

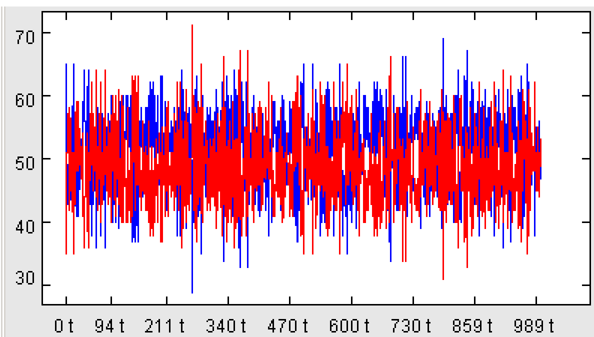

Figure 7: 100 agents and 5000 relations

The best result is in the second run (figure 7): the world rich of relations. The two lines are quite overlapped (even if there is a high variance in data).

We can now express a second simple rule coming from this analysis: a synchronous communication among the agents increases their attitude to change opinion, which is at least ten percent higher.

The proof is the third run, in which again we have an higher result when compared to the asynchronous case.

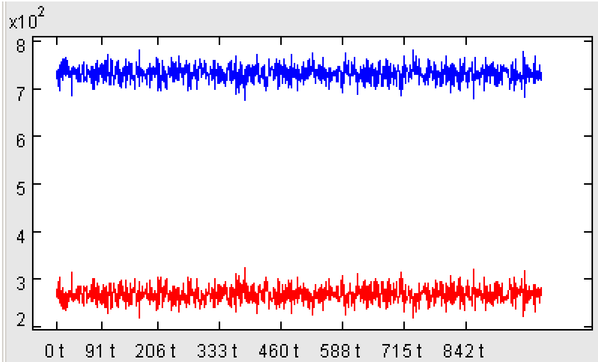

Figure 8: 1000 agents and 500 relations

FROM COMMUNICATION TO MINORITY

Kauffman (1969), firstly described a disordered dynamical system that consists of N Boolean variables or spins stable related each other (Kauffman Networks) used by gene regulatory systems, (but also for spin glasses, evolution, social sciences, economics and finance).
Each gene changes its status (active or not) depending on some other's signals.
Paczuski et al (2000) used that structure introducing a Minority Game with personal limited information resources, but with a global reward mechanism.
Kalinowski et al. (2000) described a model in which agents who are placed in a circle are able to cooperate due to self-organization. The term "Local" was introduced by Moelbert and Los Rios (2002). They depicted a one-dimensional or square lattice with communities of 3 or 5 individuals, each one interacting with two (four) nearest neighbours.

All those works are based on a bounded communication and they are generally closer to a Small Worlds like scenario. They show that space correlation becomes important.
We implement that local communication among agents, but also introduce another level of information: every agent issue a statement before acting and the decision is subsequently based on

that. At this moment we still don't consider the possibility to lie in the declaration.

Johnson et al. (1999), while describing an evolutionary version of the minority game (EMG), found that the introduction of partial information, instead of global and diffuse news, force agents to take a decision basing on inductive - rather than deductive - thinking. The result is a self-segregation of individuals.
Kirley (2004) extended this research in order to introduce small world connections in it. This spatial approach and a small degree of disorder lead to an improvement of system efficiency: the agents can coordinate more effectively their behaviour.
Local Evolutionary minority games (LEMG, Burgos et al., 2004), used an approach similar to Moelbert and Los Rios (2002) introducing a Local perspective in global EMG model. They found also a dependence on network structure and a likeness with particular spin systems.
Finally Namatame and Sato (2004) found coherent and systematic behaviours, and a macroscopic pattern arising the strategic interaction of local rules.

In literature we find a distinction about Local and Global models. A Local model contains the Global one as a particular case, in which the neighbourhood is composed by all individuals (Burgos et al., 2004).
The greater advantage in using Agent based models is to examine the dynamics of a system at a micro level, while the behaviour at the macro level is the aggregation of the micros.
We introduce here the concept of Local Minority; the same concept was referred to as "relative minority" in a previous work (Remondino, Cappellini 2004). A Local Minority is a group of individuals in minority in a (partially) close subset of the population. They can also not represent a global minority.

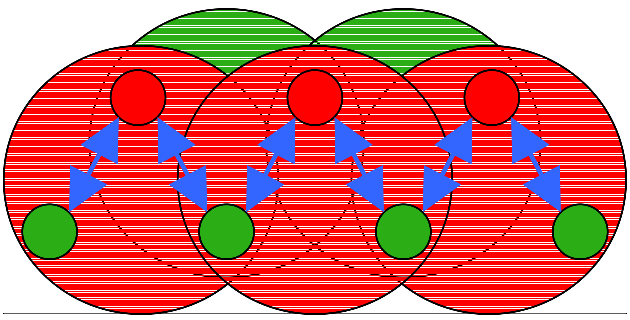

Figure 9: local minorities

In Figure 9, you can observe a population splitted into a chain of triplets (subsets of three individuals). Their rewards depend on choices of neighbours only. In this configuration every agent could potentially be in one of the minorities. In fact 5 out 7 agents are in different local minorities.

As a metaphor for local minorities we can go back to a particular case of the MG, the Bar Problem quoted before. In that framework we can represent local minorities by imaging that in the same pub there are many different rooms, with different features; for instance one of them could have live music, the other one could be a smoking room, while the last one can be a no-smoking area. Of course each of them has a limited capacity so that the time in there is enjoyable only till a certain threshold. So, it's advisable that the total amount of people is divided into local minorities (rooms), so that the time there is fine for many of them.

This perspective drives us towards some important consideration:

- The centrality of an exam at micro (meso) level of agents communities, instead of one of the total population, to understand the system dynamics.
- It respects a bounded (partial) knowledge of the world. Is this an egoistic view? Is it important to be happier than my neighbours?
- Could this be an useful framework to study "word of mouth" or NIMBY (Not In My Backyard!) problems?
- The cumulative rewards for the individuals in minority (minorities) could be higher than the half of the number of agents: this means than more than one half of the population (the majority) is included in the local minorities.

MEMORY AND REWARDING

In this section we investigate how the introduction of a simple kind of memory, based on the past turns, can change the previous results This is one of the most simple strategies implemented in the original MG; though, we are not interested in the final result, being it win or loose, but in the way the agents behave, i.e. change their original opinion, when their "mind" changes somehow.

Besides, we introduce a payoff system to reward the players in the minority. The memory is a list of length N (technically we can use the same length for all the agents or randomize it using a range from 1 to 20). In each "box" we add the last cumulate choice of the group to which the agent belongs. The value is normalized and is +1, if the sum of choices is higher than zero, or -1, if the sum is less than zero. The agent uses its memory by reading the list, and summing the last group choices. The agent choice will be +1, if the sum is lower than 0, that means the mode of the group is -1; -1 in the opposite situation; or can be random, if there is no prevailing result.

We also introduce a network graph in which we can observe the topology and the agents changing their colours, red for "+1" and green for "-1"; the relations (links) among the agents are bi-directional ones, and represented by the black arrows connecting the nodes. This means that if A can ask B, then B can in turn ask A. An example of this can be observed in figure 9.

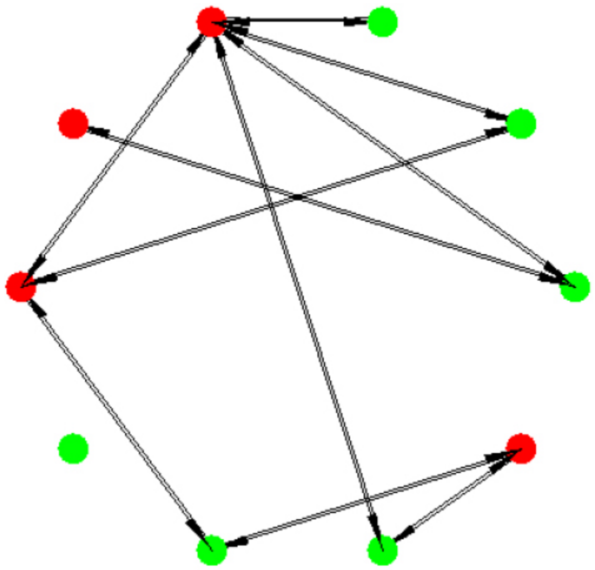

Figure 10: 10 agents and 10 relations topology

This figure depicts an interesting experiment composed of 10 agents and 10 relations, using memory and sequential communication.
Looking at the graph we can observe that every group is in equilibrium. In fact, according to bounded rationality, each agent knows only the information about his own neighbours. Observing each agent's point of view, there are triplets Green-Red-Green or Red-Green-Red in perfect equilibrium, in which every agent respects the minority rule. The agents reach an elevated global optimum (Figure 11) of eight out ten.

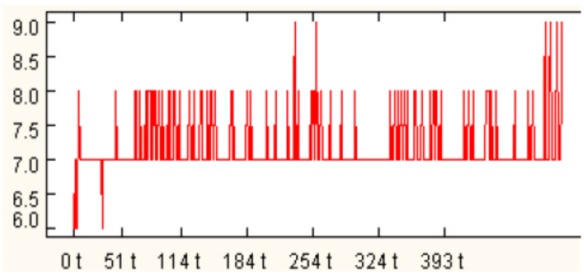

Figure 11: 10 agents and 10 relations, rewards

The stability of the system is strengthen by the steady distribution observed in figure 11. In fact, the node that changes opinion is usually the isolated one.

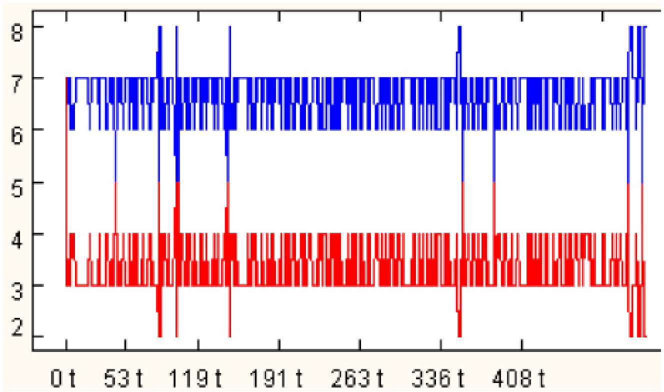

Figure 12: 10 agents and 10 relations, changed choices

The rewarding system counts one point for every agent that chooses a (local) minority option.

REAL APPLICATIONS FOR THE MODEL

Many market - or proto market - problems can be modelled by using the MG; in a sense MG gives us a powerful tool to study detailed pattern of fluctuations, the equilibrium point is trivial by design. It is the fluctuations that play the dominant role in economic activities, like the market mechanism. MG allows us to study in a precise manner how is the approach to equilibrium, how the agents try to outsmart each other, for their selfish gain, compete for the available marginal information (any deviation from the mid-point represents exploitable advantage). The introduction of a social network makes this mechanism even more realistic, since in the real world nobody is completely alone, and it's very likely that she/he can communicate with somebody else about what her/his actions will be.

Besides of the market models, typical applications of the MG, the proposed model can be applied to other important social phenomena, by considering the importance of communication into a social network of agents taking a binary decision where the minority of majority makes the difference. An example can be found in the political elections or in the way the commercials affect the public opinion about the decision of buying a product instead of another one. For every phenomenon in which the word of mouth is fundamental, and the decision can be changed according to that, then this model can be successfully applied for the analysis of the aggregate.

We can also think to some different examples, such as fashion; it is often considered important to be in a minority who owns certain luxury goods, but at the same time the word of mouth is important to spread the trend of wearing them.

CONCLUSIONS

While the original Minority Games states that the agents involved must take a decision based on the historical data, their own experience and the forecasts about what the others will choose, in this paper we introduced communication of the individual statements among them, in order to see how the decision process would change. The stress here is not on the decision taken, be it the best or the worst, but on how the agents can change their decision when they are linked into a social network; in particular, we tried to find the empiric proof to a common sense rule: with a fixed number of agents, the more the links, the higher is the probability to change opinion. We built an agent based simulation, tested some real world parameters and analyzed the results we obtained.

We examined two different communication protocols among the agents: the asynchronous one and the more realistic synchronous one, in order to see how this could affect the way the agents changed their opinions. Using the synchronous communication, the one in which an agent communicates with all the ones linked with him at the same time, we saw that the attitude to change opinion is at least 10% higher than in the asynchronous case, in which the agents act sequentially.

At last, we reintroduce a sort of memory, based on the past experiences, to act as a selection mechanism. In conjunction with communication, the so composed simple cognitive system of agents creates local stable equilibria.

The framework described here gives some interesting results about how a network of connections among the agents, who exchange their initial statement about a binary decision can change the way the aggregate behaves. Some future developments of our model include the presence of a “stronger” category of agents (e.g. opinion leaders), whose influence is higher than that of normal entities. Besides other communication protocols can be analyzed using this framework.

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ANNEX 1

In this section we show a simple statistical analysis of the results coming from our simulation.
We monitored three main variables:

- agents that changed their opinion;
- agents that chose the first option;
- total amount of rewards.

The tables were organized by agents and relations number (tables), and for typology of communication and presence/absence of memory (groups of three columns). Every experiment is based on 1000 iterations.

The parameters show a ratio between agent number and relations. So we designed four different populations by their richness in relations.

It’s important to underline that for our experiments having an odd number of agent is not critical, because of our network random structure and the topic of relative minority. The random sub-networks created could be odd or even, as you can observe in graphs 13, 14, 15 and 16, that shows the simple network structures of 10 agents and 10 relations.

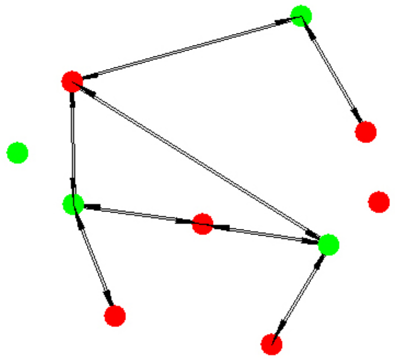

Figure 13: 10 agents and 10 relations topology

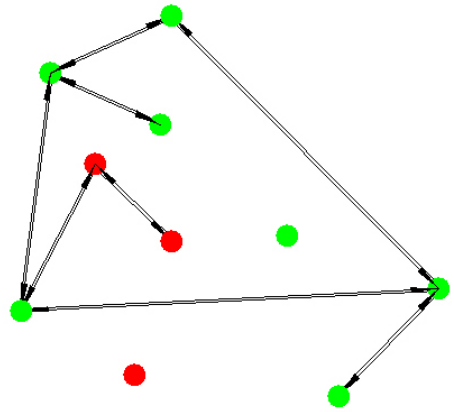

Figure 14: 10 agents and 10 with synchronous time relations topology

	changed opinion	first choice	rewards	changed opinion	first choice	rewards	changed opinion	first choice	rewards	changed opinion	first choice	rewards
	asynchronous communication			synchronous communication			asynchronous communication and memory			synchronous communication and memory		
	10 agents and 10 relations											
Mean	5,3	5,0	6,0	6,5	5,1	2,8	2,0	4,0	8,0	10,0	9,0	0,0
Minimum	2,0	2,0	1,0	2,0	0,0	0,0	2,0	3,0	6,0	6,0	5,0	0,0
First Quartile	5,0	4,0	5,0	5,0	3,0	1,0	2,0	3,0	8,0	10,0	9,0	0,0
Median	5,0	5,0	6,0	7,0	5,0	2,0	2,0	4,0	8,0	10,0	9,0	0,0
Third Quartile	6,0	6,0	7,0	8,0	7,0	5,0	2,0	5,0	8,0	10,0	9,0	0,0
Maximum	9,0	8,0	8,0	10,0	10,0	8,0	6,0	6,0	8,0	10,0	10,0	3,0
Mode	5,0	5,0	8,0	8,0	5,0	0,0	2,0	4,0	8,0	10,0	9,0	0,0
Variance	1,5	1,3	2,4	3,0	5,7	6,0	0,0	0,5	0,0	0,0	0,5	0,0
	100 agents and 500 relations											
Mean	39,7	50,0	81,4	53,2	49,0	39,1	1,9	52,8	98,2	99,6	99,6	0,2
Minimum	27,0	42,0	67,0	31,0	14,0	9,0	1,0	51,0	85,0	63,0	56,0	0,0
First Quartile	37,0	48,0	79,0	49,0	39,0	33,0	1,0	52,0	97,0	100,0	100,0	0,0
Median	40,0	50,0	82,0	53,0	49,0	39,0	1,0	53,0	99,0	100,0	100,0	0,0
Third Quartile	43,0	52,0	84,0	58,0	58,0	45,0	3,0	53,0	99,0	100,0	100,0	0,0
Maximum	54,0	59,0	93,0	74,0	81,0	70,0	47,0	54,0	99,0	100,0	100,0	27,0
Mode	40,0	50,0	83,0	55,0	49,0	39,0	1,0	53,0	99,0	100,0	100,0	0,0
Variance	19,6	6,1	17,4	40,3	175,9	82,3	3,4	0,3	1,2	7,5	11,9	4,8
	100 agents and 5000 relations											
Mean	35,6	50,1	86,5	51,3	50,4	25,5	0,1	49,0	100,0	99,9	0,1	0,1
Minimum	22,0	46,0	72,0	34,0	0,0	0,0	0,0	48,0	91,0	51,0	0,0	0,0
First Quartile	33,0	49,0	84,0	47,0	24,0	13,0	0,0	49,0	100,0	100,0	0,0	0,0
Median	36,0	50,0	87,0	52,0	52,0	25,0	0,0	49,0	100,0	100,0	0,0	0,0
Third Quartile	39,0	51,0	89,0	56,0	75,0	36,0	0,0	49,0	100,0	100,0	0,0	0,0
Maximum	49,0	56,0	98,0	69,0	100,0	74,0	38,0	52,0	100,0	100,0	49,0	48,0
Mode	35,0	50,0	88,0	51,0	13,0	35,0	0,0	49,0	100,0	100,0	0,0	0,0
Variance	18,9	2,0	16,2	36,6	840,6	239,1	1,8	0,0	0,2	4,9	2,5	2,4
	1000 agents and 500 relations											
Mean	601,5	500,2	453,6	731,0	499,9	178,3	373,4	500,8	627,6	801,0	477,2	195,9
Minimum	569,0	459,0	370,0	679,0	425,0	121,0	372,0	470,0	566,0	736,0	443,0	192,0
First Quartile	594,0	492,0	444,0	721,0	487,0	167,0	372,0	494,8	628,0	800,0	471,0	196,0
Median	601,0	500,5	453,0	731,0	499,0	178,0	373,0	501,0	628,0	801,0	477,0	196,0
Third Quartile	609,0	508,0	463,0	741,0	514,0	189,0	373,0	507,0	628,0	802,0	484,0	196,0
Maximum	633,0	534,0	549,0	781,0	557,0	230,0	588,0	535,0	629,0	807,0	511,0	205,0
Mode	600,0	501,0	454,0	724,0	502,0	181,0	372,0	501,0	628,0	801,0	479,0	196,0
Variance	119,2	152,1	223,5	251,0	400,5	266,6	53,8	90,3	7,0	7,3	92,3	0,6

Table 2: experiment results

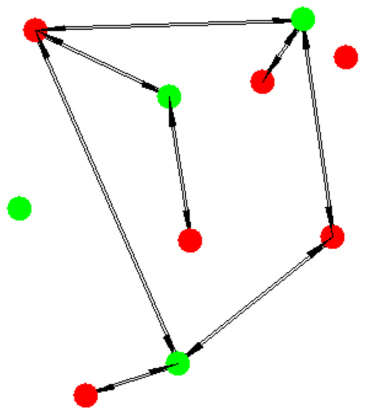

Figure 15: 10 agents and 10 with memory relations topology

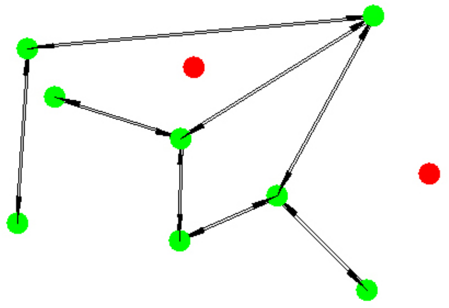

Figure 16: 10 agents and 10 with memory and synchronous time relations topology

The higher the rewards, the higher the happiness in the population, since there are more agents in the relative minorities.

The first choice is defined in the model as uniformly distributed. This is quite respected also in the results,

except in the ones where synchronous communication and memory are both present.
The presence of the memory is the more relevant variable for changing opinion, and makes the distribution change from an average one to the tails.

Obviously the lower the variance and the nearest are mean, median and mode, the more stable is the distribution: with memory we had uniform agents' behaviours.

BIOGRAPHY

MARCO REMONDINO was born in Asti, Italy, and studied Economics at the University of Turin, where he obtained his Master Degree in March, 2001 with 110/110 cum Laude et Menzione and a Thesis in Economical Dynamics. In the same year, he started attending a PhD at the Computer Science Department at the University of Turin, which will last till the end of 2004. His main research interests are Computer Simulation applied to Social Sciences, Enterprise Modeling, Agent Based Simulation, Multi Agent Systems and BDI agents. He has been part of the European team which defined a Unified Language for Enterprise Modeling (UEML). He is also participating to a University project for creating a cluster of computers, to be used for Social Simulation.

ALESSANDRO CAPPELLINI was born in Turin, Italy. He studied Economics at the University of Turin, where he obtained his Master Degree in July 2003, with a Thesis in Mathematical Economics concerning stocks market simulation with artificial and natural agents. He started attending a PhD (ending in 2006) on simulation at the University of Turin. His main research interests are Computer Simulations and Experiments applied to Finance, Economics and Social Sciences. He is also interested in behavioural finance, and is a founder of the Italian behavioural finance association ("A.I.FIN.C"). Nowadays he works in Analysis and Strategic Planning office of Sanpaolo IMI Group.