

# Minority Rule Applied to Multiple Agents Linked into a Social Network with Communication and Memory

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## ABSTRACT

In our work we simulate a community of agents, linked into a simple social network based on communication among the nodes, who must take a binary decision at every step. This resembles the original Minority Game (MG), which is a simple, generalized framework, belonging to the Game Theory field, which represents the collective behaviour of agents in an idealized situation where they have to compete through adaptation for some finite resource. It generalizes the study of how many individuals may reach a collective solution to a problem under adaptation of each one's expectations about the future. The main differences between this work and the original MG are the introduction of communication among the agents, which are now grouped basing on the common choices, and the number of players, that can also be an even number, while in the MG must be an odd number. This is done in order to generalize as much as possible the study of the choices made by agents trying to be in the minority group. Two communication protocols are implemented in the model: the asynchronous one, in which the agents act sequentially. So the first agents which act take their decision, and from then on they reply to the other agents with the new decision taken. The synchronous protocol states that the agents always communicate to the others their original opinion: they broadcast their opinion to all the agents which are linked to them. Finally, after having collected all the opinions of their friends, they reconsider their choice. After examining some random choosing agents, we embed a sort of memory into them, so that they can reason on which has been the best choice by looking at the past  $n$  results.

## INTRODUCTION

Game Theory is a distinct and interdisciplinary approach to the study of strategic behaviour. The disciplines most involved in game theory are mathematics, economics

and the other social and behavioural sciences. Game theory (like computational theory and so many other contributions) was founded by the great mathematician John von Neumann. The first important book was *The Theory of Games and Economic Behaviour*, which von Neumann wrote in collaboration with the great mathematical economist, Oskar Morgenstern. Certainly Morgenstern brought ideas from neoclassical economics into the partnership, but von Neumann, too, was well aware of them and had made other contributions to neoclassical economics.

The key link between neoclassical economics and game theory was and is rationality. Neoclassical economics is based on the assumption that human beings are absolutely rational in their economic choices. Specifically, the assumption is that each person maximizes her or his rewards - profits, incomes, or subjective benefits - in the circumstances that she or he faces. This hypothesis serves a double purpose in the study of the allocation of resources. First, it narrows the range of possibilities somewhat. Absolutely rational behaviour is more predictable than irrational behaviour. Second, it provides a criterion for evaluation of the efficiency of an economic system. If the system leads to a reduction in the rewards coming to some people, without producing more than compensating rewards to others (costs greater than benefits, broadly) then something is wrong. Pollution, the overexploitation of fisheries, and inadequate resources committed to research can all be examples of this.

In neoclassical economics, the rational individual faces a specific system of institutions, including property rights, money, and highly competitive markets. These are among the "circumstances" that the person takes into account in maximizing rewards. The implications of property rights, a money economy and ideally competitive markets is that the individual needs not consider her or his interactions with other individuals. She or he needs consider only his or her own situation and the "conditions of the market." But this leads to two problems. First, it limits the range of the theory. Wherever competition is restricted (but there is no monopoly), or property rights are not fully defined, consensus neoclassical economic theory is inapplicable, and neoclassical economics has never produced a generally accepted extension of the theory to cover these cases. Decisions taken outside the money economy were also problematic for neoclassical economics.

Game theory was intended to confront just this problem: to provide a theory of economic and strategic behaviour when people interact directly, rather than through the market. In game theory, "games" have always been a metaphor for more serious interactions in human society. Game theory may be about poker and baseball, but it is not about chess, and it is about such serious interactions as market competition, arms races and environmental pollution. But game theory addresses the serious interactions using the metaphor of a game: in these serious interactions, as in games, the individual's choice is essentially a choice of a strategy, and the outcome of the interaction depends on the strategies chosen by each of the participants. On this interpretation, a study of games may indeed tell us something about serious interactions.

In neoclassical economic theory, to choose rationally is to maximize one's rewards. From one point of view, this is a problem in mathematics: choose the activity that maximizes rewards in given circumstances. Thus we may think of rational economic choices as the "solution" to a problem of mathematics. In game theory, the case is more complex, since the outcome depends not only on my own strategies and the "market conditions," but also directly on the strategies chosen by others, but we may still think of the rational choice of strategies as a mathematical problem - maximize the rewards of a group of interacting decision makers - and so we again speak of the rational outcome as the "solution" to the game.

## THE MINORITY GAME

The Minority Game (MG) is a simple, generalized framework, belonging to the Game Theory field, which represents the collective behaviour of agents in an idealized situation where they have to compete through adaptation for some finite resource.

While the MG is born as the mathematical formulation of "El Farol Bar" problem considered by (Arthur, 1994), it goes way beyond this one, since it generalizes the study of how many individuals may reach a collective solution to a problem under adaptation of each one's expectations about the future. In (Arthur, 1994) the "El Farol Bar" problem was posed as an example of inductive reasoning in scenarios of bounded rationality. The kind of rationality which is usually assumed in economics - perfect, logical, deductive rationality - is extremely useful in generating solutions to theoretical problems, but it fails to account for situations in which our rationality is bounded (because agents can not cope with the complexity of the situation) or when ignorance about other agents ability and willingness to apply perfect rationally lead to subjective beliefs about the situation. Even in those situations, agents are not completely irrational: they adjust their behaviour based on what they think other agents are going to do, and these expectations are generated endogenously by information about what other agents have done in the past. On the basis of these expectations, the agent takes an action, which in turn becomes a precedent that

influences the behaviour of future agents. This creates a feedback loop: expectations arise from precedents and then create the actions which, in turn, constitute the precedents for the next step.

The original formulation of "El Farol Bar" problem is as follows:  $N$  people, at every step, take an individual decision among two possibilities. Number one is to stay at home; number two is to go to a bar. Since the space in the bar is limited (finite resource), the time there is enjoyable if and only if the number of the people there is less than a fixed threshold ( $aN$ , where  $a < 1$ ). Every agent has his own expectation on the number of people in the bar, and according to his forecast decides whether to go or not. The only information available to the agents is the number of people attending the bar in the recent past; this means that there is no deductively rational solution to this problem, but there can be plenty of models trying to infer the future number according to the past ones.

The other very interesting aspect of the problem is that if most agents think that the number of people going to the bar is  $> aN$  then they won't go, thus invalidating their own prevision. Computer simulations of this model shows that the attendance fluctuates around  $aN$  in a  $(aN, (1 - a)N)$  structure of people attending/not attending. The "El Farol Bar" problem has been applied to some proto-market models: at each time step agents can buy (go to the bar) or sell an asset and after each time step, the price of the asset is determined by a simple supply-demand rule.

The MG has been first described in (Challet and Zhang, 1997) as a mathematical formalization and generalization of "El Farol Bar" problem. It is assumed that an odd number of players take a decision at each step of the simulation; the agents that take the minority decision win, while the others loose. Stepping back to "El Farol Bar" problem, we can see it as a minority game with two possible actions:  $a_1 = 1$  (to go to the bar) and  $a_2 = -1$  (not to go to the bar). After each round, the cumulative action value  $A(t)$  is calculated as the sum of each value given to the single actions. The minority rule sets the comfort level at  $A(t) = 0$ , so that agent is given a payoff  $-a_i(t)g[A(t)]$  at each time step with  $g$  an odd function of  $A(t)$ .

## INTRODUCING COMMUNICATION AMONG AGENTS

The "El Farol Bar" problem, as well as the Minority game in its original formulation state that there is no communication among the agents involved in the simulation; the idea in this paper is to introduce in the model a sort of a social network, in order to see how the links among certain agents can change the results of the simulation. A social network is defined as "a set of nodes - e.g. persons, organizations - linked by a set of social relationship - e.g. friendship, transfer of funds, overlapping membership - of a specific type" (Laumann, et al., 1978).

In our case the minority rule will be very easy: a set of  $N$  agents will have to choose between (-1) and (1). Who is in the minority (denoted with  $n < N$ ) wins and gets a payoff equal to  $N/n$ : the fewer agents stay in the minority, the higher the payoff. Also the social network involved will be quite simple, just linking an agent to others with a relation limited to the possibility of asking a question: “will you choose (-1) or (1)?”. Not all the agents will be connected, though, so that some of them will have to make a prevision just considering the past few results, exactly like in the original MG. The described situation is depicted in figure 1 (in which we have twelve agents and eight links).

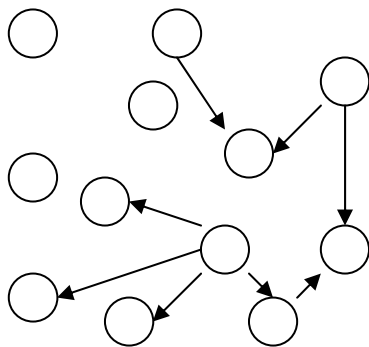


Figure 1: A Simple Social Network

In the presented model the communication among agents is bi-directional, meaning that if agent A can ask agent B, then agent B can ask agent A.

### AGENT BASED SIMULATION

In (Ostrom 1988), agent based simulation is described as a third way to represent social models, being a powerful alternative to other two symbol systems: the verbal argumentation and the mathematical one. The former, which uses natural language, is a non computable way of modelling though a highly descriptive one; in the latter, while everything can be done with equations, the complexity of differential systems rises exponentially as the complexity of behaviour grows, so that describing complex individual behaviour with equations often becomes an intractable task. Simulation has some advantages over the other two: it can easily be run on a computer, through a program or a particular tool; besides it has a highly descriptive power, since it is usually built using a high level computer language, and, with few efforts, can even represent non-linear relationships, which are tough problems for the mathematical approach. According to (Gilbert, Terna 2000):

*“The logic of developing models using computer simulation is not very different from the logic used for the more familiar statistical models. In either case, there is some phenomenon that the researchers want to understand better, that is the target, and so a model is*

*built, through a theoretically motivated process of abstraction. The model can be a set of mathematical equations, a statistical equation, such as a regression equation, or a computer program. The behaviour of the model is then observed, and compared with observations of the real world; this is used as evidence in favour of the validity of the model or its rejection”*

In Remondino (2003) we read that computer programs can be used to model either quantitative theories or qualitative ones; simulation has been successfully applied to many fields, and in particular to social sciences, where it allows to verify theories and create virtual societies. In order to simulate the described problem, multi-agent technique is used. Agent Based Modelling is the most interesting and advanced approach for simulating a complex system: in a social context, the single parts and the whole are often very hard to describe in detail. Besides, there are agent based formalisms which allow to study the emergency of social behaviour with the creation and study of models, known as artificial societies. Thanks to the ever increasing computational power, it's been possible to use such models to create software, based on intelligent agents, which aggregate behaviour is complex and difficult to predict, and can be used in open and distributed systems. The concept of Multi Agent System for social simulations is thus introduced: the single agents have a very simple structure. Only few details and actions are described for the entities: the behaviour of the whole system is a consequence of those of the single agents, but it's not necessarily the sum of them. This can bring to unpredictable results, when the simulated system is studied.

There are many toolkits and frameworks that can be used to build agent based simulations; for this work JAS was selected (<http://jaslibrary.sourceforge.net>) since it includes graph support for Social Network Analysis. In the basic model we present in this paper we only examine how many agents change their own opinion, when increasing the number of direct relations among them; further work will address some other issues, such as the correctness of the agents' choice, and so on.

### THE SIMULATION FRAMEWORK

At the beginning of the simulation, during the setup, we create a simple world populated by  $N$  agents. These agents can be considered as the vertexes of a social network and the links among them (relations) as the edges. The network is directed and every arc is composed by two edges with opposite directions. Every agent has a list of  $F$  (friends) other agents (called friendsList) to whom he can ask. This list is composed by the neighbours, i.e. the vertexes linked to the examined vertex (the agent).

Here follows a brief description of the simulation process:

- At the beginning of each simulation step, every agent has its own forecast. The forecast is absolutely random between two choices  $-1$  and  $+1$ .
- The decision taken by each agent (before communicating with others) is denoted with a “certainty index” equal to 1 (100%).
- Now an agent is randomly chosen. He starts asking to the first in the list; if this one has the same prevision, then the certainty index is increased by a value of  $1/F$ , while if the prevision is different, than the certainty index is lowered by  $1/F$
- After having asked to all the friends in his list, the agent takes the final decision: if the certainty index is equal or greater than 1, then the decision will be the other possible one, i.e. the minority one. If it's lower than 1, then the decision will be the original one.
- Another agent is then randomly chosen, and so on (the same agent can't be chosen twice during the same turn). Note that an agent that's been asked can still change his mind, basing on the agents he will in turn ask

Before starting the simulation, we can change two core parameters: the number of the agents involved and the number of the links among the agents. Here we examine three runs of the simulation, one with 1000 agents and 500 total links (an average of one link every two agents); the other one with 100 agents and 500 links (an average of five links for every agent) and the last one with 100 agents and 5000 links (fifty links for every agent). In every run we iterate the minority game for 1000 times. The model could be considered as some groups of friends that must choose between two alternatives: pub and disco. They try to select the minority one in order to avoid queues or to find some more parking lots available. They communicate the selected choice to their friends, elaborate them and then take a final decision.

In the output graph we can read the time on x-axis (1000 iterations of the game), and we plot two lines: the red one (the lower one in the graphs) depicts the decisions changed while the blue one (the upper one) is for unchanged decisions.

In y-axis we read the number of decisions (changed or not) the scale ( $10^1$ ,  $10^2$ ,  $10^3$ ) depends from agents number. We choose as standard example a world of 100 agents and 500 relations (figure 2), in which an average of 65 out 100 preserve their original decisions.

In a second run we imagine a different situation, in which the agents have many more relations among them: an average of fifty for every inhabitant (figure 3).

A simple common sense rule states that the more relations, the higher is the probability to change opinion.

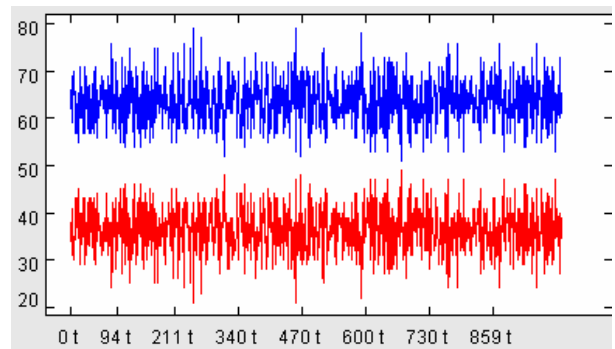


Figure 2: 100 agents and 500 relations

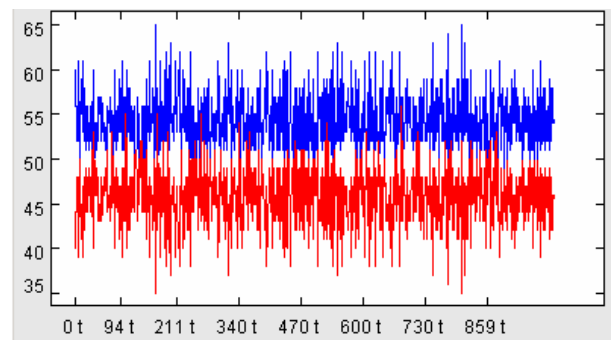


Figure 3: 100 agents and 5000 relations

This example proves the rule to be right and our model to be consistent with real world results; we can now try a counter example, i.e. a poor relations world, as the one in figure 4; one thousand inhabitants with a total of just five hundred relations.

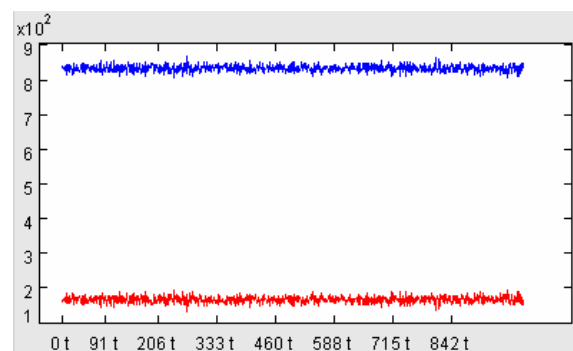


Figure 4: 1000 agents and 500 relations

Here we can observe that less than 20% of the agents changed their opinion. In order to test the extreme situation, we also imagined a world with no relations among the agents (like in the original MG).

## SYNCHRONOUS COMMUNICATION

A step further is the implementation of a different communication protocol among agents.

The first we used is an asynchronous one: the agents act sequentially. So the first agents to act take a decision, and from then on they reply to the other agents with the new decision taken. We wonder if this method can be realistic, so we decided to explore also a synchronous communication process, which seems more similar to the one we would have in a real world. Now the agents always communicate to the others their original opinion: they broadcast their opinion to all the agents which are linked to them. Finally, after they collect all the opinions of their friends, they evaluate the certainty index and reconsider their choice. We executed the simulation with the new rule and the same parameters as before.

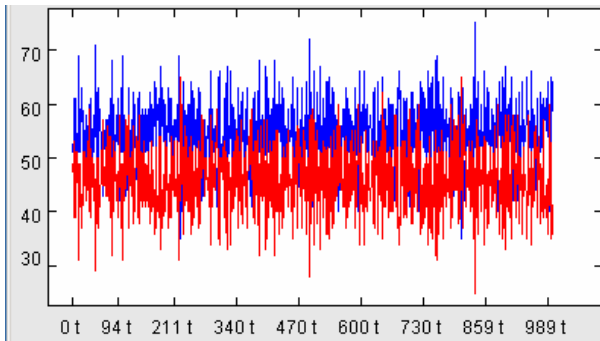


Figure 6: 100 agents and 500 relations

In the first example (figure 6) we have a ten percent more changed opinions, than we had in the sequential model.

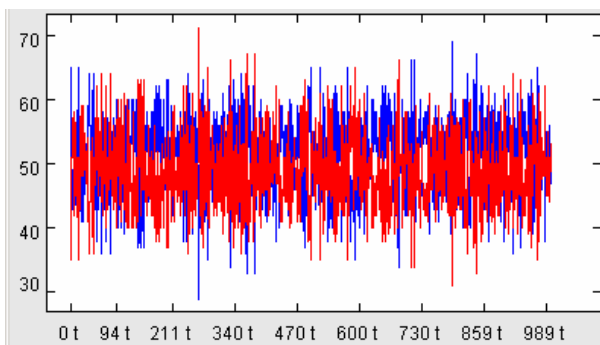


Figure 7: 100 agents and 5000 relations

The best result is in the second run (figure 7): the world rich of relations. The two lines are quite overlapped (even if there is a high variance in data). We can now express a second simple rule coming from this analysis: a synchronous communication among the agents increases their attitude to change opinion, which is at least ten percent higher.

The proof is the third run, in which again we have an higher result when compared to the asynchronous case.

## MEMORY AND REWARDING

In this section we investigate how the introduction of a simple kind of memory, based on the past turns, can change the previous results. Besides, we introduce a payoff system to reward the players in the minority. The memory is a list of length  $N$  (technically we can use the same length for all the agents or randomize it using a range from 1 to 20). In each “box” we add the last cumulate choice of the group to which the agent belongs. The value is normalized and is  $+1$ , if the sum of choices is  $>0$ , or  $-1$ , if the sum is  $\leq 0$ . The agent uses its memory by reading the list, and summing the last group choices. The agent choice will be  $+1$ , if the sum is lower than 0, that means the mode of the group is  $-1$ ;  $-1$  in the opposite situation; or can be random, if there is no prevailing result.

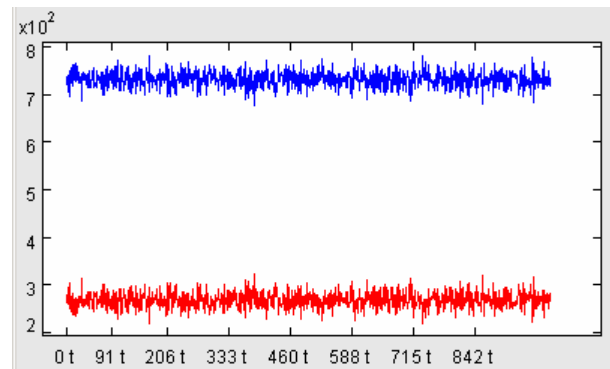


Figure 8: 1000 agents and 500 relations

We also introduce a network graph in which we can observe the topology and the agents changing their colours, red for “+1” and green for “-1”; this can be observed in figure 9.

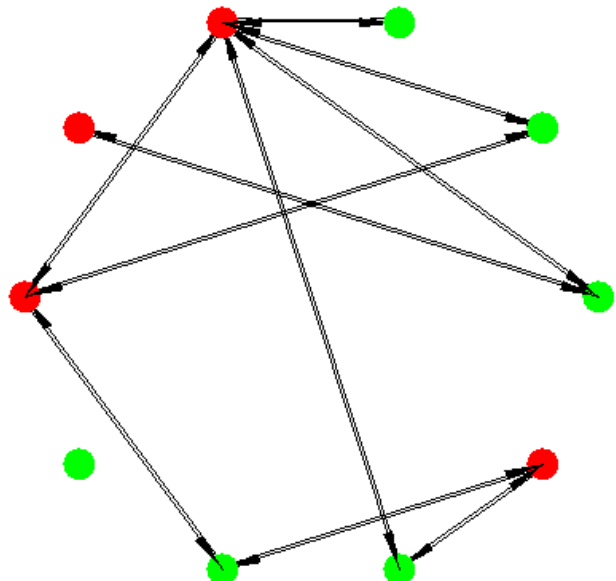


Figure 9: 10 agents and 10 relations topology

Figure 9 is an interesting experiment composed by 10 agents and 10 relations, using memory and sequential communication.

Looking at the graph we can observe that every group is in equilibrium. In fact, according to bounded rationality, each agent knows only the information about his own neighbours. Observing each agent's point of view, there are triplets Green-Red-Green or Red-Green-Red in perfect equilibrium, in which every agent respects the minority rule. The agents reach an elevated global optimum (Figure 10) of eight out ten.

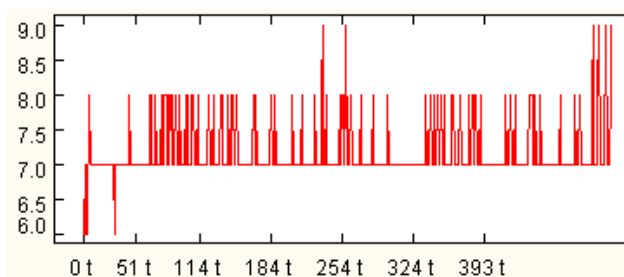


Figure 10: 10 agents and 10 relations, rewards

The stability of the system is strengthened by the steady distribution observed in figure 11. In fact, the changing opinion node is usually the isolated one.

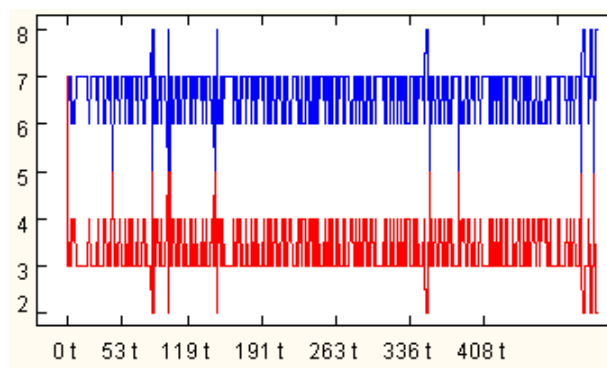


Figure 11: 10 agents and 10 relations, changed choices

The rewarding system counts one point for every agent that chooses a (local) minority option.

## CONCLUSIONS

While the original Minority Games states that the agents involved must take a decision based on the historical data, their own experience and the forecasts about what the others will choose, in this paper we introduced communication among them, in order to see how the decision process would change. The stress here is not on the decision taken, be it the best or the worst, but on how the agents can change their decision when they are linked in a social network; in particular, we tried to find the empiric proof to a common sense rule: with a fixed

number of agents, the more the links, the higher is the probability to change opinion. We built an agent based simulation, tested some real world parameters and analyzed the results we obtained.

We examined two different communication protocols among the agents: the asynchronous one and the more realistic synchronous one, in order to see how this could affect the way the agents changed their opinions. Using the synchronous communication, the one in which an agent communicates with all the ones linked with him at the same time, we saw that the attitude to change opinion is at least 10% higher than in the asynchronous case, in which the agents act sequentially.

At last, we reintroduce a sort of memory, based on the past experiences, to act as a selection mechanism. In conjunction with communication, the so composed simple cognitive system of agents creates local stable equilibria.

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